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AGARDograph No. 152

Handbook of Brittle Material Design Technology

by

W.H. Dukes

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NORTH ATLANTIC TREATY ORGANIZATION

ADVISORY GROUP FOR AEROSPACE RESEARCH AND DEVELOPMENT

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HANDBOOK OF BRITTLE MATERIAL DESIGN TECHNOLOGY

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W.H.Dukes

This AGARDograph has been sponsored by the Structures and Materials Panel of AGARD

FOREWORD

The Structures and Materials Panel of the NATO Advisory Group for Aerospace Research and Development (AGARD), is composed of engineers, scientists and technical administrators from industry, governmental establishments and universities in the NATO nations. Our concern for the advancement of aerospace technology is reflected in the Panel activities which cover more than 20 different technical subjects, one of which deals with problems and progress in the field of Brittle Materials application.

The development of re-entry and other high speed flight vehicles, in which structure is exposed to a severe thermal environment beyond the capability of most metallic materials, gave rise to our interest in brittle materials. In turning to the study of refractory inorganic, non-metallic materials, designers face a novel situation. Firstly, at temperatures of interest, these materials show no plastic deformation before failure; yet designers have implicitly relied upon material ductility to accommodate local deviations from the average conditions assumed in conventional design practice. Secondly, brittle materials exhibit a scatter in properties far greater than that normally encountered in conventional metals.

A need was identified to provide designers, meeting these materials for the first time, with the best statement possible of design practices and of the quantitative data to apply in these practices. The achievement of this objective required not the view of one man, no matter how eminent, nor the collected disparate views of several such men. The Panel sought to achieve its ends by the collection of opinions and data from a wide field and their distillation into a Handbook which could reasonably be held to represent a consensus of opinion amongst a group of practitioners in the field.

A co-ordinator, appointed in 1965, first outlined the Handbook and its requirements. He made contact with other designers with experience of design in brittle materials. Reporting to the Panel's Working Group he agreed with them a series of questions and problems requiring resolution and a small, carefully programmed symposium was held in 1967 to obtain answers to the problems he had framed. By this process and subsequent extensive consultation between the co-ordinator and other NATO experts, a Handbook has been constructed which represents the best statement of practices and data which can be made available at the present time.

December 1970

Anthony J.Barrett
Chairman,
Structures and Materials Panel

ACKNOWLEDGMENTS

The author wishes to thank A. Krivetsky, J. Witsel and D. Dupree for their assitance in making computations and preparing figures, B. Murawski for typing the text and Dr. Gellatly, S. Jordan and F. Anthony for technical advise.

The material in this publication has been repr... ed
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Published February 1971

666.76

Printed by Technical Editing and Reproduction Ltd
Harford House, 7-9 Charlotte St. London. W1P 1HD

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I. INTRODUCTION

1.1 INTEREST IN BRITTLE MATERIALS

Interest in the structural use of brittle nonmetallic refractory materials in aerospace vehicles arises as a result of interest in re-entry vehicles and the continuing need for propulsion systems of increased performance. In both of these situations performance is dependent on the temperature capability of the structural materials. The presence of temperatures beyond the capability of most metallic materials has encouraged designers to study "ceramics" but, with the possible exception of inserts in the throats of some solid rocket motors, significant applications have not developed. Characteristically these materials show no plastic deformation before failure at temperatures of interest, and they have little toughness to arrest crack growth. These characteristics have resulted in a lack of confidence by designers, in such materials, and an unwillingness to use them.

Conventional methods of structural design involve approximations which limit typical engineering analyses to the study of average conditions. Local departures from these average conditions, including the effect of inherent and fabrication induced material defects, are accommodated by material ductility. When completely brittle materials are used these design practices are insufficient to produce a reliable structure. In the past this problem has been avoided by using brittle materials at very low stress levels and restricting their use to applications where the resulting weight penalty was of no concern. For airborne and space vehicle applications, however, this weight penalty is not acceptable and, hence, there is now interest in developing design techniques for brittle materials which will produce structures with a weight efficiency and a reliability comparable with metallic structures.

This brittle material technology is new and will be unfamiliar to most designers who have occasion to use nonmetallic refractory materials. Furthermore, in the numerous areas where the technology departs from conventional design practice, the designer is in need of quantitative design data. This handbook is intended to satisfy these needs by providing both a comprehensive description of brittle material design technology, in terms of differences from conventional design, and also an assembly of the best and most generally accepted and available practical data. This latter is presented in the form of the typical "Structures Manual", involving, whenever possible, step-by-step procedures with graphs and charts to facilitate numerical evaluations.

The materials to which this handbook is intended to apply include oxides, carbides, borides and similar compounds. Graphite, in its many forms, is also included. The important characteristic of these materials is refractoriness, which permits them to be used in applications where the more structurally efficient metals are useless. Such materials have been used extensively in the past for high temperature applications, such as furnace linings, but these have involved ground installations where weight was generally not important. Currently the interest involves the high temperature applications generated by re-entering space vehicles and rocket engine compounds, but since these are extremely weight critical applications, substantial improvements in structural efficiency and reliability over those typical of the furnace type application must be obtained.

Among the applications which are of current interest for this class of material are numerous components for winged re-entry vehicles or hypersonic atmospheric vehicles, and these include leading edge elements, nose caps, surface panels, which may or may not include insulative elements, control surface structural parts, and engine intake structural parts. Rocket engine nozzles and chambers are the obvious propulsion applications.

These applications may appear limited and insufficient to justify the development of a whole new technology. Experience shows, however, that when a new technology is developed its applications become much more extensive than was originally anticipated. There are already indications, in fact, of application of these techniques to nonaircraft commercial uses of the materials, where the interest is not in less weight but in more efficient design, leading to lower initial cost, less frequent replacement, etc.

1.2 THE DESIGN PROCESS AND THE EFFECT OF BRITTLE MATERIALS

The process of structural design involves two distinct parts which generally are used consecutively and repeatedly in an iterative process as the design is refined and finalized. The first of these two parts involves the selection of material and type of construction and the establishment of the structural dimensions to support the applied loads. To make these selections use is made of parametric studies, approximate stress analysis for the more obviously critical design conditions, and optimization techniques, and consideration is given to methods of fabrication, cost, and the program schedule.

The second aspect of design involves the process of verifying, by analysis and test, that the structure has the strength and stiffness to maintain its integrity under all of the required loadings, temperatures, and other environmental conditions. Often this analysis requires minor adjustments in structural dimensions as local deficiencies in the initial design are revealed.

The process of structural analysis involves a number of distinct steps, including the determination of critical loads, pressures, temperatures, etc., the determination of stresses due to static, repeated and vibratory applied loads and temperature gradients, determination of the mechanical properties of the material under the appropriate environmental conditions and after exposure to an appropriate stress history, and the comparison

of the mechanical properties with the applied stresses to establish structural integrity. In many cases the permissible stresses are controlled by instability of the structure, requiring predictions of buckling. Determinations of structural stiffness or displacements under loading conditions where dynamic effects are potentially critical may also be required.

The introduction of brittle materials into a structure as primary load carrying elements requires no change in the design process as described above. Changes are required, however, in the degree of design refinement which must be used in conducting some of these steps. Conventional design with ductile metallic materials involves a number of approximations which are made in the interests of analytical and experimental simplicity, and which generally involve the use of average or mean values of applied stresses, applied loads, etc. Such methods have, indeed, been very successful because structures built with ductile materials are not sensitive to local effects and are, consequently, forgiving of approximations made by the designer. Examples where these methods begin to break down are well known, they generally involve complex stress situations where the effective ductility of the material is reduced, often in combination with high strength, low ductility materials and often under repetition of the critical loading condition so that the locally yielding material is rapidly fatigued. With metals even these situations, however, can be handled successfully by small corrections to and modest modifications of, the conventional design methods.

With brittle materials these simplifications can no longer be tolerated and additional refinement and realism must be introduced into the standard design procedures. The areas where these refinements are most important include stress analysis, where a much more precise understanding of the stress distribution is necessary; mechanical property testing, where the peak stresses must be measured, or avoided by appropriate test specimen and apparatus design; specification of material mechanical properties, where the variability in these properties from one sample of material to the next must be considered; and the establishment of design criteria, which should be based on probability of load occurrence rather than specified singular conditions with arbitrary safety factors. In every case these changes represent a refinement of technique so that the present practice with ductile materials becomes a particular case of a more general technology.

Brittle material design technology involves, however, more than refinement of analytical and experimental method. These are the tools necessary to assure a "safe" structure; to achieve such a structure with an efficiency suitable for airborne and aerospace applications also requires changes in design concept in certain areas, particularly at joints and connections. Again, such changes represent refinements, which are mandatory for brittle materials, but which could probably be used to advantage, to the extent which increased weight and manufacturing cost justify, with ductile materials.

1.3 HANDBOOK OBJECTIVES, SCOPE AND FORMAT

In view of the growing interest in the use of brittle nonmetallic refractory materials for structural purposes and in view of the changes required in design technology for their successful use, this handbook has been prepared to summarize, for the designer, the best current practice. It is further intended that the material be presented in a form most useful for the direct application by the designer in his work. This handbook is not intended, however, to assist the engineer concerned with material development or material processing, or component fabrication, except to the extent that such an individual can benefit from a knowledge of the designer's problems. For this reason any discussion of specific materials, their mechanical and physical properties or the effect of variations in composition or structure or the effect of various processing parameters, is avoided.

This handbook is also confined to those areas of design where differences with conventional design technology are necessary. No attempt is made to present any of the standard stress analysis methods which can be found in text books, and reports and papers, and for which convenient design charts are available at any organization which practices structural design.

The handbook presents, first, a general discussion of those areas of design technology where different techniques or different concepts are required for applications involving brittle materials, and then each of those areas is treated in detail, in individual sections. Within these sections each topic is discussed to present concepts, assumptions, theories, etc. and also the recommended approach. Emphasis is placed on those techniques recommended for design purposes and a variety of ideas or approaches is discussed only when a unanimous opinion on the best technique is not available. Associated past developments are not discussed unless necessary for an understanding of current practice. The review of the most generally accepted current practices includes indication of their shortcomings and limitations, together with as much quantitative data as can be found. Where possible this has been presented in the form of charts, tables and curves to facilitate its use in routine design exercises.

Throughout this volume it has been recognized that the designer is usually faced with the need to produce hardware, and that the demand cannot generally be deferred because analytical technique is not as advanced as it could be. Therefore, wherever useful information has been found on a particular branch of the technology it has been included, regardless of limitations or of differences of opinion within the technical community on its validity. Such limitations or differences of opinion are, however, clearly indicated, so that the designer can establish test programs for design verification where doubt exists.

Certain aspects of brittle material design require digital computer techniques to achieve sufficient analytical refinement. Complete details of the computer programs required are not included, however, since these will generally be built up, by each organization, over a long period of time, to suit the facilities and requirements of that organization.

Assembly of the handbook has been chiefly the work of one individual, based on information available in the literature or on research work in progress. No attempt has been made to advance the art by the addition of original work not otherwise published except on the subject of design criteria, where no data exists in the literature. However, material has generally been rearranged to present it in a more convenient form for the designer. Again, recognizing the uncertainty and lack of practice in many areas, and the need for supplementary opinion and judgment, much of the material has been reviewed by recognized authorities in the individual subjects. To further pursue this process a symposium was held during 1967, under the sponsorship of the Structures and Materials Panel of AGARD for the purpose of reviewing the more basic unresolved questions associated with the technology. Many of the foremost research investigators and design practitioners in the various subjects were present and the symposium was used to obtain agreement on the best available answers to these questions. The methods and data presented in the sections of the handbook which are affected by the results of this symposium are consistent with these agreements.

1.4 DEFINITION OF TERMS

The term brittle materials is used in this handbook to describe materials which show no plastic deformation under stress but which, instead, deform only elastically until failure is reached. Furthermore, since the elastic modulus of the materials of interest is high, the resulting total deformation at failure is small. This is in contrast to most ductile metals of structural usefulness where the plastic deformation before fracture may be from ten times the elastic deformation in the case of high strength, low modulus, materials such as titanium alloy to 500 times for a low strength carbon steel. To further define terminology, distinction must be drawn between brittleness as defined above, and involving zero inelastic deformation, and the conventional use of the term to describe materials such as the ultra high strength steels, steels operating below the ductile to brittle transition temperature, beryllium, tungsten, etc. Each of these materials shows some ductility which, though small compared with more conventional metals, is nevertheless sufficient to make the methods of this handbook unnecessary.

2. PRINCIPLES OF BRITTLE MATERIAL DESIGN

2.1 GENERAL

Experience shows that structural design techniques which have been developed and used successfully for structures fabricated from ductile metallic materials cannot be used without modification with completely brittle materials, if the same degree of structural reliability is required. In the previous section the reasons why conventional design techniques are not satisfactory with brittle materials were reviewed in general terms. In this section the principles and practices which must be followed to achieve reliability with structures constructed with brittle materials are described in detail. However, it should be recognized that experience with structures in which brittle materials support significant tension stresses without the benefit of metallic reinforcement is presently very limited, so that many of the concepts to be presented in this section are based on theoretical considerations. Where these practices have been followed the results have been satisfactory, but whether all of these practices are necessary or whether they are sufficient for all materials and applications can only be determined by experience.

All of the considerations involved in brittle material design result from the condition that the material shows no yielding prior to failure, that is, that the stress-strain curve is a straight line of constant slope to the stress level where failure occurs. The consequences of this assumption are illustrated and summarized in Figure 2.1, which shows a series of dependences and the design consequences that follow from each. The initial assumption of no yielding leads to failure at points of maximum stress, regardless of the fact that these high stresses might be localized. As a consequence additional requirements are imposed on the stress analysis methods since they must define these localized maximum stresses. New concepts must be introduced into the design of joints and connections and important changes are required in the methods for experimentally determining the mechanical properties of the materials.

One of the sources of very localized high stresses may be flaws within the material, and thus the apparent strength of the material becomes dependent on the size, type and frequency of such flaws. This requires an improved understanding of material fracture mechanisms and the establishment of appropriate material fracture theories, particularly under complex stress conditions.

Since the material strength is affected by the presence of flaws, which are in themselves random phenomena, a variability in material strength among supposedly identical specimens can be expected. Such a variability is present in all materials, but the inability to relieve high local stresses by local yielding makes the variability in brittle materials sufficiently great that it must be considered in the design. It thus becomes necessary to use a statistical rather than a deterministic definition of material strength. This variability also requires attention, by the designer, to the material processing and quality control to minimize such variability, and hence maximize the strength that can be expected with a specified level of reliability. Variability also introduces the consideration of other methods of achieving high working stresses in conjunction with high reliability, and these methods will generally affect the design processes.

With such fundamental changes in the material failure characteristics, the established methods of specifying structural design criteria also require revision. For example, the conventional factor of safety loses meaning if a single value cannot be specified for the material strength. Finally, the conventional optimization techniques for defining minimum weight or minimum cost structures also require revision to introduce reliability considerations.

Figure 2.1 shows ten areas where changes in design practice are necessary when brittle materials are used. The remainder of this section discusses each of these areas in detail while the remainder of this manual provides charts and other information to facilitate the application of these practices.

2.2 REFINED STRESS ANALYSIS

Conventional stress analysis involves a number of simplifying assumptions, including the assumption of no sudden change of cross section, freedom from end effects, idealized boundary conditions, etc. Such methods neglect changes in the stress distribution which occur at sudden changes in structural cross section or at the locations where external loads are applied. Changes in cross section may arise for a number of reasons; gradual or abrupt changes in the basic cross section in an attempt to minimize weight by matching the component strength to the internal forces and moments, abrupt changes in cross section at the ends of a component; local changes due to the presence of stiffening members; holes, splices and joints; changes due to the presence of material provided for attachment and support of the component or for attachment of adjacent structural components. The application of concentrated loads, such as occurs at attachments between structural components, and the presence of distributed external loads which vary appreciably across a component, are also a source of changes in the stress distribution.

All of these effects will generally produce local strains which may be greater, at some point, than those predicted by simple "strength" theory. Generally, with a ductile metallic material, such localized strains can be absorbed by yielding and redistribution of stress and, since the average strength across the section will be sufficient to support the applied loads, no failure results. Sometimes, under repeated loads, fatigue cracks may develop at

these points where yielding occurs. Where it is necessary, as a consequence, to examine such effects analytically it can still be done sufficiently accurately by applying correction factors to the simple stress distribution. These correction factors have been computed, using the theory of elasticity, for a number of simple cases and the results are sufficiently accurate for application to metallic structures.

If the material is completely brittle, so that stress is a linear function of strain to fracture, it can be expected that if these local effects include the regions of maximum strain they will be the source of failure. As a consequence, they cannot be neglected, however localized they may be. These considerations are particularly important if the stresses are produced by temperature gradients. Typically the thermal expansion of a brittle nonmetallic refractory material at the temperatures where such materials will frequently be used is greater, in the absence of yielding, than the strains required to produce failure. Thus the success of a structure of brittle materials, in a high temperature environment, depends on the control of temperature gradients, i.e. the differences of temperature throughout the component, and on the external restraints against thermal deformation. Hence an accurate thermal and stress analysis is necessary for structural reliability. In a typical metallic structure, on the other hand, temperature gradients will rarely produce static failure, since they represent a self-balancing internal load system which is immediately relieved by local yielding at points of excessive thermal strain. Thermal gradients may thus produce buckling of unstable sheet metal parts, or fatigue cracking from repeated yielding if the environmental conditions are repeatedly applied, but they will rarely produce short time fracture.

From considerations such as these, a most important principle of successful design with brittle materials is believed to be the use of stress analysis methods which avoid the limitations of both the "simple" strength theory and the classical theory of elasticity; the former limitation involves extremely simple component geometry and loading conditions and the latter, while giving precise results, is limited by mathematical complexity to a few specialized loading conditions and geometric shapes. Fortunately an analytical tool which meets these requirements is available. It involves division of the structure into a very large number of elements of sufficiently simple shape that relationships between loads, stresses, and deformations of each element can be easily written, and of sufficiently small size that all complexities of the component geometry and loading can be accurately represented. Equations expressing equilibrium and displacement compatibility at the junctions of the elements are then written in matrix form and solved for the internal displacements and stresses. Since a large number of elements are required for accurate representation of a typical structure a computer is generally required for solution of the matrix. Similarly it is necessary to computerize the stress-deformation relationships of the various types of elements that are used in the analysis so that the matrix can also be assembled by the computer. This technique is explained in some detail in Section 4, although a specific procedure for establishing the computer programs is not given. Generally, such a capability requires an appreciable amount of time and effort for its establishment and it must be related to the specific needs, capabilities and equipment of each organization.

For the benefit of those organizations who are active in the use and development of brittle material technology, but who do not have access to an appropriate computer program, Section 4 also includes curves of correction factors to be applied to stresses calculated by the simple bending theory, and to account for some of the local effects mentioned previously. These curves are similar to those which can be found in any book on elastic theory and which give stress concentrations due to holes, fillets, etc. However, by the application of the matrix methods of stress analysis described above, it has been possible to present correction factors for a wider range of concentration effects than is possible using the classic theory of elasticity.

2.3 ADDITIONAL DESIGN RULES AT JOINTS

The previous considerations, involving refined stress analysis, clearly apply at splices and joints between elements of a brittle material structure, and at the points where brittle components are attached to metallic structure, since these areas will normally involve severe changes in cross section, the presence of holes, fillets, etc. and points of application of concentrated loads. There are in addition, however, a number of other principles which must be followed in order to design a successful joint with brittle materials.

Typically, in metallic construction, joints and connections contain multiple fasteners, and a relatively accurate assessment of the ultimate strength of such a joint can be made by assuming loads divided between the fasteners in proportion to the cross sectional area of each fastener, and to its position with respect to the center of rotation of the connection. With brittle materials, and the absence of yielding, the load path through a multiple connection will be dependent on the deformations in the various structural parts of the connection, and on the respective fits between the bolts and the holes. Because of tolerances on bolt and hole diameters, and on the positions of the holes in the adjacent connected elements, the loads or moments will be transferred through those bolts which make contact first with the material surrounding the bolt hole. Consequently the load distribution cannot be calculated. The distribution may also be modified by the deformations of the joint elements, but these deformations are difficult to calculate, even with the methods discussed in Section 2.2, because of the complexity of the geometry and the stress distribution in a typical joint, and because of the presence of deformations due to bearing in the bolt holes and shear in the bolt.

Generally the load is likely to pass through one or two bolts that contact first, and these will probably fail before there is sufficient deformation for the other bolts to pick up a proper share of the load. Thus multiple connections are likely to be ineffective, in a joint, and will represent useless weight.

This discussion is not intended to imply that the use of redundancy in a properly designed connection is undesirable. Indeed, in a later section the use of redundant load paths to increase structural reliability without reduction in allowable stresses is discussed and recommended. The significant point is, however, that the redundancy must be of the type which permits the loads through each load path to be accurately determined. If a connection is reduced to a single pin, to avoid the above problems, the parts may deform relatively, and then the same principles must be applied. A single pin will generally provide rotational freedom only about one axis. Unknown moments may be introduced about the other two axes and critical stresses, which would be relieved by yielding in a metallic component, may be introduced. Section 7 discusses the design of joints in more detail and presents examples of joints which avoid the problems mentioned above, and also charts to facilitate their design.

One additional principle is involved when connections are made between brittle components and supporting metallic structures. It will generally be found that a brittle component operating at high temperatures and subjected to temperature gradients will need to be supported in a manner permitting free thermal deformation. For most applications complete restraint against overall thermal deformation must be avoided, otherwise the thermal strains at the temperatures where brittle refractory materials will normally be used are greater than the strain required to produce fracture. Furthermore, it will generally be necessary to provide not only for thermal growth but for changes in curvature due to temperature gradients. Thus, again, the connections to the supporting metallic structure must provide both translational and rotational freedom in all directions. A similar requirement exists if the supporting metal component is subjected to loads which produce significant deformation and change in curvature. Such deformations must be prevented from inducing unknown forces in the brittle components by using connections which provide complete translational and rotational freedom. Again, Section 7 discusses how these conditions may be achieved in practice.

Instances have been found where partial restraint against deformation due to temperature gradients can be beneficial. In materials which have much greater compression strength than tensile strength, which is true of essentially all full density "brittle" materials, restraints which increase compressive thermal stresses, but thereby reduce tensile thermal stresses, could improve reliability. Whether it is practical to control restraints to such a degree is not known.

Another problem which requires special consideration concerns thermal expansion differences at the point of connection between a metallic and a nonmetallic component. Depending on the materials used and the difference in thermal expansion characteristics, conventional bolted connections may become loose or alternatively sufficiently tight to fracture the brittle material component as the temperatures are raised. Techniques to avoid this problem are available and are discussed, together with appropriate design charts, in Section 7.

2.4 IMPROVED MECHANICAL PROPERTY TEST METHODS

The basic mechanical property data used for structural design is the uniaxial tensile and compressive stress-strain curve, which is normally obtained from axially loaded specimens. With brittle materials problems result in attempting to conduct tensile and compressive strength test because of the inability of the test materials to absorb local stress concentrations. To obtain the true strength of the material, therefore, very uniform stress distributions must be developed across the test sections. Specimens and apparatus must permit extremely accurate load alignment and maintenance of alignment during testing, specimens must be fabricated to close tolerances, and specimen shapes must be developed which will accommodate the stress concentrations that normally arise at load application points. Experience shows that these requirements are difficult to meet, but the testing problem is further complicated by the high cost of fabricating many of the refractory nonmetallics due to extreme hardness. Thus the requirement for extremely accurate close tolerance specimens, together with the need for statistical strength data requiring a large number of such specimens, can result in extremely expensive programs.

Many types of tensile test have been tried to avoid the problems described above, generally involving different types of load application grips, with arrangements for alignment and adjustment, and soft spacing materials to accommodate local surface imperfections. Strain gages on the test section, in combination with adjustable grips, have also been used to obtain accurate load alignment. None of these methods has proven particularly satisfactory except perhaps the use of strain gages. This technique again is too expensive to consider for large numbers of specimens. In some programs the bending strains on tensile test specimens resulting from load misalignment have been measured and the values are surprisingly high. Values between 20 and 35% of the tensile strain are apparently common when special precautions to obtain accurate alignment are taken, and values of 50% and above are easily obtained in the absence of such precautions. Probably the magnitude of this difficulty is related to the size of the test specimen. These are generally small in the interests of conserving material, and the small test section increases the difficulty since many of the tolerances in the test equipment are of a fixed magnitude and independent of specimen size.

Tests with photoelastic specimens to check misalignment, eccentricity, and stress concentration effects are commonly conducted but the evidence is that such tests are not sufficiently sensitive to determine the suitability of test methods for brittle ceramic materials.

Numerous other types of tensile specimen have been examined, including ring specimens, and the "Theta" specimen, but where these are at all satisfactory they are extremely expensive to manufacture. Compression testing also has problems; first, the requirements for perfectly flat and parallel loading surfaces to prevent the concentration of load on some local unyielding high spot and, secondly, the elimination of friction at the load application surface. Friction prevents lateral strain due to the Poisson's ratio effect, and produces highly complex stress distributions which lead to premature fracture.

Because of the difficulties of producing a satisfactory tensile specimen the most common test is the bend test, but these, too, have problems. Such specimens are generally made with increased cross section at the point of load application so that the resulting stress concentrations do not initiate fracture. However, friction effects under the load application points have been shown to produce errors of over 25% in the measured stresses. Various devices such as loading jigs with ball bearings at the load application points have been developed to minimize friction, but the literature contains no comments on their performance.

The problem of accurate alignment of load application devices also exists with bend specimens and most of the methods which have been tried introduce twist, in addition to bending, across the test section.

All of the above problems, of course, are additionally complicated by the fact that extremely high temperature testing will generally be required with brittle materials, since such materials are most commonly used for high temperature applications.

From the above discussion it will be evident that conventional testing methods cannot be used with brittle materials and that special techniques must generally be developed. Furthermore, even with special techniques, the effects of brittleness show up in many unexpected ways. This subject is, therefore, discussed in more detail in Section 8 where the best of current practice for mechanical property testing of brittle materials is defined in a form suitable for application by the structural designer.

2.5 FAILURE MECHANISMS

At present any discussion of material failure can take one of two forms; we can discuss fracture mechanics, which attempts to explain in physical terms the details of the initiation and progression of failure in the material, or we can discuss failure theories, which define material strength under complex stress conditions, but which are based on gross considerations. The study of failure mechanisms involves a description of the failure in terms of atomic rupture or slip, the effect of dislocations in the atomic structure, microcrack formulation and growth, the effect of grain boundaries, precipitates, etc. on crack propagation, etc. This subject is generally limited to the consideration of simple uniaxial tension. It does not, at present, provide the designer with quantitative information, but it is nevertheless desirable that he understand material fracture mechanics in a general manner, so that the failure theories that he does use can reflect the proper parameters. This subject is therefore covered in the present section.

Failure theories, on the other hand, provide a quantitative statement of the stress conditions which will result in material failure, whether it be yielding or fracture. Such theories are based on overall consideration which are assumed to control material behavior, such as the assumption that fracture will occur when the maximum tensile stress reaches a limiting value or that yielding will occur when a limiting value of deformation energy is reached. These failure theories require empirical verification but they do provide quantitative data which can be used in design. This subject is therefore discussed in the next section.

The study of the fracture mechanics of metals is complex because many types of fracture are involved. It is commonly assumed that metals are ductile, involving slip and low in the atomic structure, but this picture is modified by the complexity of many metallic materials, and by the fact that many such materials show a sensitivity to temperature, strain rate and stress-state which can produce brittle, or combinations of brittle and ductile fracture. This sensitivity, in turn, may be critically dependent on such factors as heat treatment, work hardening, residual stresses, etc. so that it is unsafe to make general statements about the failure mechanics of a particular material in a particular application, in the absence of experimental data. On the other hand it is not practical to take the conservative view and assume that all metals are brittle, since this would greatly limit the strength levels which can be quite safely developed in very many cases.

The situation with brittle materials such as oxides, carbides, borides is believed to be more simple since, although such materials may show ductility at temperatures near the melting point, brittle failure will almost always be the critical strength consideration. The study of the fracture mechanics of these materials is nevertheless extremely limited at present.

The best description that can presently be assembled from the literature in terms that will be useful to a designer is as follows. The material is made up of grains within which the atoms are arranged in a particular order. At some points this order is disturbed by an

excess or deficiency of atoms producing local internal stresses in the material structure. Upon the application of external stress these so-called "dislocations", which are the weak points in the material structure, will move until they meet obstructions. At these obstructions they collect eventually in sufficient numbers to form microcracks. Various mechanisms for obstructing the movement of dislocations, and thereby initiating crack nucleation, have been postulated including grain boundaries, the intersection of two slip systems, etc. The stresses which cause the dislocations to move and collect may be due to external loads, or temperature gradients produced by external thermal environments, but they may also be produced by material processing or by the presence of different phases or impurities within the material, with associated differences in thermal expansion. Accordingly, microcracks may exist in the material before the component is put into service. Furthermore, the microcracks can be expected to form at points of stress concentration within the material such as are caused by pores, voids between adjacent material grains or other flaws.

With cracks present, large stress concentrations will be formed at the tips. The stress distribution around such a region may be computed by using elastic theory and from such computations it is known that the stresses may be as large as the theoretical atomic bond strength. When this situation occurs the atomic or molecular bonds at the crack tip will be ruptured and the crack therefore grows. As the crack is lengthened elastic strain energy in the surrounding material is released, but lengthening of the crack requires the addition of surface energy. A spontaneous increase in the length of crack will occur when the decrease of strain energy is greater than the corresponding increase of surface energy. Initially this propagation may be periodically disturbed by obstructions within the material and may in fact be stopped by voids, but generally the stress concentration increases as the crack lengthens, the energy balance can no longer be maintained, and the crack propagates rapidly through the material, leading to complete failure. The rate of crack propagation will reach a limiting value for a particular material and, when this velocity is reached, any extra strain energy released must find other dissipation mechanisms. In a completely brittle material some of this excess energy is transformed into extra surface energy by producing multiple branching of the running crack.

At present, within the literature, there is no consideration of stress condition, that is whether the stress is tensile or compressive or uniaxial or multiaxial, and particularly there is little understanding of the significance of the type of material. As the application of these materials increases, however, studies of failure mechanics can be expected to increase in the effort to develop materials of improved mechanical properties. The application of such technology to the prediction of quantitative mechanical property data cannot, however, be foreseen.

2.6 ESTABLISHMENT OF MATERIAL FAILURE MODES

For a simple uniaxial tension stress in a ductile material the establishment of allowable stresses, whether for initiation of yielding, or fracture, is a simple matter of pulling tensile specimens. It is not essential that the designer know the mode of material failure, in such terms as atomic slip or cleavage, dislocation pile up, crack propagation, the effect of grain boundaries, etc., in order to use this data for design purposes, although such knowledge is useful in assessing the significance of stress concentrations, fatigue, etc. When a more complex biaxial or triaxial stress situation exists, the material strain in a specified direction is no longer related to the stress in that direction in the manner indicated by the uniaxial stress-strain curve. Consequently it is no longer obvious at what stress level the material will show yielding or at what point it will fracture. One can ask, for instance, whether these conditions are controlled by a limiting stress, or a strain, or an energy limitation, etc.

In order to have some basis for determining allowable stresses under conditions of combined stress, which are often encountered in design, various strength theories have been advanced for metallic materials. The purpose of these theories is to establish laws by which, from the behavior of a material in simple tension or compression tests, the condition of failure, either yielding or rupture, can be predicted under any kind of combined stress. Such theories are strongly empirical, with a basis in a very general understanding of material failure, but the actual failure mechanics, on an atomic or microscopic basis, are not involved.

Among the theories which have been proposed for yielding of ductile materials are the maximum stress theory, the maximum strain theory, the maximum shear theory, the maximum strain energy theory and the distortion energy theory, of which the latter best fits experimental data.

For brittle materials a different approach is required, since the lack of shear slip and yielding make the shear and energy theories inappropriate. Consequently the maximum stress theory is most commonly used. This theory postulates that fracture will occur when the maximum tensile stress in the body reaches a limiting value. This theory predicts fracture to be independent of the other two principal stresses, which is contrary to observation, and it neglects both compressive stresses and the compression strength of the material. There is also the so-called stress invariant theory which does realistically predict compression strength values six times the tension strength, although there is no evidence in the literature of its application.

The Griffith crack theory and the Weibull theory have also been extended to complex stress conditions so that they become failure theories relating strength values. The Griffith theory predicts failure under biaxial tension when the maximum normal stress acts upon a flaw of critical size. It is thus equivalent to the maximum stress theory in the tension-

tension quadrant, but it also predicts a uniaxial compressive strength equal to eight times the uniaxial tensile strength, so that it also defines the compression quadrant of the failure envelope. In the compression-tension quadrant, however, the theory does not predict an increase in allowable tensile stress due to a normal compressive stress, though limited test data suggests that this may be the case.

The Weibull theory predicts values less than the maximum stress theory in the tension-tension quadrant, and in the compression-tension quadrant it predicts that the presence of compression will increase the allowable tension in the normal direction.

The Griffith theory suffers from the limitation that the assumed flaws are sharp whereas many brittle materials show flaws which are not sharp. Recent work by Sines has considered flaws of various shapes and the result is a predicted uniaxial compressive stress that varies with the shape of the critical flaw. In this respect the theory is a better fit with experimental data, although in the tension-tension quadrant it predicts an increase in strength due to a biaxial stress state, which seems very unlikely.

Very little quantitative experimental data is available to verify which failure theory should be used for brittle materials. In much of the experimental work, variability in mechanical properties between supposedly identical samples has not been considered, and this effect alone may be greater than the difference between the predictions of the various theories. Furthermore, very few materials have been examined under complex stress states, so that it is not known whether different failure theories are required for different types of brittle materials. Certainly it must be anticipated that the material manufacturing process will have a significant effect, since it controls such factors as porosity, grain size, and the absence, presence and degree of microcracks. Nothing is yet known, for example, about the effects of anisotropy under complex stress conditions. Such data as is available is presented, in conjunction with the predictions of the various theories mentioned above, in Section 5, and this data may be used for preliminary design purposes. In view of the severe limitations mentioned above, however, it is recommended that material tests be conducted for any significant application, until a substantial body of data is built up.

For metallic structures the use of a failure theory, as described above, has been found inadequate in many important instances. As already explained, these theories are established generally on the basis of a ductile material, while experience shows that many metals fail in a brittle manner under certain conditions of temperature, loading rate, etc. Under such conditions flaws in the metal, which act as local stress raisers, become critical and can result in failures at average stresses much less than would otherwise be expected. This problem is rather like the fatigue problem in metallic structures; it results from simplified analytical treatments which consider only average stresses and which depend upon ductility and local yielding to take care of local concentrations. Under conditions where the metal will not yield, trouble results and special treatment is called for.

It is assumed that no parallel to this situation will exist in brittle material design, since yielding is never assumed. Local stress concentrations due to component geometry and loading configuration are accounted for in the stress analysis (see Section 4), while cracks and other material flaws are accounted for in the determination of material properties.

As will be discussed in some detail in the next section, the mechanical strength of otherwise identical samples of a brittle material shows considerable scatter, since failure is precipitated by the stress concentrations present at flaws, and the flaws are random in size, direction and distribution. Thus any mechanical property test conducted on a sample of material with the expectation of defining a point on the failure envelope, does not specify a single-valued material property, but rather is associated with a probability of occurrence. Each point on the failure envelope should therefore be determined by conducting a series of identical tests, defining a strength distribution curve under the particular stress ratios and other environmental conditions of interest, and then selecting a strength value associated with a certain probability of failure. Thus the failure envelope becomes, not a single envelope as in the case of ductile materials, but a series of envelopes each associated with a certain probability of failure. This in turn requires the combination of a failure theory and a statistical theory. This again is a subject which currently has received no attention in the literature.

Again, because of sensitivity to stress concentration, the strength of a particular material, such as aluminum oxide, is not a single value or even a single distribution curve, if strength variability is accounted for. The strength, and the mode of failure will also be dependent on the material structure, such as porosity, grain size, the number and character of the various secondary phases, etc. Empirical relationship that account for some of these effects are given in Section 5, but whichever failure theory is used, specific values must be determined for the particular material and the particular material characteristics.

Also obscure, from the literature, is the significance of shear as a potential failure mode. The subject receives no discussion and yet compression failure is accepted and certainly can be demonstrated experimentally. Assuming a compression test method which does not introduce extraneous stresses, a questionable assumption in many cases, it should be anticipated that a compression failure, particularly in a very dense material, might be produced by a shear failure within the material.

With respect to failure modes under repeated loads, again the literature contains very little data. There is very limited evidence of flaws growing with repeated stresses so that fatigue as a failure mode must be anticipated. In Section 6 some suggestions are given with respect to the extension of the fracture mechanics concept to the prediction of fatigue in brittle materials, and the strength variability considerations are appropriately introduced. However, there is at this time no experimental verification of this approach.

2.7 STATISTICAL DEFINITION OF FRACTURE STRENGTH

Typically, in design with ductile metallic materials, the critical strength of the material, whether it be fracture or yielding, is defined by a single value for a given set of environmental conditions. Metallic materials produced to the standards of quality and process control typical of the aircraft industry show little variability in mechanical properties, either throughout a single piece of material or between batches of material produced at substantially different times, or between material produced to the same specification by different suppliers. This situation is due partly to the refined process control techniques which have been developed by the material suppliers and partly due to the ductility of the materials, which accommodates very localized stress concentrations caused by microscopic flaws and defects in the material.

Because of these characteristics a consideration, by the designer, of possible variability in the properties of metallic materials is not generally necessary. Reductions in allowable strength properties, from the mean value of a very large number of samples from many different sources, to a value which will ensure an extremely low probability that material of lower strength will be encountered, are very small and are normally made in compiling the standard tables of minimum properties such as those presented in MIL-HDBK-5.

The processing of nonmetallic inorganic materials which are of potential interest for structural applications has not generally reached the stage of development and refinement associated with the processing of metals, so that substantially greater variations in the mechanical properties of supposedly identical batches of material is apparent. Of much more significance, however, is the sensitivity of the mechanical properties to flaws, as a consequence of the lack of ductility. These flaws include pores, microcracks, inclusions of foreign materials, etc., and will generally be random in size and distribution. They will produce local stress concentrations which will depend on the size and configuration of the flaws and, since the apparent material strength will be controlled by the peak stress, it follows that a randomness will exist in the strength of the material.

Experience shows that the total variability in the mechanical properties of brittle materials is great enough that a single strength value cannot be assigned, but rather that the usable strength level must be associated with an acceptable probability of failure.

By conducting tests on a large number of samples, strength data can be obtained from which a curve can be plotted to show an expected failure rate against stress level. This is typically an S-shaped curve and for structural applications, where the probability of failure must be extremely low, only the extreme lower tail of the curve is of interest. The definition of material strength as a matter of probability rather than certainty is perhaps one of the most important differences between brittle material design technology and the technology used with ductile materials.

In addition to the analytical techniques required to calculate the probability of failure for a complex structure, this statistical concept introduces changes in design criteria philosophy, with the need to define an acceptable failure probability, and the possibility of trade-off between allowable stress level and failure probability. Testing to obtain material property data is greatly complicated, since a single characteristic requires numerous test specimens to define the curve describing the variability of that characteristic, and the lower part of the curve, which is of design interest, is difficult to obtain experimentally, since only a small proportion of a batch of test specimens will give extreme values of mechanical strength. As will be shown in the section devoted to statistical theory, there is also introduced the dependence of the strength level on volume of material, since the greater the volume the greater the probability of a critical flaw. Obviously, this statement is tempered by the stress distribution, and the prediction of strength becomes dependent not on the maximum stress at some particular location but on the summation of the failure probabilities of each element of the component. For instance, a low stress distributed over most of the volume of component may contribute significantly to the failure probability despite very localized high stresses, since the low stress is more likely to be associated with a larger flaw.

Attempts to specify the strength of brittle materials as a function of failure probability were made by Weibull in 1939 and, until recently, little additional work had been conducted. Weibull established a series type model to describe a material containing flaws of random size and distribution, and he selected the simplest mathematical relationship which would fit the model. The expression contains parameters which must be evaluated experimentally from samples of the material of interest. Despite the apparent inadequacies of the assumed material model and the simplicity of the mathematical expression, the Weibull description of the failure probability of the material has been reasonably well confirmed in practice, although experimental work is presently quite limited.

Weibull limited his considerations to uniform uniaxial and biaxial tension stresses and simple bending. Under conditions of uniaxial tension and simple bending he also considered the possibility that the material could have a threshold strength level below which there

was no probability of failure. He assumed further that compression stresses and shear stresses do not contribute to the probability of failure and he gave no consideration to anisotropy or to the effect of repeated loadings.

In the application of statistical theory to the design of structural components, an assessment of the probability of failure under the most critical loading conditions must be made and compared with the acceptable failure probability. This process contrasts with the calculation of maximum stresses and the comparison of these stresses with the ultimate or yield strength of the material, which is the procedure for ductile materials. In Section 3, procedures and charts are given to facilitate the calculation of failure probability. They are based on the methods of Weibull, since these currently are the only ones available. Weibull's methods have been extended to include, in combination, triaxial stresses and the possibility that the material has a zero probability of failure stress. The method can also be readily applied to a component of any shape with any stress distribution, and it recognizes that the variability of the material may be different in different parts of the component, due to environmental conditions such as a variation of temperature. While the methods given represent a very simple technique for the assessment of failure probability, the numerous influential factors which Weibull neglected make it only a temporary expedient, and require either design verification testing or the use of large factors of safety in assessing the acceptable failure probability. Furthermore, the modest extensions of Weibull's work which have been made in the preparation of this handbook indicate fundamental limitations in the method as the design condition departs more and more from a uniform uniaxial tensile stress.

2.8 IMPROVED MATERIAL CHARACTERIZATION, PROCESS CONTROL AND INSPECTION

The large reductions from the average strength of a brittle material, which are necessary to establish allowable stresses which will give a very low probability of failure, make it necessary that the designer give attention to minimizing this variability in order to avoid unnecessary structural weight. There are ways in which this can be done; control of processing, control of inspection methods, and control of material characteristics. Again, this contrasts with practice with metallic materials where the simple reference to a material specification is often all that is required of the designer. The purpose of this control is not only to maximize the strength of the material but also to minimize the variation in strength, so that the controls are concerned less with what is actually done during processing, and more with ensuring that each step is repeated identically for each piece of material. In this respect it is also important that the process be identical between the structural components and the test specimens used to establish the strength characteristics of the material.

The problem of process and inspection control is complicated, with respect to nonmetallic, inorganic refractory materials, by proprietary considerations of the supplier. Most of these materials have been developed to their present state by a long process of trial and error, and as a result the details of some of the constituents and the values of some of the important process parameters are often closely guarded. Probably, in time, this situation will change, particularly with the application of Government research funds to the development of such materials. Certainly it is desirable that the material obtained from different sources be as nearly identical as possible, and that the material suppliers assume the responsibility for rigorous process control. Unfortunately, the present demand for such high quality material is insufficient to interest the manufacturers to this extent, so that the designer must assume responsibility.

Another problem is the fact that the significance of various processing parameters on the variability of the mechanical properties of the finished material is not generally known, so that it is not generally possible to impose close control on one or two processing factors and thereby ensure reproducible material characteristics. Until much more is known, with respect to individual materials, the only approach seems to be to control every processing step and every processing variable that can be controlled. This will include control of the raw ingredients, the weight and chemical purity of each, their source, and processing and handling, to the point where they are ready for mixing. Similarly with the material mixing; consideration should be given to the control not only of mixing time, but rate, atmosphere, temperature and humidity, cleanliness of mixing equipment, type of construction material of this equipment, etc. If compacting is involved there will be parameters such as pressure, temperature, time, atmosphere, storage time and conditions before and after compacting, etc. Similar lists of possible control parameters can be established for sintering, subsequent machining, etc.

Consideration should also be given to inspection techniques for use both during processing and for inspection of the finished material. In addition to the more conventional methods, consideration might be given to various crack detection techniques; die penetrant for the surface, and acoustic for internal cracks; X-ray inspection for density variability and porosity, etc. Obviously, also, for every inspection step a criteria of acceptance or rejection must be established. Experience with nondestructive inspection methods for the quality control of "ceramic" materials is also very limited. Attempts have been made to correlate NDI results with physical and mechanical properties of the material but only very limited success has been achieved. This subject is also discussed further in Section 5.

In addition to controlling the processing parameters, the inspection techniques and the inspection criteria, it is necessary to define, so far as is possible, the material characteristics to be achieved. Again the relationship between such characteristics as grain size, surface finish, material phases, porosity, etc. and the required mechanical and physical properties is very incompletely known and the general practice is to

specify any characteristic that might be significant and that can be measured. Guidance is given in Section 5.

Clearly the design task, with materials of this class, is greatly increased by the requirement for the preparation of detailed material processing and inspection specifications. Preferably, also, these should be compiled before extensive mechanical property testing of the material is undertaken, so that such testing can be performed on a true specification material. Ideally, also, test specimen material should be processed to the limits of the tolerances on the various parameters so that the maximum variability in the material from this source can be examined. However, this will not generally be practical, because of the large number of parameters involved, and reliance will generally be placed on obtaining a satisfactory statistical distribution from numerically large samples.

It should be emphasized that the significance or the necessity of adopting all of the above procedures on the variability of the material mechanical properties is not yet understood. It is not generally known, for any materials of this class, how much of the variability is due to processing variations, and how much is inherent in the brittleness, in combination with microscopic and atomic scale flaws. Furthermore, the controls mentioned will be costly; nevertheless, until these materials are much better understood, there seems to be no other alternative.

2.9 METHODS OF ACHIEVING HIGH ALLOWABLE STRESSES

The significance of the variability in the mechanical properties of nonmetallic refractory materials on the allowable stress level that can be used, if a specified level of structural reliability is required, has been discussed in previous sections. The importance to the designer of applying close control of material processing and inspection techniques to minimize variability has also been mentioned. In the basence of ductility,however, it is not expected that variability of mechanical properties can be reduced to the level where it has no significant effect on allowable stress, and hence on component weight. It therefore becomes necessary, during design, to consider other steps which can be taken with a material of given variability to maximize allowable stresses without sacrifice of reliability.

A promising technique for achieving increased strength levels is the use of the proof test, which involves subjecting each component to a predetermined stress to eliminate by destructive testing the occasional sample of low strength and thus raise the allowable stress level for a given failure probability. In terms of the Weibull strength distribution curve, proof testing has the effect of truncating the curve by cutting off the long tail on the low strength side. The proof stress in effect becomes a "zero probability of failure" stress but with the advantage that it is determined directly by test. The distribution curve thus has a definite and experimentally determined end point and the use of the curve to predict stress levels for failure probabilities greater than zero becomes a matter of interpolation between available test data rather than extrapolation well beyond the limits of the experimental results. Thus, not only is the allowable stress level for a given failure probability increased, but the confidence in the value is also substantially improved.

In describing the strength distribution curve that applies after proof testing and after rejection of the low strength samples which are destroyed during proof testing, two methods are available. Either a new distribution curve can be determined, based on the remaining samples, or the original distribution curve can be truncated analytically to include the effect of the proof test. The latter method is considered preferable since it uses more experimental data, although it cannot be used where proof testing causes material damage in the remaining samples. Methods are available to make this adjustment when the Weibull distribution curve is used. The appropriate analytical techniques are included in Section 3.

Numerous studies are already available in the literature to show quantitatively the benefits of proof testing and these generally predict very significant improvements in permissible stress levels. However, such studies generally consider failure probabilities between 0.01 and 0.10, which are completely impractical for airframe structural applications. If the probability of failure is to be reduced to 1×10^{-6} or 1×10^{-7}, for instance, it will generally be found that the allowable stress is nearly coincident with the proof stress. Thus, in the simple case where both the applied stresses and the proof stresses are uniform throughout the specimen, and where in addition the proof test does not damage the material, it appears that the statistical considerations are eliminated. In the more general case,however, which will be discussed below, this is not so.

The introduction of the statistical approach in expressing the failure of a brittle material makes the probability of failure dependent on both the state of stress and the strength properties, throughout the part. As a result a low stress level applied over most of the volume of the component might contribute as much to the probability of failure as the maximum stress, which may be present only in very localized areas. By extending this reasoning, any proof stress distribution will result in improved allowable stresses by elimination of low strength components, and it is not necessary that the proof stress match the applied stress in distribution. Thus, a proof test can be selected on the basis of convenience and low cost. A leading-edge component, for instance, which is stressed in service by a complex system of aerodynamic loads may be effectively proof tested by inducing thermal stresses by heating and subsequently cooling. In determining the effect of a given proof stress distribution on the probability of failure, statistical methods must

again be used. The necessary analytical treatment, together with charts which facilitate numerical computations, is given in Section 3 for the case of a Weibull distribution curve.

Proof testing raises the question of material damage, and limited studies which have been made, particularly with graphite, show that damage in the form of microcracks can be produced by a proof test. Presumably this question of damage will be one of the considerations involved in selecting the proof stress level. This will require test specimens which can be examined before and after the application of proof stress to check for material damage. As a second method of checking whether the selected proof stress level is causing significant material damage, tests to failure can be conducted on a series of proof-tested specimens to ensure that the test points lie on the truncated distribution curve predicted by the methods in Section 3. In practice the material damage due to proof testing must be kept small, either by the proper selection of material or proof stress level, since it is not practical to design a structural component such that extensive material damage is occasioned by a single load application.

The other consideration in selecting the proof stress level is one of economics. The higher the proof stress the higher will be the resulting allowable stress for the selected failure probability, and hence the lower will be the component weight. High proof stresses, however, increase the proportion of components which will be destroyed. It has been postulated that the allowable stress in a component can be raised to any level by proof testing at a sufficiently high level, and accepting a very large rejection rate. On this basis it is considered that weight optimization loses its usual meaning and becomes an economic consideration with a trade-off between weight and the cost of the number of components which must be made to achieve one having the selected strength level. More realistically, in brittle material design, optimization will probably involve the usual determination of geometric characteristics to sustain the necessary loads with minimum weight, but this will be supplemented by a study of allowable stress level, and hence weight, against cost in terms of the type of proof test and the component rejection rate.

To date, proof-testing is the only method of improving allowable stresses, for a given structural reliability and with a material of given variability, that has been studied, but there are two other considerations which can be mentioned although no data exists on their significance. The first of these is the use of redundant load paths which increase allowable stresses by permitting a higher failure probability in any single load path, for the same overall component failure probability. Redundancy, however, must not violate the rule, which has been discussed in an earlier section, and which requires that there be no external restraints to brittle material components which introduce unknown or undefinable loads. It is also necessary that redundancy be provided by disconnected load paths so that failure of one path does not produce failure of the other as, for instance, by crack propagation. In other words, the construction should be redundant but fail-safe and determinate. Considerations of redundancy are discussed further in the section on joints and connections, since it is in the design of joints where the problem of achieving redundancy without introducing unknown loads becomes most apparent.

2.10 NEW DESIGN CRITERIA CONCEPTS

The conventional practice in establishing airframe design criteria is to determine limit loads, the maximum loads which are expected to arise in service, and to design so that the airframe will function satisfactorily under these conditions. This is generally interpreted as designing so that the yield strength of the material is not exceeded or so that instability of compression elements does not occur. It is also conventional practice to increase these loads by an arbitrary but experience proven factor of safety, and design so that failure of the airframe does not occur under these ultimate loads. Continued use of the airframe without repair, after exposure to ultimate loads, is not required.

Despite the probabilistic nature of essentially all loads, it has been customary to establish limit loads on a deterministic basis, selecting levels which, from experience, would be unlikely to be exceeded during the life of the vehicle. However, adequate statistical data to determine quantitatively the risk involved, as a function of the selected load level, is rarely available.

More recently the probabilistic approach has been used in examining the effect of turbulence and gusts on the fatigue life of the structure. In space vehicle design also, it is the practice to treat more of the loads probabilistically. In space vehicles most of the loads are induced by atmospheric effects or mechanical subsystems, and it is easier to obtain sufficient statistical data, than it is for a manned aircraft of long life. Even with space vehicles, however, the probabilistic treatment of loads is far from adequate.

When thermal effects are introduced the extension of the deterministic approach becomes much more difficult. The combination of extreme values of loads, heat transfer rates, temperatures, etc. would be simple, but generally unacceptably conservative. Alternatively, the selection of combinations of various levels of the many parameters involved, to try to find critical combinations, is often somewhat arbitrary, involving much judgment.

Extension of the arbitrary safety factor practice also becomes difficult when thermal effects are important and while there is general agreement that it is unnecessary to apply a safety factor to every parameter, there is little agreement on just what should be factored.

The question of repeated loads, with associated thermal effects is even more complex, if the approach is by extrapolation of deterministic methods of load specification, and very little consideration of criteria for this situation, particularly where thermal effects predominate, has presently been made.

Current practice in establishing allowable material properties for airframe design does consider the statistical distribution of material strength. The variability, with metals, is small, and the values that the designer selects from approved handbooks such as already allow for this variability.

However, the same procedure is not followed with other sources of data which are determined experimentally and used for design. Generally, the degree of conservatism to apply to measured aerodynamic coefficients, heat transfer coefficients, etc., is left, in an uncontrolled manner, to the individual specialist.

When brittle materials are used as part of the airframe, the question of structural criteria requires review for a number of reasons. First, the emphasis on specifying material allowables on the basis of failure probability, requires a corresponding expression of loads and thermal effects on a probabilistic basis, since the significant consideration is the failure probability of the airframe; a combination of structural failure probability and load occurrence probability. For the same reason the conventional safety factor approach must be examined; it is desirable to substitute a rational, probabilistic loading condition for the current arbitrary factor of safety.

The importance of thermal effects for structural components requiring "ceramic" materials also justifies at least an attempt to rationalize such conditions both with respect to defining the important design considerations and with respect to the question of safety factors. Next, it becomes necessary to establish a basis for selecting material allowables, involving now the selection of an allowable failure probability, either for the material or the entire airframe, and it becomes necessary to consider how repeated loads, with the associated thermal effects and allowable material properties, should be treated probabilistically.

Finally, the question of qualification testing, to demonstrate that the airframe meets the requirements of the design criteria, needs examination. The present practice of static and repeated load ground testing of a complete airframe can certainly not be conducted on a statistical basis and neither does it seem practical to conduct pre-operational flight testing other than by flying to discreet, pre-selected and specific conditions.

This subject of structural criteria is examined in some detail in Section 6. It is clear that in a Handbook of this type it is not practical to change the present approach to the determination of loads and thermal effects. To require that these be established on an entirely probabilistic basis at this time would not be acceptable to the industry and is premature because of insufficient statistical data. Furthermore, there are significant legal implications if the allowable operating limits of the vehicle cannot be clearly defined. The concept of limit loads and associated thermal effects is therefore retained with the understanding that these are conditions which are very likely to occur in the life of the vehicle, and that where probabilistic data can be used for their definition a probability of occurrence of about 1% should be used. Similarly, ultimate conditions are retained with factors proven by experience with the understanding that the probability of occurrence of these conditions in the life of the vehicle is extremely remote but unspecified quantitatively.

It now becomes necessary to treat material failure probability separately since load occurrence probability cannot be well defined. Accordingly, the approach is to select material allowable stress levels which will provide an extremely low probability of failure under limit load conditions and a much higher value under ultimate conditions. The former is chosen to be nominally zero, and in practice proof testing of each component will be required to meet this condition. The material failure probability under ultimate loads is permitted to increase to 10^{-2} which is consistent with the value used to determine mechanical properties in metal structures under ultimate loads. The effect of this criteria should be to achieve the same levels of structural integrity as has been the practice in metallic structures while minimizing the need for statistical data of either the environment or the structural materials.

With respect to qualification testing, it is considered that flight testing must still be conducted by flying at predetermined critical conditions, and with brittle components its chief function would be to verify with the use of strain gauges the loads used for design and the predicted stress distributions. The structural ground test, however, can be retained, and in Section 6 it is shown how the design of a brittle material component can be verified with ground testing despite material variability and without requiring large numbers of tests.

2.11 OPTIMIZATION CONCEPTS

Present optimization techniques applied to metal structures are concerned principally with structures which fail by compression instability, and in which the strength is controlled by both material properties and geometry. The techniques involve selection of the geometric proportions for a given type of construction and a given metal material, such that the applied load can be supported with the minimum weight of structure. An early approach was to assume that minimum weight is attained when all elements of the structure collapse at the same stress level, a condition which will provide sufficient equations to

define the geometry of the structure. For a given loading condition and given external component shape, minimum weight structures can be designed in this manner, in various materials and for various types of construction, so that an optimum with respect to material, type of construction and structural geometry can be determined. Similar studies can be made at different temperatures for structures subjected to heating.

For many of the loading intensities which are common in airframe practice these techniques lead to structural proportions which are impractical to fabricate, so that it is necessary to introduce practical constraints on minimum material thickness, minimum edge distance from rivet lines, minimum stiffener spacing dimensions, etc. It is also necessary to introduce the fact that most structures are subjected to multiple loading conditions, with different loading conditions being critical for different elements of the structure. Recently the so-called structural "synthesis" methods have been under development and these will define a minimum weight structure of given material and type of construction including practical constraints and multiple load conditions. These structural synthesis methods are also useful where the structure is redundant. They permit the determination of the minimum weight structural proportions where the structure contains multiple load paths but where it is also subjected to multiple loading conditions.

For structures constructed from brittle nonmetallic materials the same techniques could be used to define structures which are critical with respect to compression instability. Generally, however, structural elements constructed from this class of material will be relatively bulky and stable, so that tension and bending loads become critical. When only a single load path exists and the structural element is subjected to tension or stable compression loadings, the design procedure with metallic structures involves simply selecting areas and thicknesses to match the local loadings without exceeding specified stress levels. With brittle materials the procedure is basically the same, with allowable stresses selected on the basis of an acceptable material failure probability and, again, the structural synthesis methods can be used to obtain minimum weight proportions if the structure contains multiple load paths. The problem is slightly more complex than the metal structures, however, since, for a given failure probability the allowable stresses are dependent on material volume, which in turn is dependent on allowable stress. Thus, an iterative procedure, or at least the establishment of a number of designs, using preselected values of allowable stress, to span the required failure probability, is indicated. There are, however, some aspects of optimization which are not present with metallic structures.

The first is to minimize stress concentrations by appropriate choice of local geometry as, for instance, at fillets, corner radii, changes in cross section, etc. High local stresses contribute substantially to the component failure probability and reduction of these concentrations by small local additions of material or by local changes in shape will permit, for a given component failure probability, a general increase in stress level over the remainder of the material. Hence, local small additions of material can result in relatively large weight savings. Since the weight increments involved will generally be very small special optimization studies are not justified.

Many refractory materials will be used for high temperature applications in which temperature gradients will be present through the structural component. In many such applications the resulting thermal stresses will be a critical design condition. The type of optimization mentioned above is again required to reduce local stress peaks and increase the stress level throughout the bulk of the component material. Now, however, the variations in geometry affect the heat transfer characteristics of the component, and hence temperature gradients. Stresses are therefore affected, both directly and indirectly, by geometric changes. There are no methods of establishing the minimum weight component geometry systematically and the result must be obtained by repeated analytical trials, seeking trends from which the optimum can be deduced.

Finally, optimization can be introduced into the definition of the structural configuration of a component in the selection of proof-test stress level. For a given component reliability or failure probability, higher allowable stresses result from the application of higher proof stress, so that weight becomes directly related to the proof-stress value. On the other hand, as proof stress is increased the proportion of components destroyed in proof-testing will increase, so that a trade-off between cost and weight results. In some applications it may be possible to realistically evaluate the value of each pound of weight saved and, from such analysis, together with the cost of component fabrication, a proof stress level based on minimum total cost could be determined. In practice, however, it is not likely that a component failure rate during proof-testing of more than about 10% would be acceptable, since higher values imply damage to the material of the remaining components. Thus again, optimization studies peculiar to brittle material structures are not justified.

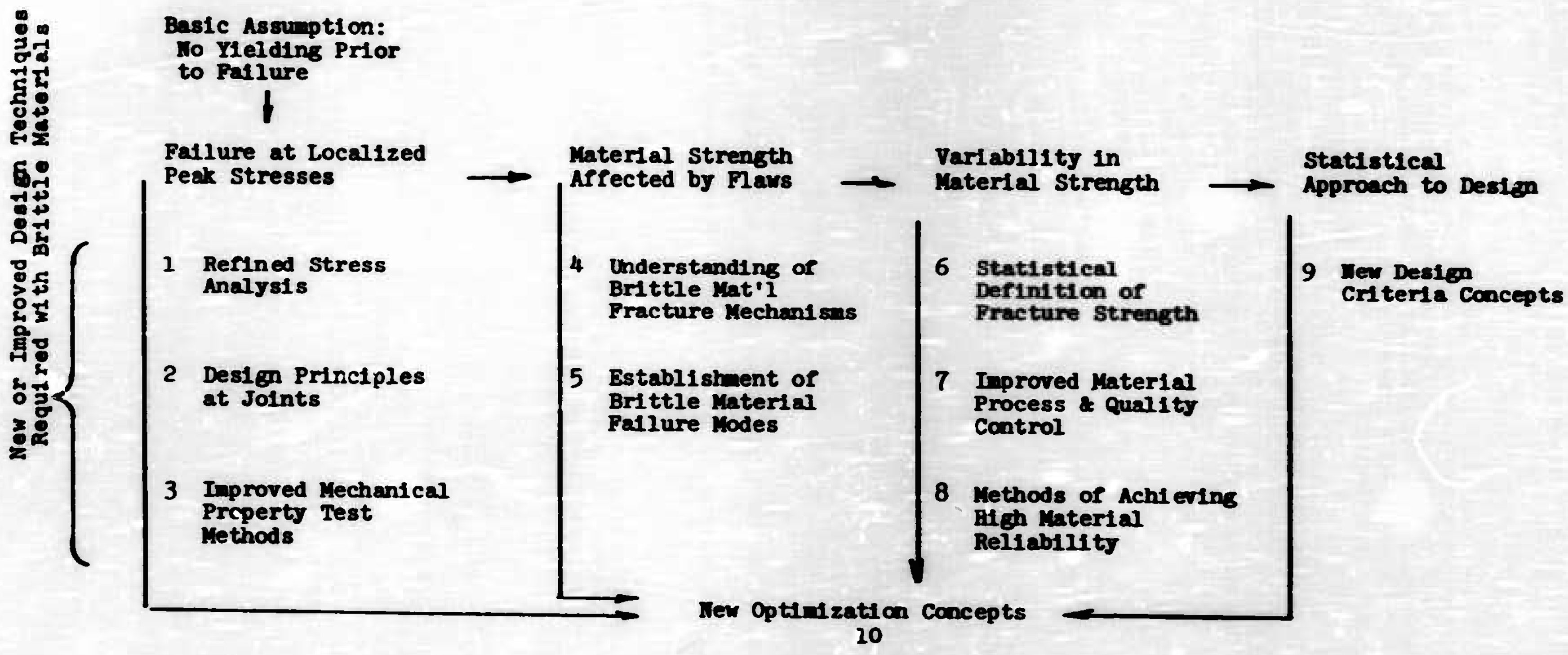


FIGURE 2.1 BRITTLE MATERIAL DESIGN PRINCIPLES

3. ESTABLISHMENT OF DESIGN STRESSES

3.1 THE STATISTICAL APPROACH

Conventional structural design with metallic materials involves a number of basic assumptions related to the response of the material. Those which are important when structural design is extended to brittle materials are as follows:

(a) It is assumed that the strength characteristics of a complex structure subjected to a complex system of internal stresses can be predicted from experimental data on material properties determined from small simple specimens subjected to simple stress systems. A number of strength theories such as the distortion energy theory and the maximum shear stress theory have been developed for this purpose and have proven generally satisfactory with metallic structures.

(b) It is assumed that the mechanical characteristics of the material for any given set of conditions can be defined specifically by a single unique value, i.e. characteristics under tension loads by ultimate tensile strength.

(c) It is assumed that the strength of a structural component is determined by the stresses at some critical point and that the strength of the structure can be determined by an examination of the stress system at this single point.

When brittle, nonmetallic materials are used the same facility for determining material properties from small simple test specimens, and using the information to predict the characteristics of large complex components subjected to complex loadings, must be available, otherwise structural design with such materials becomes impractical. A material failure theory which is generally accepted for this purpose, with brittle nonmetallic materials, is a maximum stress theory, which assumes that the component will fail when the maximum principal stress equals the strength of the material as determined in simple tension. That is, the state of stress is assumed to have no effect on the limiting value. Extensive verification of this theory has not been accomplished, particularly for stress states involving compression, but there is no more satisfactory theory available at this time.

With brittle materials, however, the mechanical characteristics are not expressible as a single number. Due to the wide variability in any particular mechanical property, the most that can be done is to predict the probability of failure for any particular stress condition. Variability in mechanical properties is assumed to be due to the presence of flaws, which produce local stress concentrations that cannot be relieved by yielding, due to the complete lack of ductility. Whether a sample of material will fail under a given stress condition will depend, therefore, on the size and distribution of flaws, and since the latter is random the only statement that can be made about failure is the probability of its occurrence. Furthermore, since the probability of experiencing a flaw of critical size will increase as the size of component or the valume of material under consideration increases, the strength becomes a function of component size.

The introduction of a statistical approach to the expression of mechanical characteristics, and the dependence of failure probability on volume, also means that the strength of a component cannot be determined by the examination of a critical point. For example, an applied stress distribution may have a very localized peak, so that the probability of this peak combining with a flaw sufficiently severe to produce failure is small. On the other hand the same stress distribution may subject a large volume of material to a lower stress, and although a more severe flaw is needed to produce a failure there may actually be a greater probability of meeting such a flaw. Thus the probability of failure of the entire component must be expressed, and this involves some type of summation of the failure probability at all points within the component. Again it must be pointed out that the dependence of failure probability on volume is based on the assumptions of the flaw concept. Its validity remains to be positively demonstrated, and experimental evidence to date is limited, and in some cases conflicting. Similar statements can be made about most of the procedures to be presented in this Section, but they remain the best that are presently available, and the overall application of these procedures to component design, while very limited, has nevertheless proved satisfactory.

Additional details of the flaw theory can be found in References 3.1 and 3.2 and substantiating data for the size effect in References 3.1, 3.3, 3.4, 3.5, 3.6, 3.7, 3.8, 3.9, 3.10, 3.11.

The variability in mechanical properties of a brittle material, from supposedly identical specimens, implies that there is a distribution function which shows the relative frequency with which a particular value of some mechanical property may be expected to occur. If frequency of occurrence is plotted, for instance, against tensile strength, the plot will show the frequency rising to a maximum value at some strength level, and falling off on either side. Extreme values of strength, either high or low, do not occur as aften as those near the center. Furthermore, the curve may or may not be symmetrical about the maximum frequency. Such a curve is called a frequency distribution; it is not presently known whether all brittle materials have a similar frequency distribution, nor is it possible to analytically determine such a function. The most that is presently possible is to assume various frequency distributions and seek a best fit with experimental data.

By normalizing of the frequency distribution it is possible to make the total area under the curve equal to unity, and by integrating the resulting expression the distribution function is obtained. This function expresses the probability of a given value of the variable, such as tensile strength in the example given above, occurring. When plotted, this function is typically an approximately "S" shaped curve, approaching or passing through zero (absolute impossibility) at one extreme and asymptotically approaching unity (absolute certainty) at the other.

To construct a distribution curve, defining a particular mechanical property for a particular material, generally requires a very large number of test specimens since the structural designer is interested in the extreme lower portion of the curve, where the probabilities of failure are very small, while at the same time the rareness of extreme events precludes a good definition of this part of the curve. In order to avoid very extensive testing programs it is desirable to find a general mathematical description of the distribution function applicable to all brittle materials. As already stated, there is, at present, no theoretical basis for completely defining the distribution curve, and to establish such a curve on an empirical basis introduces the same problem of lack of test data at the lower extremities, the only part of the distribution curve of real interest. As a result there is no complete agreement on the type of curve to be used, although a relationship established by Weibull is accepted as the most satisfactory to date. The Weibull distribution function has been found to fit a wide variety of experimental data, covering not only material strengths but many other statistical data. It must always be remembered, however, that adequate data in the range of interest is always lacking, although a few programs have been conducted in which very large numbers of test specimens have been deliberately used in an attempt to verify the general usefulness of the Weibull curve (Reference 3,12).

The Weibull distribution function is based on the flaw theory, with the assumption that the flaws are distributed at random with a certain density per unit volume. The strength of the specimen is then determined by the weakest point in the specimen. The result is a representation of the material as a series model, or a chain in which failure depends on the weakest link. This is perhaps the major limitation of the Weibull theory since a real material is perhaps better represented as a series of chains in parallel, the so-called series-parallel model. Again, however, a convenient, practical, mathematical representation of such a model is not available.

The Weibull distribution function also assumed that the test specimens or structural components under investigation are all of the same population, so that all data points fit one distribution curve. It also assumes, when data from small simple specimens are used to design larger, complex components, that the material in both cases is of the same population. In general these are not good assumptions, but in order to use nonmetallic refractory materials for the design of aircraft or space vehicle components, much tighter processing controls must be used, both for material processing and selection, than is the customary practice with ceramic materials. This is necessary, both to reduce variability in mechanical properties as much as possible, in order to maximize the design stresses for a given reliability, and also to ensure that indeed the mechanical characteristics determined on simple specimens are representative of more complex components. If such procedures are followed any adverse consequences from the two assumptions just mentioned are minimized.

The procedures presented in this Section will be based only on the Weibull distribution function, since this function appears to be more appropriate for brittle materials than other available functions. Its limitations must be recognized, however, and it is possible that it may not be particularly appropriate for some particular material or material combination. There is however no other procedure currently available. Methods for minimizing the importance of an accurate definition of the distribution curve will be discussed later.

The Weibull distribution function is expressed as the probability of fracture, which is given by

$$S = 1\text{-exp}\left[-V\left(\frac{\sigma - \sigma_u}{\sigma_o}\right)^m\right]$$

where V is the volume of the element under consideration, σ is the applied uniform tension stress, and σ_u, σ_o and m are material constants. If the Weibull function is truly applicable to the material under consideration, and if the materials used for small test specimens and for the structural components are truly identical, then σ_u, σ_o and m will indeed be material constents and will not change with volume. In the following section a procedure is given for the evaluation of the material constants from experimental data and a table to facilitate this process is included.

The above expression can be generalized by writing the exponent as an integral over the volume of the component, and expressing σ as a variable in terms of V. In the completely general case a component can be divided into a large number of elements, each sufficiently small that the stress within the element can be considered uniform. The above equation can be applied to each element, and by proper summation, the probability of failure of the component can be determined. Curves to facilitate such a computation are given. Furthermore, the equation can be evaluated directly to give failure probability, for a few cases where the stress distribution is simple and can be expressed in simple mathematical form.

It will be evident that the use of brittle nonmetallic materials in structural components introduces two basic difficulties if efficient reliable structures are to be obtained. These difficulties are:

(a) Variability in mechanical properties, and the infrequent but real possibility of low values, particularly in large volumes of material, demands low stress levels relative to mean strength of the material, and hence high weight, if high reliability or extremely low probability of failure is to be achieved.

(b) Accurate definition of the important lower part of the distribution curve, which is necessary if a reliable prediction of failure probability is to be made, still requires, with present theoretical knowledge of the problem, extensive test programs to obtain statistical data on mechanical properties. Furthermore, these programs must be repeated for each environmental condition of interest, such as temperature.

It is considered that the most promising method of circumventing these difficulties is by subjecting each component to a proof-test. A proof-test will reveal those components which are low in strength and will have the effect of experimentally fixing the lower extremity of the distribution curve. Thus it becomes possible to define much more accurately the lower branch of the curve with relatively few experimental data. Furthermore, the elimination of the few extreme values substantially increases the permissible stress level for a given probability of failure. It should be noted that the proof stress distribution need not match the applied stress distribution if the proper relationships are available to determine the probability of failure of an element of the component, when subjected to a given applied stress and a given proof stress. Accordingly, proof-testing may be conducted in any convenient manner. For instance, a component designed by aerodynamic loadings may be proof-tested by introducing thermal stresses by heating and cooling, if such a procedure is convenient or economical. It should be noted also that the proof stress may be less than the applied stress, for any particular element, while still producing significant improvements in failure probability for a given applied stress, or conversely improvements in permissible stress level for a given failure probability. Maximum benefits are produced if the proof stress equals the maximum applied stress, but the selection of the proof stress level must also consider the economics of the possible component failure rate and the degree of material damage imposed on acceptable components. More detailed discussions of these various aspects of proof-testing can be found in Reference 3.13.

Methods for introducing any type of proof stress distribution into the assessment of component failure probability are given in the following sections.

3.2 ANALYTICAL RELATIONSHIPS

From the original work by Weibull (Reference 3.7) the probability of failure of a structural component or element is given by

$$S = 1 - \exp\,[-B]\,, \tag{1}$$

where B is the risk of rupture and is given by

$$B = \int n(\sigma)\, dV. \tag{2}$$

Weibull uses experimental data as the basis for the assumption that, for the case of an element under uniform, uniaxial tension,

$$n(\sigma) = K\sigma^m. \tag{3}$$

For this simple case, therefore,

$$B = K\sigma^m V, \tag{4}$$

where σ is the uniaxial stress, V is the volume of the element, and K and m are material constants.

K can be conveniently replaced by another constant $(1/\sigma_o)^m$ so that

$$B = V\left(\frac{\sigma}{\sigma_o}\right)^m \tag{5}$$

and

$$S = 1 - \exp\left[-V\left(\frac{\sigma}{\sigma_o}\right)^m\right], \tag{6}$$

We recognize the possibility that some materials may be able to support a certain stress level, σ_u, without the possibility of failure. To introduce this stress level, we rewrite Equation (6), as

$$S = 1 - \exp\left[-V\left(\frac{\sigma - \sigma_u}{\sigma_o}\right)^m\right]. \tag{7}$$

If an element is subjected to triaxial stresses, and the material is isotropic, we consider σ_x, σ_y and σ_z as mutually perpendicular principal stresses acting at a point. The x, y, and z axes for a potential element are to be oriented so that σ_x is the maximum positive principal stress. We can now consider the failure probability of a small spherical volume around this point. The normal stress at any point on the surface of this sphere is

$$\sigma = \sigma_x \cos^2\phi \cos^2\psi + \sigma_y \cos^2\phi \sin^2\psi + \sigma_z \sin^2\phi, \tag{8}$$

where ϕ is an angle with respect to the xy plane and ψ is an angle with respect to the xz plane.

The risk of rupture is still defined by Equation (2), where

$$dV = \left(\frac{r^3}{3}\right) \cos\phi d\phi d\psi \tag{9}$$

If the material can support a stress level σ_u without the possibility of failure, the stress contributing to failure probability becomes

$$\sigma - \sigma_u = \sigma_x \cos^2\phi \cos^2\psi + \sigma_y \cos^2\phi \sin^2\psi + \sigma_z \sin^2\phi - \sigma_u.$$

Substituting this expression and Equation (9) into Equation (2), and assuming that the form of $n(\sigma)$ is the same as for the uniaxial case, that is $n(\sigma) = K\sigma^n$, and assuming also that the integration includes only that part of the stress which contributes to the probability of failure, gives

$$B = K_1 \frac{r^3}{3} \iint (\sigma_x \cos^2\phi \cos^2\psi + \sigma_y \cos^2\phi \sin^2\psi + \sigma_z \sin^2\phi - \sigma_u)^{m_1} \cos\phi d\phi d\psi \tag{10}$$

or

$$B = \frac{K_1 r^3}{3} \sigma_x^{m_1} \iint \left[\cos^2\phi \cos^2\psi + \left(\frac{\sigma_y}{\sigma_x}\right) \cos^2\phi \sin^2\psi + \left(\frac{\sigma_z}{\sigma_x}\right) \sin^2\phi - \left(\frac{\sigma_u}{\sigma_x}\right)\right]^{m_1} \cos\phi d\phi d\psi \tag{11}$$

or

$$B = \frac{K_1 V}{4\pi} \sigma_x^{m_1} \iint \left[\cos^2\phi \cos^2\psi + \left(\frac{\sigma_y}{\sigma_x}\right) \cos^2\phi \sin\psi + \left(\frac{\sigma_z}{\sigma_x}\right) \sin^2\phi - \left(\frac{\sigma_u}{\sigma_x}\right)\right]^{m_1} \cos\phi d\phi d\psi \tag{12}$$

or

$$B = \frac{K_1 V}{4\pi} \sigma_x^{m_1} I. \tag{13}$$

Subscript 1 has been used with K and m to permit the possibility that they may have different values from the values in the previous derivation for the uniaxial case.

$$I = \iint \left[\cos^2\phi \cos^2 x + \left(\frac{\sigma_y}{\sigma_x}\right) \cos^2\phi \sin x + \left(\frac{\sigma_z}{\sigma_x}\right)^2 \sin^2\phi \left(\frac{\sigma_u}{\sigma_x}\right)\right]^{m_1} \cos\phi d\phi dx \tag{13a}$$

and the surface integration is limited to the area where the stress is positive (tension).

For the uniaxial case, Equation (13) becomes

$$B = \frac{K_1 V}{4} \sigma_x^{m_1} I_{(\sigma_u = \sigma_z = 0)}. \tag{14}$$

But

$$B = KV \sigma_x^m \left(1 - \frac{\sigma_u}{\sigma_x}\right)^m. \tag{15}$$

These two equations are of different form, but Equation (14) has been evaluated numerically for values of m_1 from 2 to 10 and values of σ_u/σ_x from 0 to 0.8 and is found approximately equal to Equation (15) if we write

$$m_1 = m - 1 \quad \text{and} \quad K_1 = \frac{4\pi \; \sigma_x K}{I_{(\sigma_u = \sigma_y = \sigma_z = 0)}}.$$

Then

$$B = KV \sigma_x^m \left[\frac{I}{I_{(\sigma_u = \sigma_y = \sigma_x = 0)}}\right]. \tag{16}$$

The term in brackets can be evaluated from Equation (13a), for values of m_1. m_1 can be determined for values of m from $m_1 = m - 1$. Values of the term in brackets are given in Figures 1 to 6 for various values of σ_y/σ_x, σ_z/σ_x, σ_u/σ_x and m_1.

Relationships are not available, at present, for anisotropic materials. However, in most brittle materials of interest the anistropy results because the material is a composite and is fabricated specifically to give different properties in different directions. In such a case the differences in the properties will probably be large and it is suggested, without proof, that stresses be resolved along the principal material axes and failure probabilities be calculated for each direction individually, i.e. it is assumed that if the material is strongly anisotropic, the failure probabilities in the principal material directions are independent.

If the stress distribution or the material properties vary throughout the structural component, it can be assumed to consist of a large number of elements, each subjected to a different, but uniform, stress. Then the previous expressions can be generalized to give the failure probability of the component as follows.

For elements subjected to uniaxial stresses only,

$$S = 1 - \exp\left\{-\sum\left[\left(\frac{\sigma - \sigma_u}{\sigma_o}\right)^m V\right]\right\} . \tag{17}$$

For elements subjected to triaxial stresses,

$$S = 1 - \exp\left|-\Sigma B\right| , \tag{18}$$

where B is given by Equation (16), V is the individual element volume and the summations are extended only to those elements of volume where they have positive values.

In practice we are interested only in very small values of S, such as 10^{-5} or 10^{-6}, so that we can write

$$S = \log_e\left(\frac{1}{1-S}\right) .$$

Substituting into Equations (17) and (18) gives, respectively,

$$S = \sum\left[\left(\frac{\sigma - \sigma_u}{\sigma_o}\right)^m V\right] \tag{19}$$

$$S = \Sigma B . \tag{20}$$

If the component is subjected to a proof-test, resulting in a proof-stress distribution expressed by σ_p, then Equations (19) and (20) become,

when σ and σ_p are both uniaxial stresses,

$$S = \sum\left[\left(\frac{\sigma - \sigma_u}{\sigma_o}\right)^m - \left(\frac{\sigma_p - \sigma_u}{\sigma_o}\right)^m\right] V, \tag{21}$$

when σ_x and σ_{px} are maximum principal stresses in a triaxial stress system,

$$S = \Sigma(B_{(\sigma_x)} - B_{(\sigma_{px})}) . \tag{22}$$

These equations express the probability of failure S of a component subjected to a uniaxial or triaxial stress distribution and having variability characteristics defined by m, σ_u and σ_o and subjected to a uniaxial or triaxial proof stress distribution which may be different from the applied stresses. The equations also recognize that the material constants may vary throughout the component, depending on temperature or other environmental conditions.

In the application of brittle nonmetallic materials, it is customary to determine the material strength characteristics by conducting bending tests to failure on small, rectangular cross-section bars. A four-point loading system is used, so that the center portion of the bar is subjected to a constant bending moment.

Applying the Weibull expression to this condition and considering, conservatively, only the volume of material subjected to the maximum bending moment, gives the following expression for failure probability:

$$S = 1 - \exp\left\{-\frac{V}{2(m+1)}\left[\frac{(\sigma_b - \sigma_u)^{(m+1)}}{\sigma_o^m \sigma_b}\right]\right\} , \tag{23}$$

where σ_b is the maximum bending stress. If N bars are tested and are arranged in order of increasing failure stress, and if σ_{bm} is the failure stress of the nth bar, as measured in the test, then the failure probability corresponding to the fracture stress σ_{bm} is

$$S_n = \frac{n}{N+1} \qquad (24)$$

Equations (17) and (18) can be used to determine the Weibull constants for the material. To do this, Equation (17) is rewritten, for the nth test bar, as follows:

$$\log \log \left(\frac{1}{1 - S_n}\right) + \log \sigma_{bn} = (m + 1) \log (\sigma_{bn} - \sigma_u) + \log \left(\frac{V}{2(m + 1)}\right) - m \log \sigma_o \qquad (25)$$

Equation (25) shows that a plot of the Weibull distribution function will be linear in a system of rectangular coordinates in which $\log \log [1/(1 - S_n)] + \log \sigma_{bn}$ is the coordinate and $\log (\sigma_{bn} - \sigma_u)$ the abscissa. The slope will be $(m + 1)$ of the distribution function in these coordinates.

A graphical method for determining the Weibull constants σ_u, m, and σ_o is presented in Section 3.3.

3.3 PROCEDURE FOR THE DETERMINATION OF FAILURE PROBABILITY OF A BRITTLE COMPONENT

3.3.1 General

The procedure given is an analysis procedure, not a design procedure, and as such it determines the probability of failure under a given applied stress distribution. Probability of failure for brittle materials is similar to ultimate strength for metallic materials, and a safety factor or margin of safety can be expressed if a criterion expressing the required probability of failure is available. Clearly the procedure can be used for design by repeated application in conjunction with adjustments of the design.

The procedure involves two steps: (i) the determination of material characteristics from experimental data on small simple test bars exposed to a simple loading situation, and (ii) the application of these material characteristics to the determination of failure probability. The material characteristics are the constants in the Weibull expression, and must be determined separately for each environmental condition of interest. Such conditions will generally involve at least different temperatures.

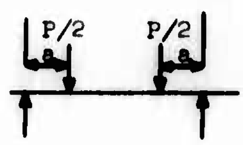
The procedure covers the completely general case, in which the component may be of any size and shape, the applied stresses may have any distribution, and the proof test, if used, may produce any stress distribution.

3.3.2 Determination of Weibull Parameters

(i) For each temperature or other environmental parameter of interest conduct four-point bending tests to failure, on rectangular or square section test bars. To properly define the material characteristics for each condition of interest, the application of the material must be considered.

(ii) From the measured failure loads, determine the extreme fiber stresses at failure using the simple bending stress formula

$$\sigma_b = \frac{3Pa}{bd^2} ,$$



where σ_b is maximum bending stress at failure,

P is total applied load at failure,

a is dimension shown,

b is width of test bar,

d is depth of test bar,

A more refined expression for failure stress is given in Section 8.

(iii) Determine, for each test specimen, the value of the terms

$$\left[\log \log \left(\frac{1}{1 - S_n}\right) + \log \sigma_{bn}\right] \text{ and } \log \sigma_{bn}$$

To facilitate the evaluation of $\log \log [1/(1 - S_n)]$ for test specimen groups of 10, 20, and 30, Table 3.1 is presented.

(iv) Plot values of $\left[\log \log [1/(1 - S_n)] + \log \sigma_{bn}\right]$ against the corresponding values of $\log \sigma_{bn}$. If the resulting curve is a straight line, the material does not have a finite zero strength σ_u. A concave downward curve indicates the material does have a finite zero strength and a tentative value is taken for σ_u and $\log (\sigma_b - \sigma_u)$ is calculated and the test data replotted. If the tentative value for σ_u is too large the resulting curve will be concave upwards and smaller values for σ_u must be taken until the resulting curve approximates a straight line. At this point use the

method of least squares to determine the slope of the straight line and the goodness of fit of the test data to the straight line for values of σ_u above and below the graphically determined value. Select the value for σ_u which gives the best fit of the test data go a straight line. Having selected the best curve, compute m from

$$\text{slope} = (m + 1) \ .$$

(v) With σ_u and m established, σ_o is calculated from Equation (19), using the intercept of the straight line on the ordinate and the volume of the test specimen which was subjected to the applied stress. The value of the intercept is equated to $\log [V/ |2(m + 1)|] - m \log \sigma_o$ and the equation solved for σ_o .

Further details of the graphical method of determining σ_u, m and σ_o may be found in References 3.14, 3.15 and 3.16.

(vi) Repeat the above procedure for each environmental condition of interest.

3.3.3 Evaluation of Failure Probability for a Structural Component

(i) Determine the distribution of principal stresses and temperature throughout the component for the design condition of interest, and determine also the distribution of principal stresses for the selected proof test. Methods for determining these stresses are given in Section 4.

(ii) Divide the component into elements, the size of each to be selected so that the applied principal stresses and temperatures can be considered uniform, and equal to the maximum values over the element. Orient the x, y and z axes for each element so that σ_x is the maximum positive principal stress.

(iii) Calculate, for each element, the following:

(a) If the material is isotropic, and the stresses are uniaxial

$$\left[\left(\frac{\sigma - \sigma_u}{\sigma_o}\right)^m - \left(\frac{\sigma_p - \sigma_u}{\sigma_o}\right)^m\right] V \ .$$

(b) If the material is isotropic and the stresses are triaxial

$$\left[B(\sigma_x) - B(\sigma_{px})\right] \ ,$$

where
$$B = KV \sigma_x^{\,m} \frac{I}{I(\sigma_u = \sigma_y = \sigma_z = o)}$$

and $K = 1/\sigma_o^{\,m}$ and the term in brackets is evaluated from Figures 3.2 to 3.6.

(c) If the material is strongly anistropic the stresses must be resolved in the directions of the principal material axes and the following value calculated for each direction:

$$\left[\left(\frac{\sigma - \sigma_u}{\sigma_o}\right)^m - \left(\frac{\sigma_p - \sigma_u}{\sigma_o}\right)^m\right] V \ ,$$

where σ_u, σ_o and m will have different values for the different directions.

(iv) Sum the above values over the entire component and the result is the failure probability.

3.4 SAMPLE PROBLEM

An example of the method of computing failure probability of a structural component, as defined in the previous section, is presented below. A relatively complex application has been chosen in order to bring out all details of the method. The component selected is a segment of wing leading edge of a lifting re-entry vehicle. Figure 3.7 shows this segment schematically, together with principal dimensions and the integral lugs for attachment to the wing structure. The leading edge is subjected to both aerodynamic loadings and nonuniform heating which produces temperature gradients and thermal stresses. The total stresses have been calculated by the methods described in Section 4 which involves the division of the structural component into a number of small elements. Figure 3.8 shows the leading edge skin, developed into a flat plate, and it shows also the elements which were used for the stress analysis, and which will be used for the failure probability analysis. The volume of each element, together with the average three principal stresses acting at the center of each element, are listed in Table 3.2. Figure 3.9 presents the material properties m and σ_u as a function of temperature and appropriate values corresponding to the temperature of each element are listed in Table 3.2. The remaining material constant σ_o, which is used in the calculation of K, has been assumed constant with the value of 15,000 lb/in^2. The calculations required by the previously described procedure

are completed for each element in Tables 3.2 - 3.4 and summed to give a failure probability of 0.010. In practice this value would be much too high. An m value of approximately 3 indicates a material with considerable variability in mechanical properties, and the use of a peak stress of over 11,000 lb/in^2 in a material which has a mean strength of only 12,000 to 13,000 lb/in^2 would not be expected to lead to high reliability. Therefore, for this particular example some modification of the design is indicated.

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TABLE 3.1

EVALUATION OF log log $\left(\frac{1}{1 - S}\right)$ FOR 10, 20 AND 30 SPECIMENS

n	N = 30	N = 20	N = 10
1	-1.84645	-1.16703	-1.38307
2	-1.53812	-1.36300	-1.05973
3	-1.35451	-1.17380	-0.85917
4	-1.22183	-1.03851	-0.70709
5	-1.11694	-0.92795	-0.57965
6	-1.02952	-0.83475	-0.46544
7	-0.05406	-0.75319	-0.35721
8	-0.88725	-0.68275	-0.24851
9	-0.82696	-0.61510	-0.13056
10	-0.77171	-0.55181	-0.01761
11	-0.72946	-0.49161	
12	-0.67239	-0.43469	
13	-0.62689	-0.37889	
14	-0.58347	-0.32216	
15	-0.54172	-0.26469	
16	-0.50128	-0.20648	
17	-0.46185	-0.14329	
18	-0.42314	-0.07488	
19	-0.38487	0.00763	
20	-0.34677	0.12016	
21	-0.30855		
22	-0.26988		
23	-0.23037		
24	-0.18953		
25	-0.14672		
26	-0.10099		
27	-0.05087		
28	0.00624		
29	0.07580		
30	0.17381		

N = Total number of specimens tested.

n Rank of specimen fracture stresses when arranged in increasing order from 1 to N.

TABLE 3.2

SAMPLE PROBLEM

ELEM	TEMP	σ_x	σ_y	σ_z	m_1	σ_u	σ_u/σ_x	VOL.
1_i	490	9300	-1400	-4000	2.02	5960	0.642	0.0469
1_o	540	11400	+1400	-4000	2.03	5940	0.520	0.0469
2_i	550	9750	-2000	-2900	2.03	5930	0.608	0.0469
2_o	620	7450	-1600	-2000	2.05	5910	0.794	0.0469
3_i	690	4000	0	-4000	2.06	5900	1.472	0.0469
3_o	740	-1000	0	-7100	2.08	5880	-5.88	0.0469
4	940	-3800	0	-11400	2.16	5800	-1.526	0.1406
5	1275	-3500	0	-10300	2.38	5610	-1.602	0.1406
6	1800	4300	0	-4300	3.00	5700	1.187	0.1875
7_i	2210	6300	0	-3600	3.85	4400	0.700	0.0937
7_o	2270	8800	+3100	0	4.07	4280	0.487	0.0937
8_i	2300	6000	0	-3000	4.18	4200	0.700	0.0703
8_o	2360	7600	+4500	0	4.45	4050	0.534	0.0703
9_i	2240	5600	-1000	-3600	3.97	4350	0.775	0.1170
9_o	2340	9200	+300	-1000	4.35	4100	0.445	0.1170
10_i	2160	8000	-2000	-4600	3.72	4510	0.565	0.1562
10_o	2250	9700	-400	-2000	4.0	4320	0.446	0.1562
11	1790	5600	0	-5600	2.96	5120	0.915	0.3125
12	1275	0	-4000	-4000	2.38	5610	∞	0.234
13	940	0	-6000	-6000	2.16	5800	∞	0.234
14_i	700	2900	0	-2600	2.06	5890	2.03	0.0782
14_o	750	5000	0	-4200	2.08	5880	1.176	0.0782
15_i	570	6700	-1000	-3600	2.04	5930	0.885	0.0782
15_o	630	8200	-1000	-3900	2.05	5900	0.720	0.0782
16_i	480	7000	-2000	-2900	2.02	5950	0.850	0.0782
16_o	550	9700	-1000	-2000	2.03	5930	0.612	0.0782
17_i	510	5400	0	-5300	2.02	5940	1.10	0.0625
17_o	560	7800	0	-5300	2.03	5930	0.760	0.0625
18_i	600	7000	0	-4800	2.04	5920	0.846	0.0625
18_o	640	10700	0	-7200	2.05	5900	0.551	0.0625
19_i	720	7000	0	-4700	2.07	5880	0.841	0.0625
19_o	760	9700	0	-7400	2.09	5870	0.605	0.0625
20	940	7200	0	-4000	2.16	5800	0.805	0.1875
21	1275	8600	0	-4400	2.38	5610	0.653	0.1875
22	1780	9700	0	-5250	2.95	5130	0.529	0.250
23_i	2120	7000	-4000	-4450	3.62	4600	0.658	0.125
23_o	2240	6300	-3350	-4000	3.97	4340	0.688	0.125
29_i	2190	3100	-3000	-3100	3.80	4450	1.435	0.0937
29_o	2310	6000	-3000	-5850	4.22	4180	0.696	0.0937
					(m-1)	Fig. 3.9		Fig. 3.8

TABLE 3.3

SAMPLE PROBLEM

ELEM	m_1	σ_u/σ_x	σ_y/σ_x	σ_z/σ_x	I	$I_{(o)}$	$I/I_{(o)}$
1_i	2.02	0.642	-0.1506	-0.430	0.08	2.47	0.0323
1_o	2.03	0.520	0.1228	-0.351	0.217	2.45	0.0886
2_i	2.03	0.608	-0.205	-0.298	0.103	2.45	0.0420
2_o	2.05	0.794	-0.215	-0.268	0.0137	2.43	0.00563
3_i	2.06	1.472	0	-1.0	0	2.43	0
3_o	2.08	-5.88	0	-7.1	0	2.42	0
4	2.16	-1.526	0	-3.0	0	2.36	0
5	2.38	-1.602	0	-2.94	0	2.20	0
6	3.00	1.187	0	-1.0	0	1.78	0
7_i	3.85	0.700	0	-0.571	0.007	1.44	0.00486
7_o	4.07	0.487	0.352	0	0.0246	1.37	0.01796
8_i	4.18	0.700	0	-0.50	0.00291	1.33	0.00219
8_o	4.45	0.534	0.592	0	0.00832	1.26	0.00660
9_i	3.97	0.775	-0.1781	-0.643	0.00066	1.39	0.000475
9_o	4.35	0.445	0.0326	-0.1089	0.00831	1.28	0.00649
10_i	3.72	0.565	-0.250	-0.575	0.029	1.48	0.01958
10_o	4.0	0.446	-0.0413	-0.206	0.057	1.39	0.0410
11	2.96	0.915	0	-1.0	0.0031	1.81	0.001711
12	2.38	∞	$-\infty$	$-\infty$	0	2.20	0
13	2.16	∞	$-\infty$	$-\infty$	0	2.36	0
14_i	2.06	2.03	0	-0.897	0	2.43	0
14_o	2.08	1.176	0	-0.840	0	2.42	0
15_i	2.04	0.885	-0.1492	-0.538	0.0074	2.44	0.00303
15_o	2.05	0.720	-0.122	-0.475	0.041	2.43	0.01687
16_i	2.02	0.850	-0.286	-0.415	0.0085	2.47	0.00344
16_o	2.03	0.612	-0.103	-0.206	0.116	2.45	0.0473
17_i	2.02	1.10	0	-0.982	0	2.47	0
17_o	2.03	0.760	0	-0.680	0.025	2.45	0.0102
18_i	2.04	0.846	0	-0.685	0.0078	2.44	0.0032
18_o	2.05	0.551	0	-0.672	0.150	2.43	0.0617
19_i	2.07	0.841	0	-0.671	0.0105	2.42	0.00434
19_o	2.09	0.605	0	-0.763	0.1002	2.41	0.0416
20	2.16	0.805	0	-0.555	0.0138	2.35	0.00586
21	2.38	0.653	0	-0.512	0.0157	2.20	0.00714
22	2.95	0.529	0	-0.540	0.101	1.80	0.0560
23_i	3.62	0.658	-0.572	-0.635	0.0646	1.50	0.04307
23_o	3.97	0.688	-0.532	-0.635	0.00266	1.39	0.001911
24_i	3.80	1.435	-0.967	-1.00	0	1.45	0
24_o	4.22	0.696	-0.50	-0.975	0.00226	1.31	0.001723
					Fig. 3.2 to 3.6	Fig. 3.1	

TABLE 3.4

SAMPLE PROBLEM

ELEM	m	V	σ_x	σ_x / σ_o	$(\sigma_x / \sigma_o)^m$	$I/I_{(o)}$	B_1
1_i	3.02	0.0469	9300	0.620	0.235	0.0323	0.000356
1_o	3.03	0.0469	11400	0.760	0.440	0.0886	0.001828
2_i	3.03	0.0469	9750	0.650	0.275	0.0420	0.000541
2_o	3.05	0.0469	7450	0.497	0.120	0.0056	0.000031
3_i	3.06	0.0469	4000	0.267	0.019	0	0
3_o	3.08	0.0469	-1000	0	0	0	0
4	3.16	0.1406	-3800	0	0	0	0
5	3.38	0.1406	-3500	0	0	0	0
6	4.00	0.1875	4300	0.287	0.007	0	0
7_i	4.85	0.0937	6300	0.420	0.015	0.0049	0.000006
7_o	5.07	0.0937	8800	0.587	0.067	0.0180	0.000113
8_i	5.18	0.0703	6000	0.400	0.009	0.0022	0.000001
8_o	5.45	0.0703	7600	0.507	0.027	0.0066	0.000012
9_i	4.97	0.1170	5600	0.373	0.007	0.0005	0.0000004
9_o	5.35	0.1170	9200	0.613	0.072	0.0065	0.000054
10_i	4.72	0.1562	8000	0.533	0.051	0.0196	0.000156
10_o	5.00	0.1562	9700	0.646	0.110	0.0410	0.000704
11	3.96	0.3125	5600	0.373	0.018	0.0017	0.000009
12	3.38	0.234	0	0	0	0	0
13	3.16	0.234	0	0	0	0	0
14_i	3.06	0.0782	2900	0.193	0.007	0	0
14_o	3.08	0.0782	5000	0.333	0.032	0	0
15_i	3.04	0.0782	6700	0.447	0.090	0.0030	0.000021
15_o	3.05	0.0782	8200	0.547	0.160	0.0169	0.000211
16_i	3.02	0.0782	7000	0.467	0.105	0.0034	0.000027
16_o	3.03	0.0782	9700	0.647	0.270	0.0473	0.000998
17_i	3.02	0.0625	5400	0.360	0.046	0	0
17_o	3.03	0.0625	7800	0.520	0.140	0.0102	0.000089
18_i	3.04	0.0625	7000	0.466	0.100	0.0032	0.000020
18_o	3.05	0.0625	10700	0.713	0.350	0.0617	0.001349
19_i	3.07	0.0625	7000	0.466	0.100	0.0043	0.000026
19_o	3.09	0.0625	9700	0.647	0.260	0.0416	0.000676
20	3.16	0.1875	7200	0.480	0.100	0.0059	0.000110
21	3.38	0.1875	8600	0.573	0.150	0.0071	0.000199
22	3.95	0.250	9700	0.647	0.180	0.0560	0.002520
23_i	4.62	0.125	7000	0.466	0.030	0.0431	0.000161
23_o	4.97	0.125	6300	0.420	0.013	0.0019	0.000003
24_i	4.80	0.0937	3100	0.207	0.0005	0	0
24_o	5.22	0.0937	6000	6.400	0.009	0.0017	0.000001

Fig. 3.9 σ_o = 15,000 $\Sigma = 0.010234$

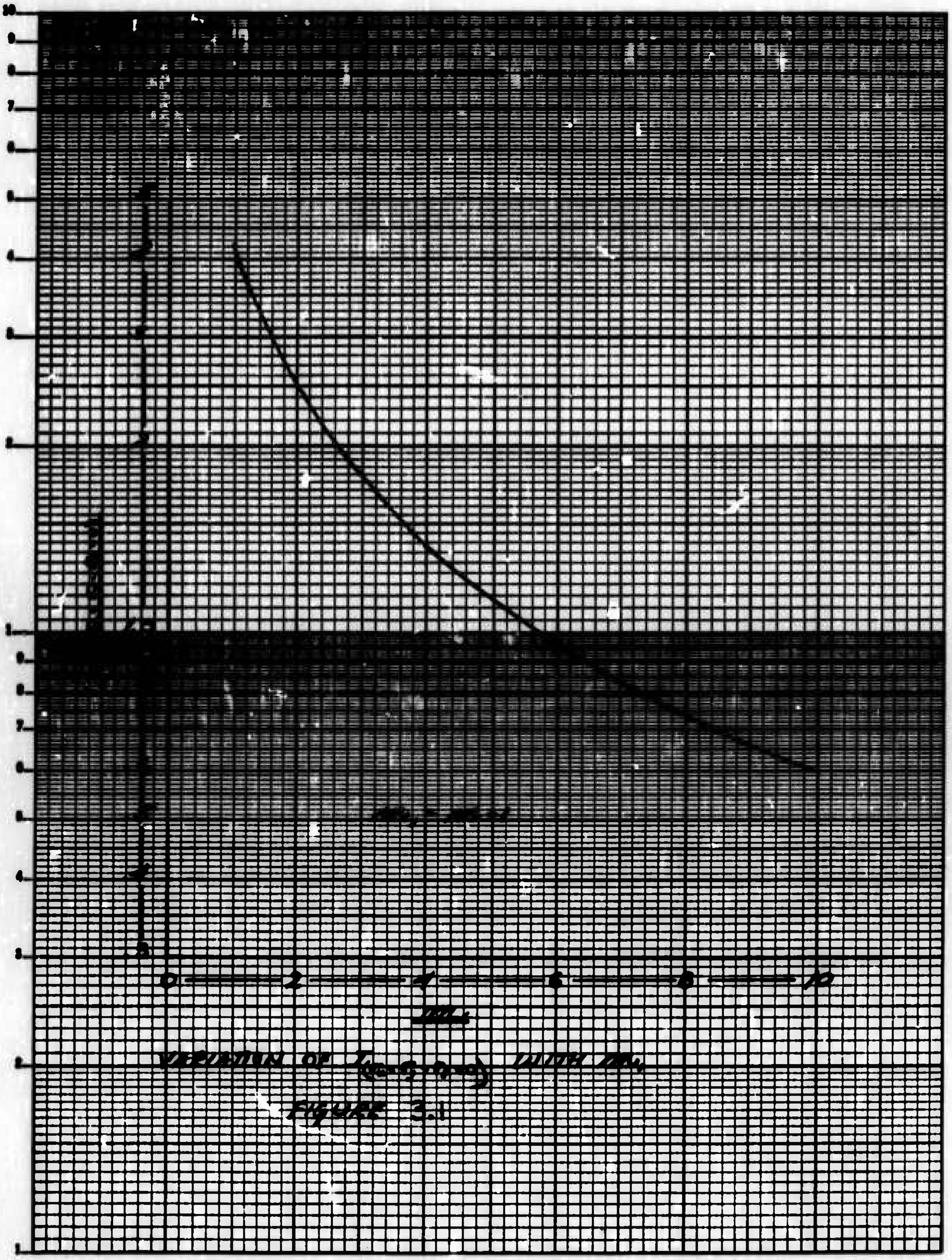


Fig.3.1 Variation of $I_{(\sigma_u=\sigma_y=\sigma_z=0)}$ with m_1

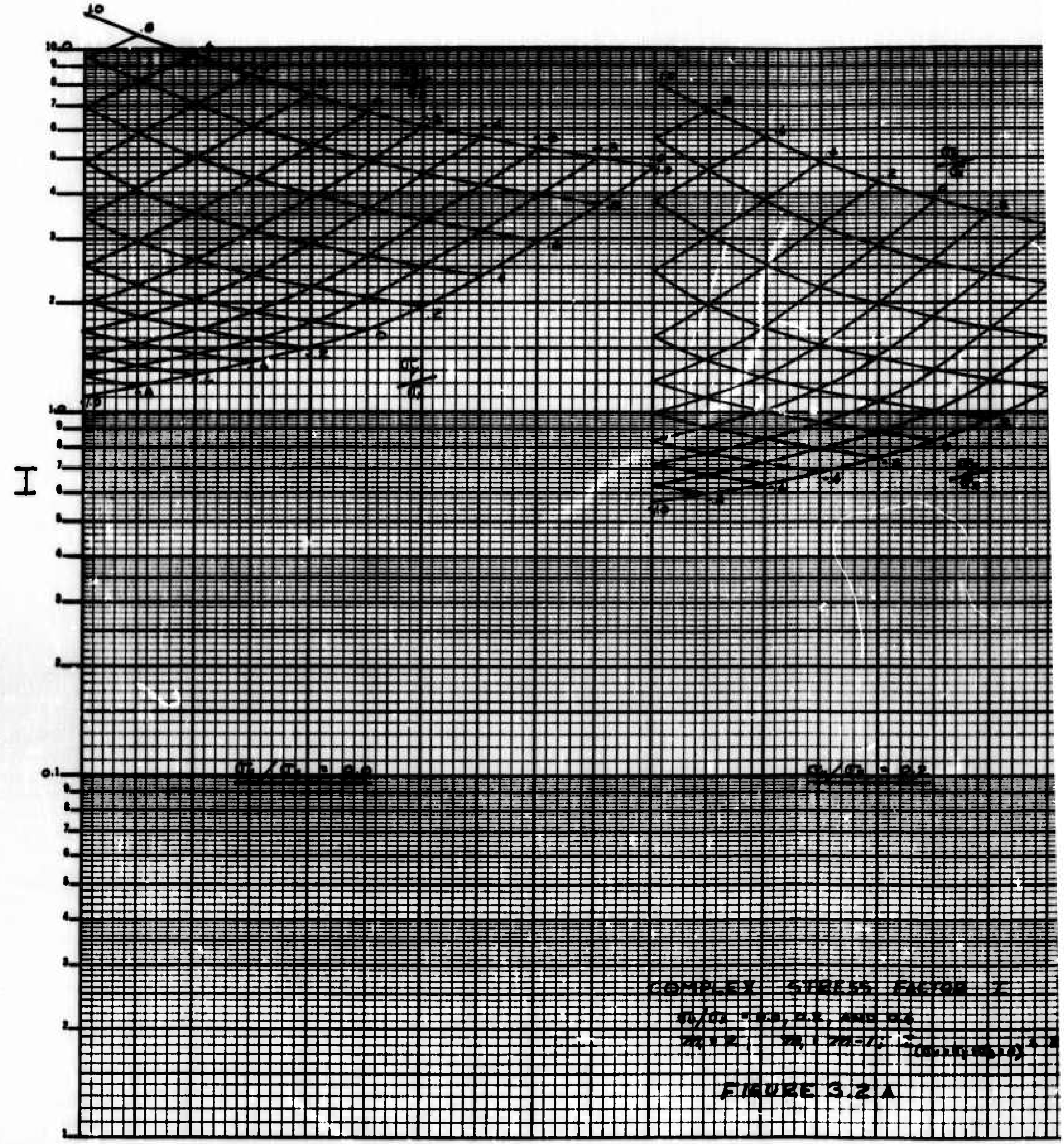
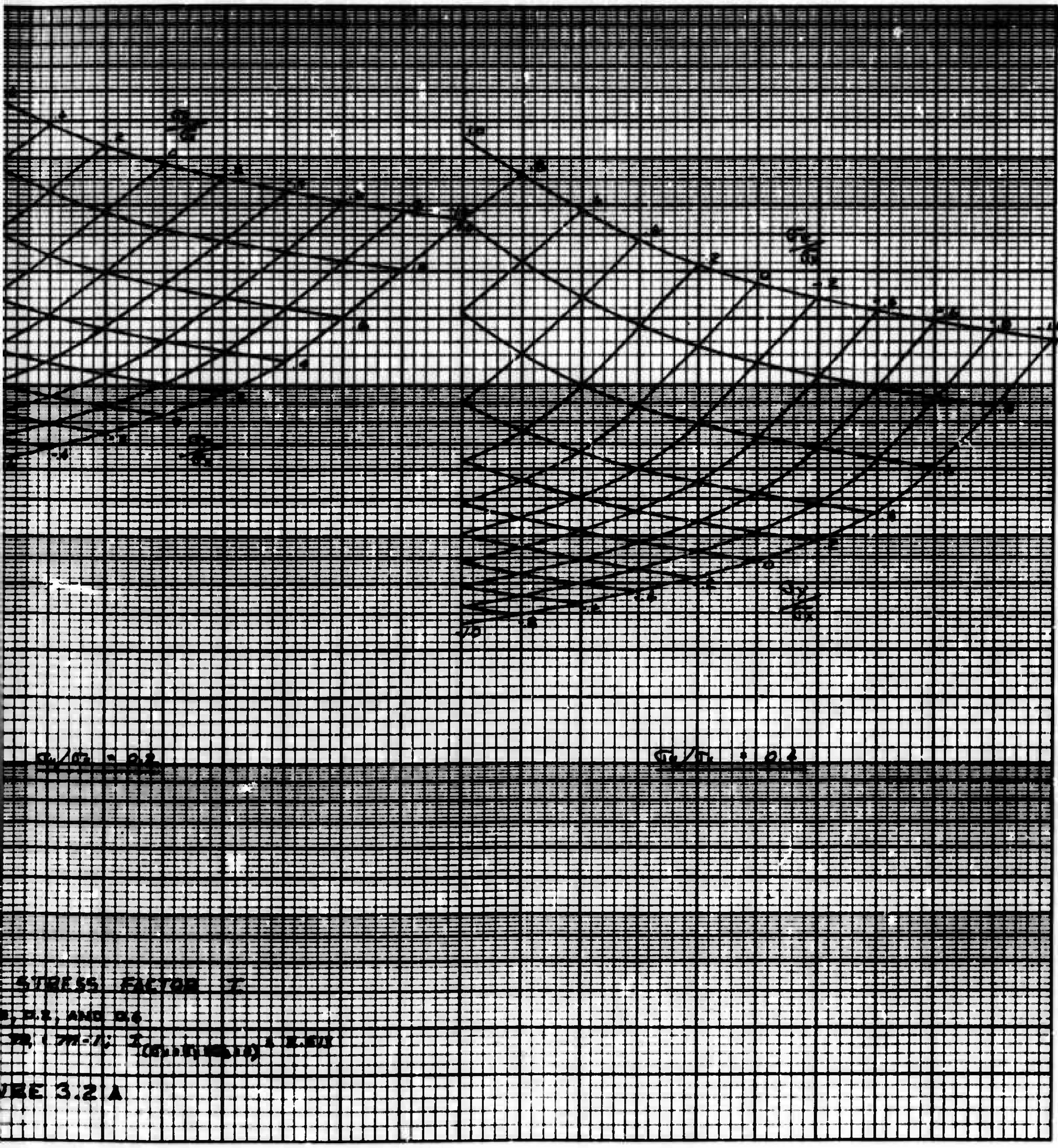


Fig.3.2(a) Complex stress factor I
σ_u/σ_x = 0.0, 0.2, and 0.4
$m_l = 2$; $m_l = m - 1$; $I_{(\sigma_u = \sigma_y = \sigma_{...}}$



Complex stress factor I
σ_u/σ_x = 0.0, 0.2, and 0.4
$m_1 = 2$; $m_1 = m - 1$; $I_{(\sigma_u=\sigma_y=\sigma_z=0)} = 2.513$

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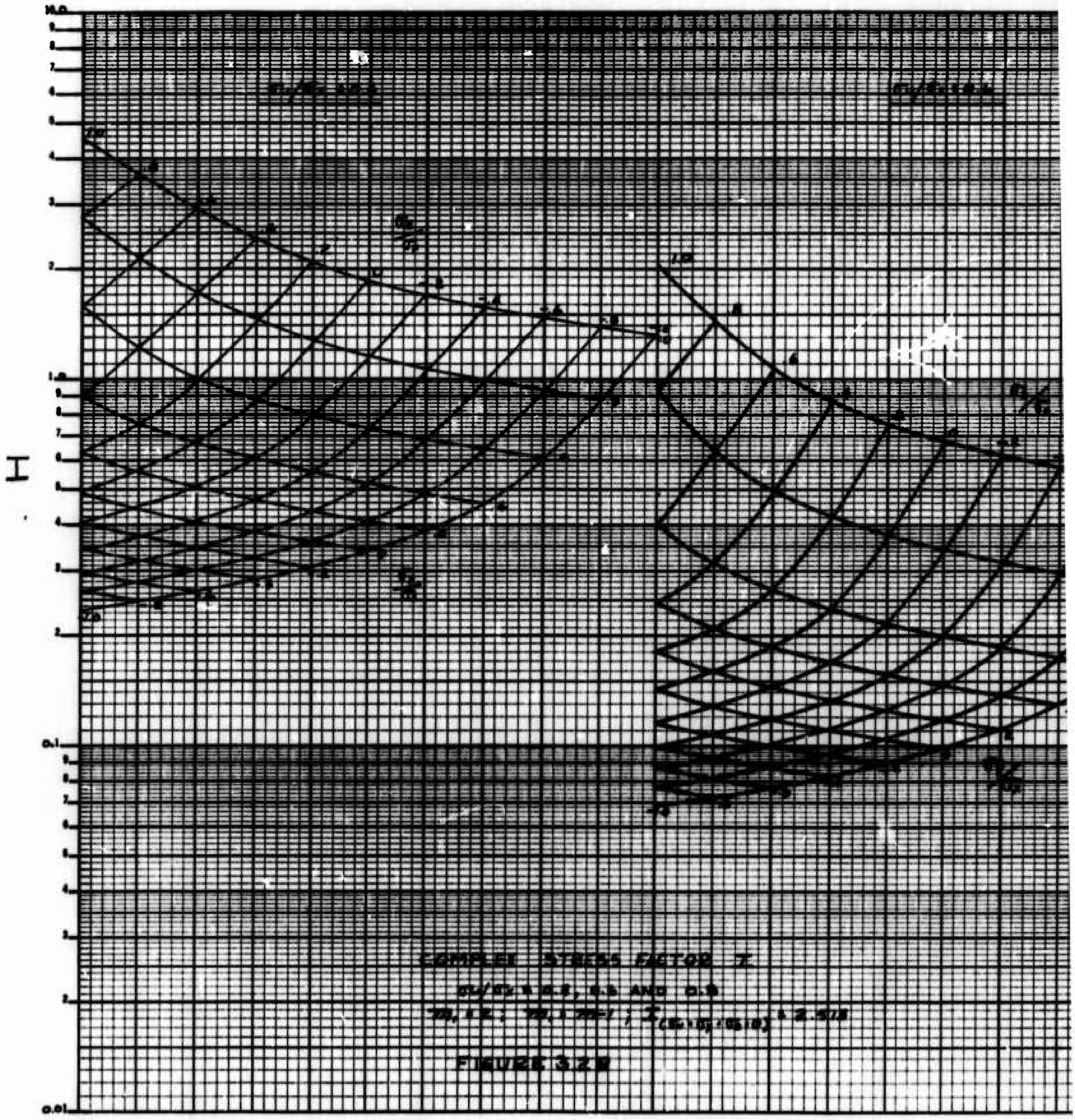
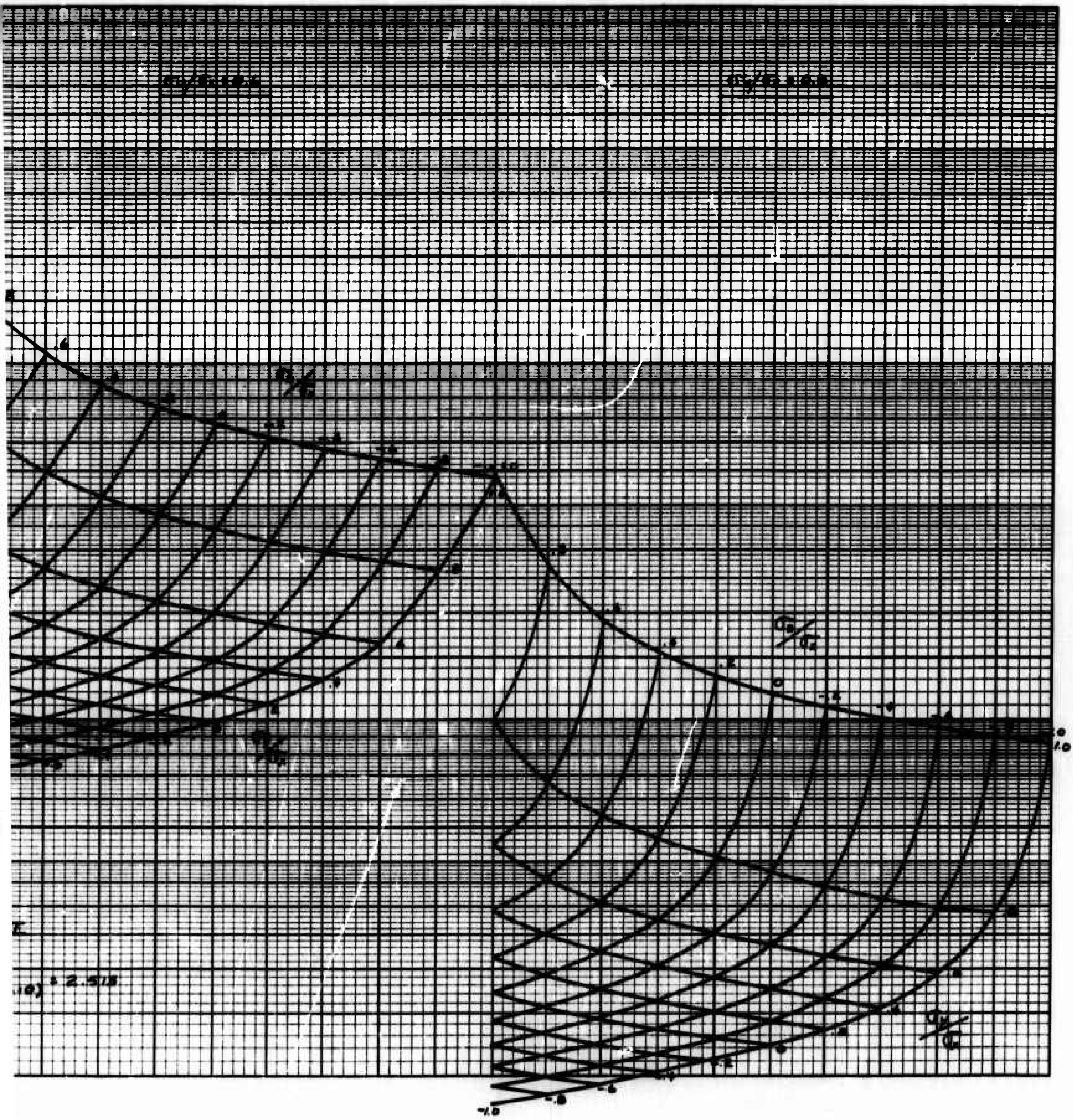


Fig.3.2(b) Complex stress factor I
σ_u/σ_x = 0.4, 0.6 and 0.8
m_1 = 2; m_1 = m − 1; $I_{(\sigma_u=\sigma_y}$



.2(b) Complex stress factor I

σ_u/σ_x = 0.4, 0.6 and 0.8

$m_1 = 2$; $m_1 = m - 1$; $I_{(\sigma_u = \sigma_y = \sigma_z = 0)} = 2.513$

B

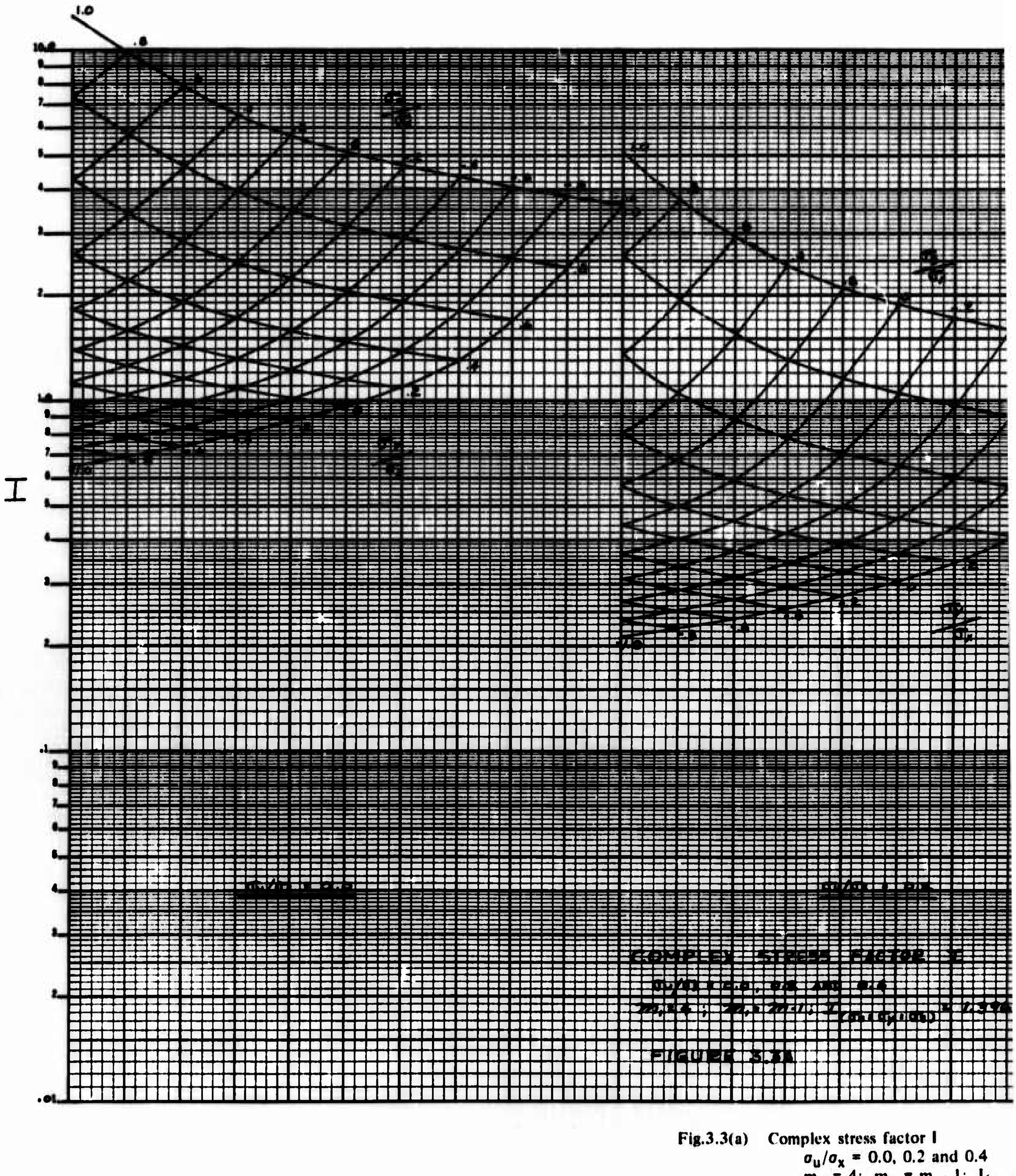
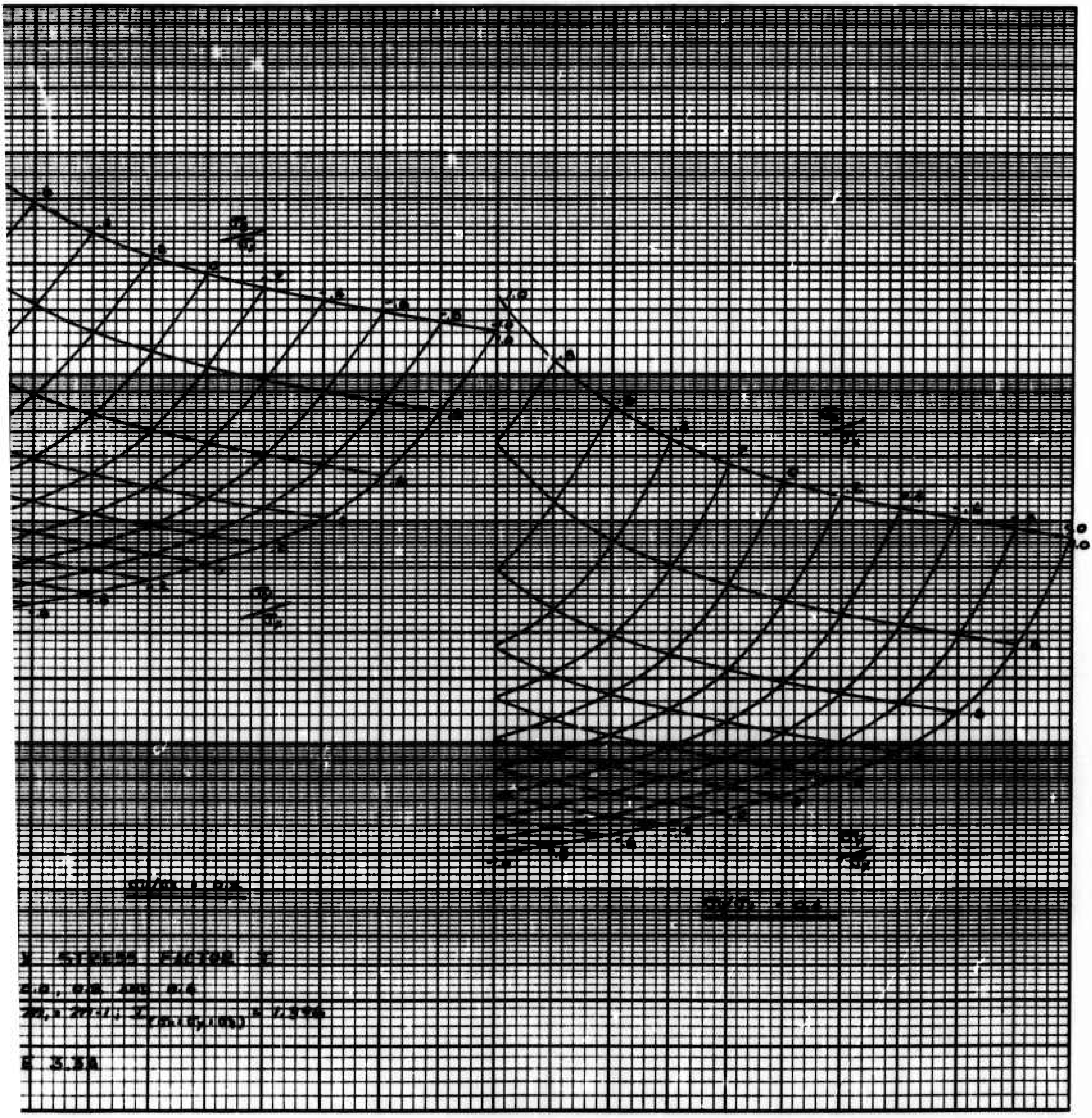


Fig.3.3(a) Complex stress factor I
σ_u/σ_x = 0.0, 0.2 and 0.4
m_1 = 4; m_1 = m − 1; $I_{(\sigma_u}$



3(a) Complex stress factor I

σ_u/σ_x = 0.0, 0.2 and 0.4

$m_1 = 4$; $m_1 = m - 1$; $I_{(\sigma_u=\sigma_y=\sigma_z=0)} = 1.396$

B

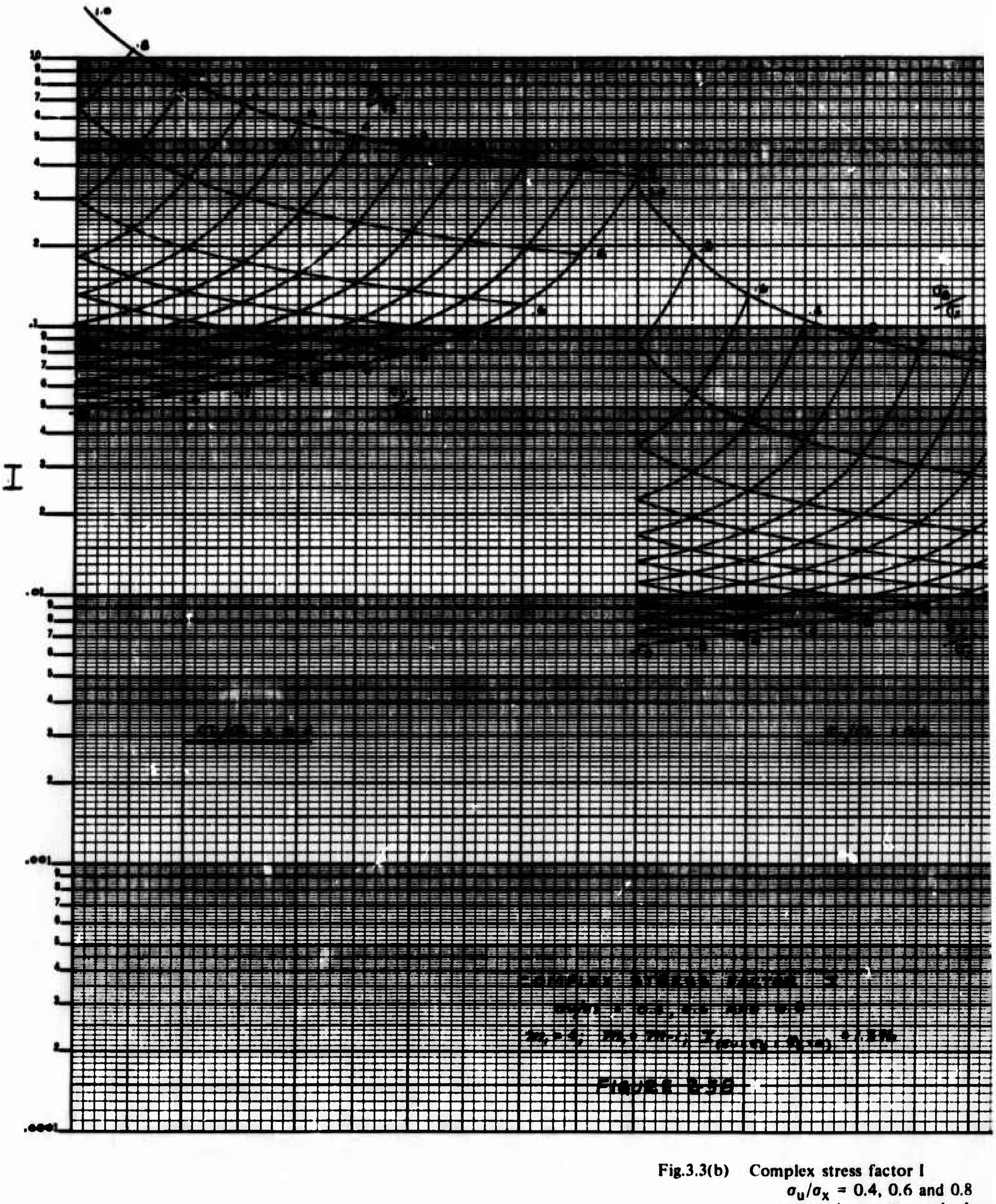
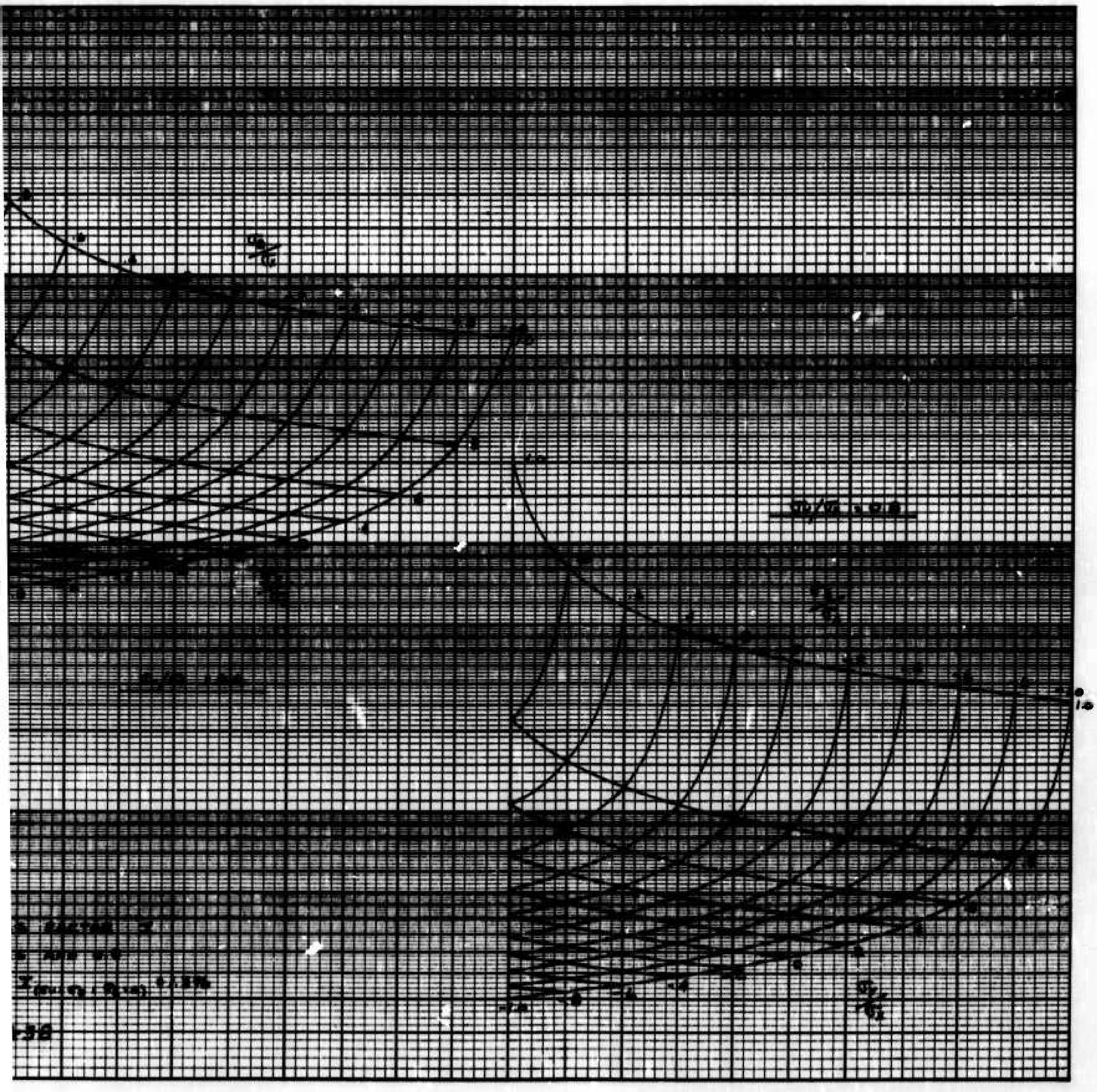


Fig.3.3(b) Complex stress factor I
σ_u/σ_x = 0.4, 0.6 and 0.8
m_1 = 4; m_1 = m − 1; $I_{(\sigma_u}$

A



.3(b) Complex stress factor I
σ_u/σ_x = 0.4, 0.6 and 0.8
$m_l = 4$; $m_l = m - 1$; $I_{(\sigma_u=\sigma_y=\sigma_z=0)} = 1.396$

B

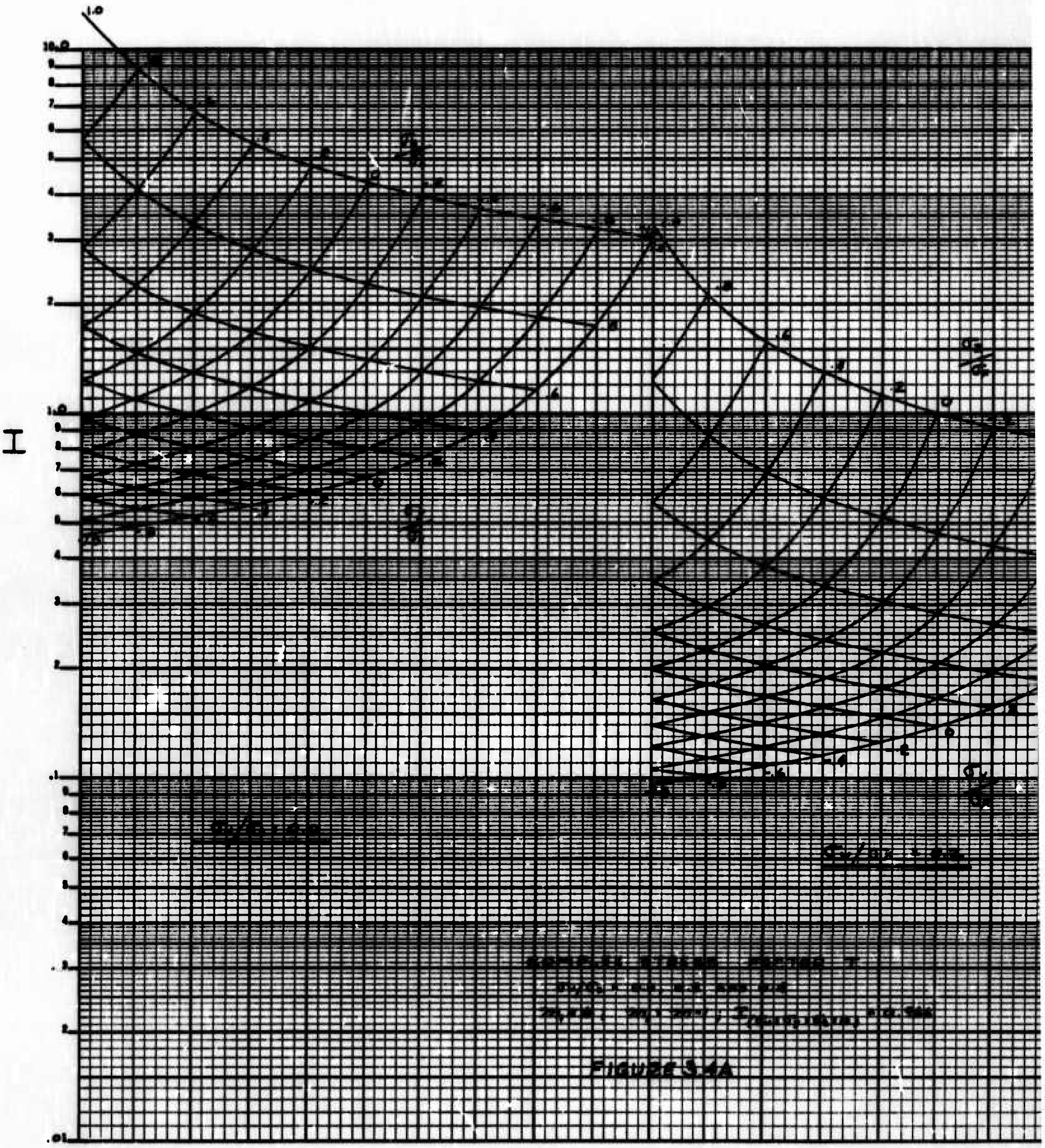
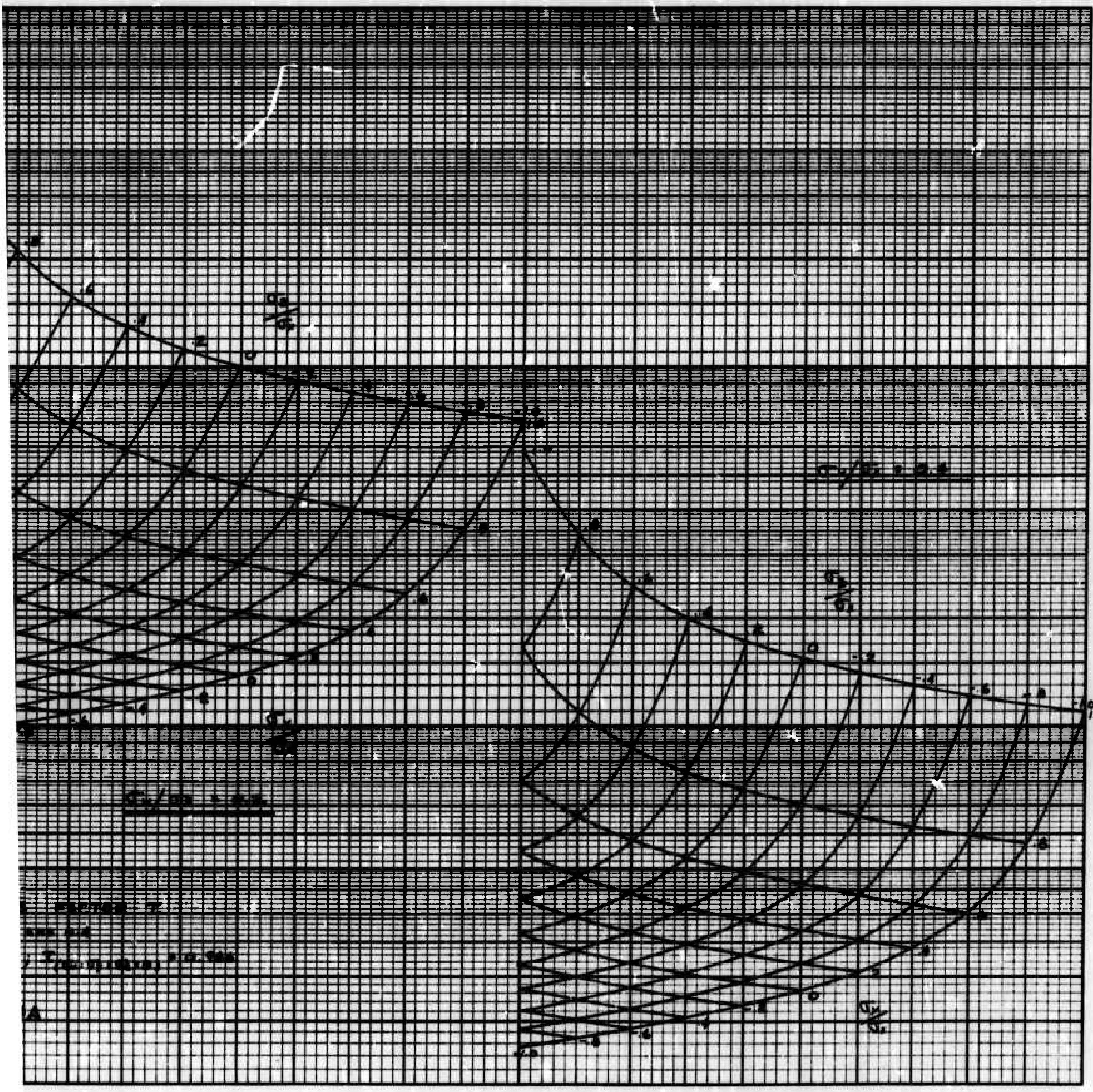


Fig.3.4(a) Complex stress factor I
σ_u/σ_x = 0.0, 0.2 and 0.4
m_1 = 6; m_1 = m – 1; $I_{(\sigma_u=\sigma_{...}}$

A



.4(a) Complex stress factor I
σ_u/σ_x = 0.0, 0.2 and 0.4
$m_1 = 6$; $m_1 = m - 1$; $I_{(\sigma_u=\sigma_y=\sigma_z=0)} = 0.966$

B

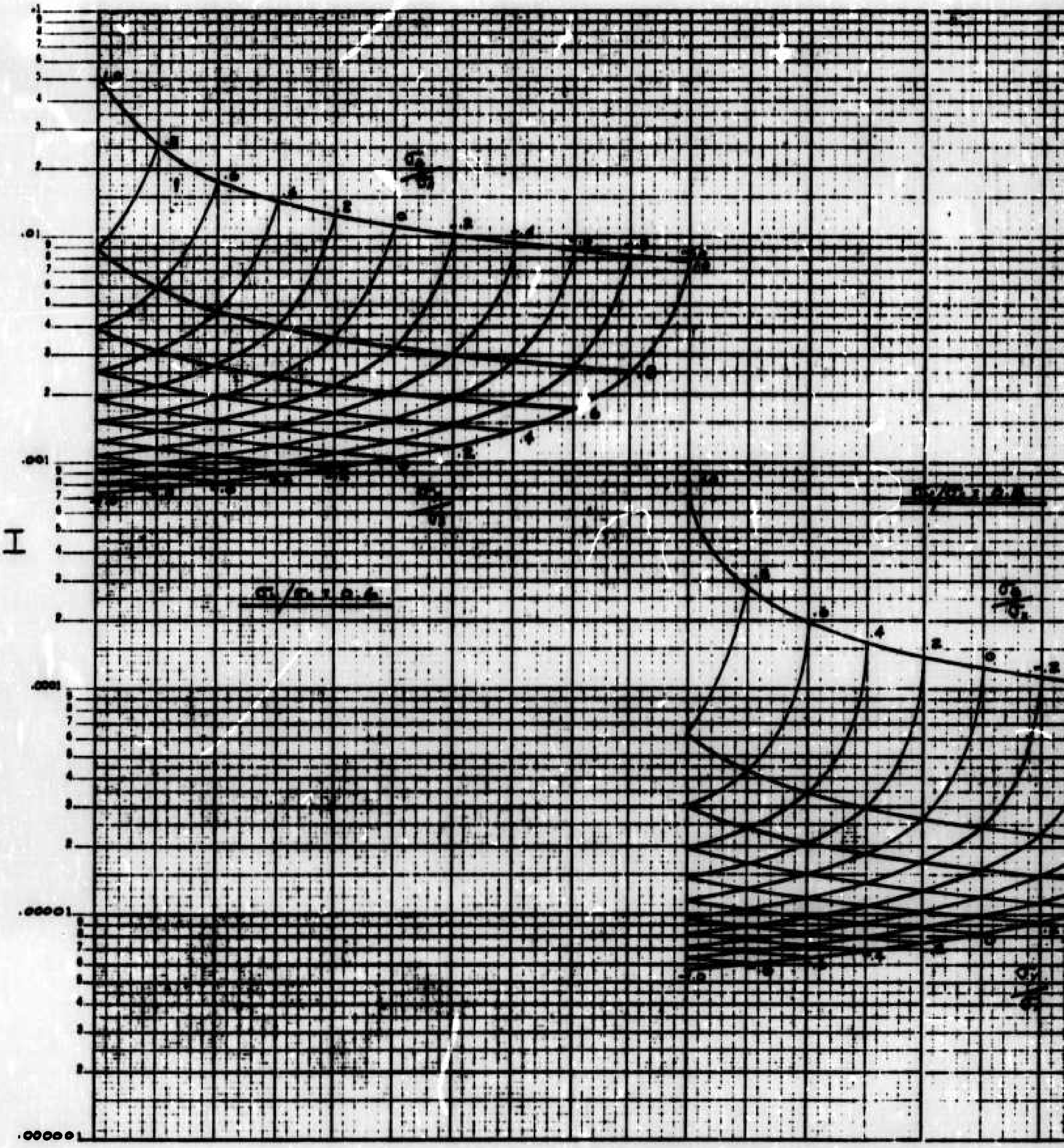
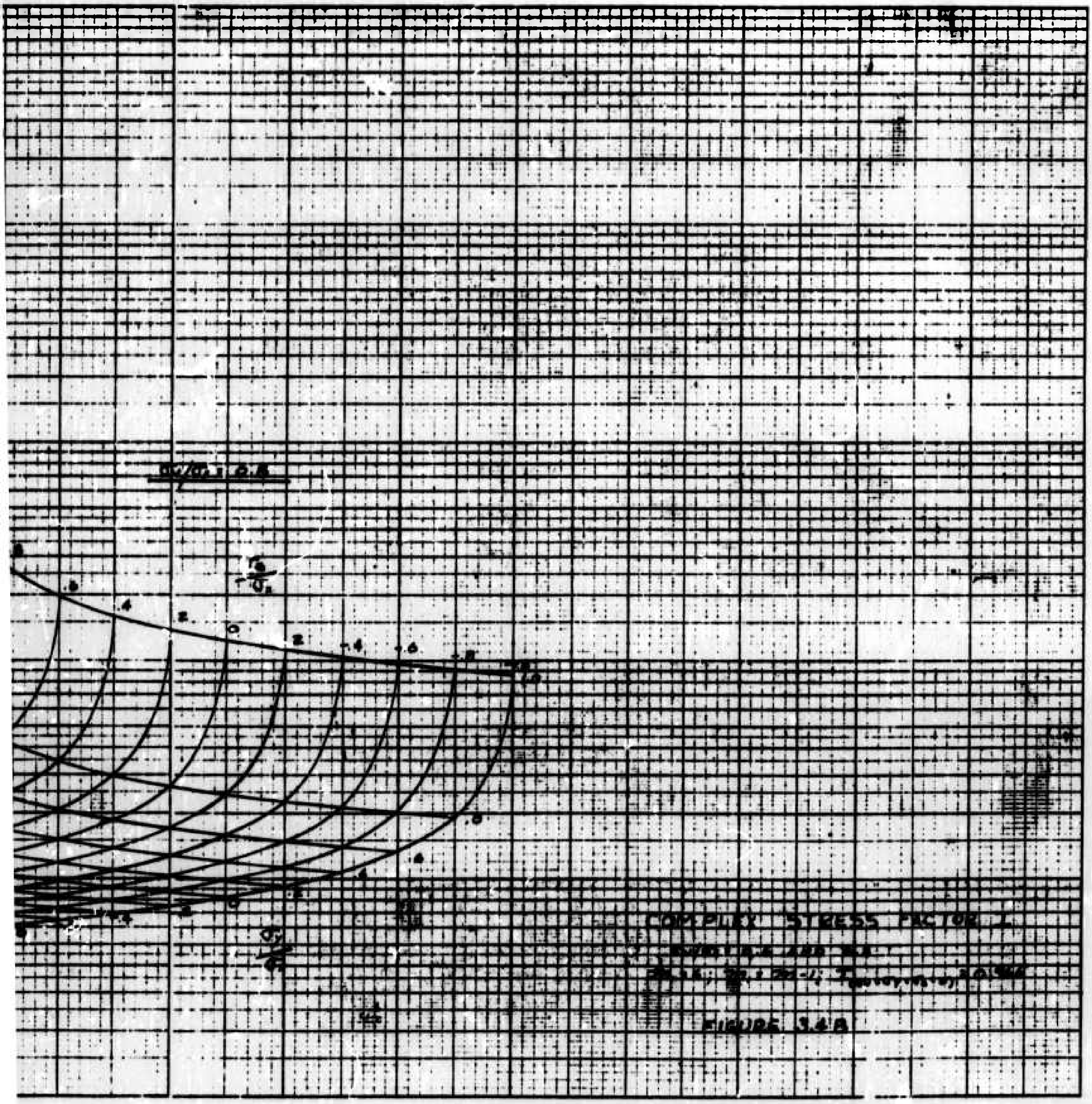


Fig.3.4(b) Complex stress factor I

$\sigma_u/\sigma_x = 0.6$ and 0.8

$m_1 = 6$; $m_1 = m - 1$; $I_{(\sigma_u = \sigma}$

A



Complex stress factor I

$\sigma_u/\sigma_x = 0.6$ and 0.8

$m_1 = 6$; $m_1 = m - 1$; $I_{(\sigma_u = \sigma_y = \sigma_z = 0)} = 0.966$

B

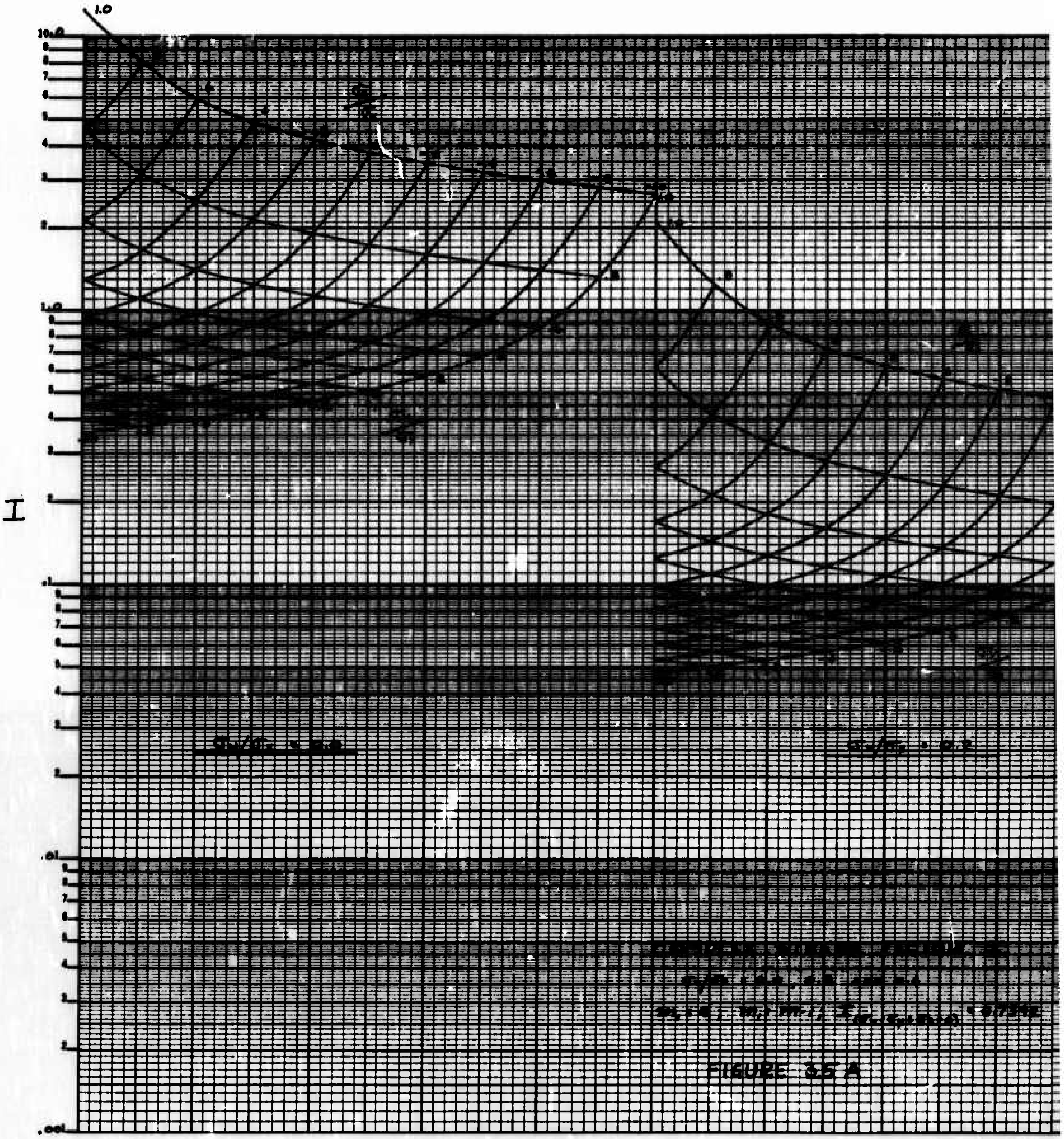
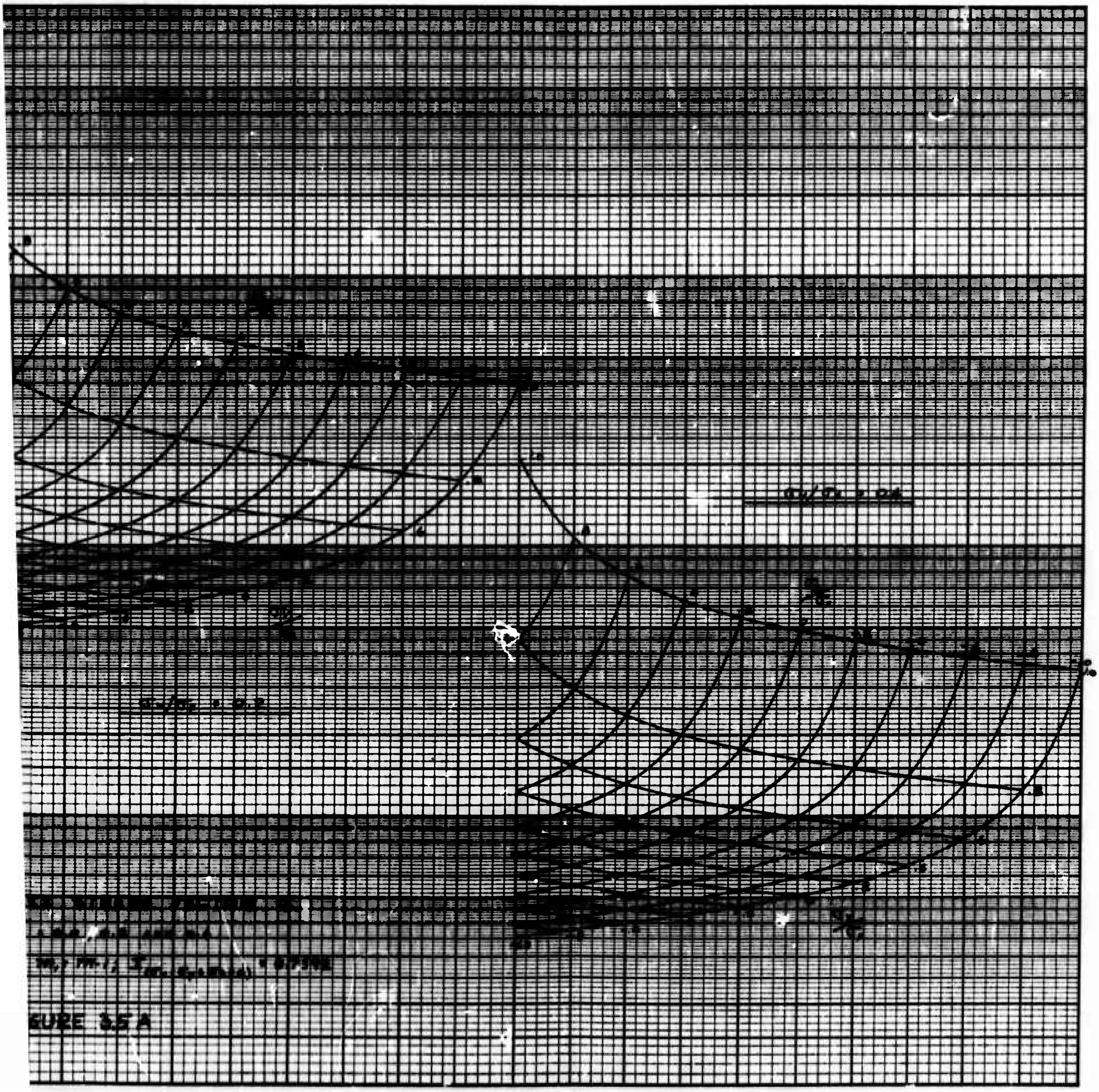


Fig.3.5(a) Complex stress factor I

σ_u/σ_x = 0.0, 0.2 and 0.4

m_1 = 8; m_1 = m − 1; $I_{(\sigma_u = \sigma_y = \sigma_z}$



Complex stress factor I

σ_u/σ_x = 0.0, 0.2 and 0.4

$m_1 = 8$; $m_1 = m - 1$; $I_{(\sigma_u=\sigma_y=\sigma_z=0)} = 0.7392$

B

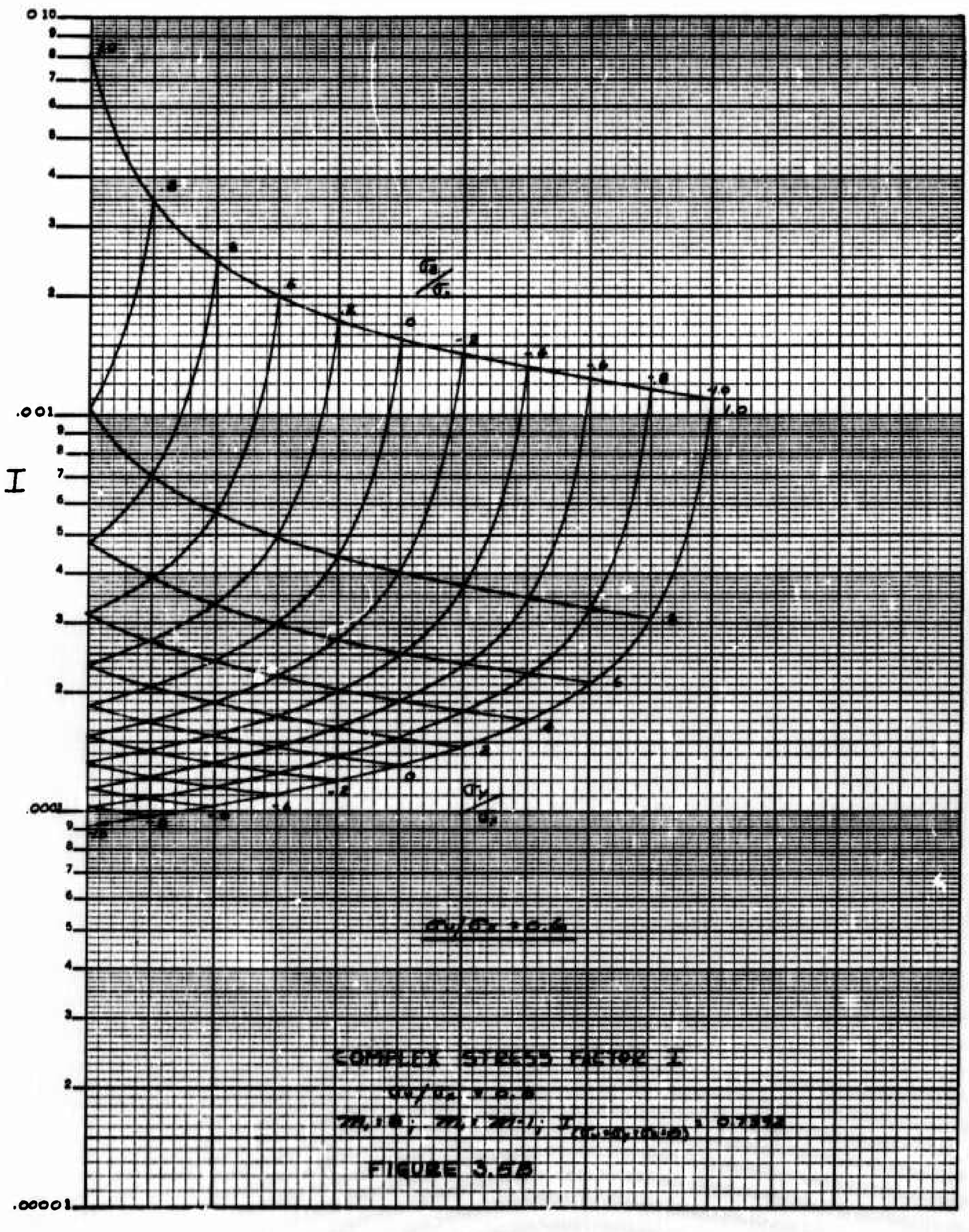


Fig.3.5(b) Complex stress factor I
$\sigma_u/\sigma_x = 0.6$
$m_1 = 8;\ m_1 = m - 1;\ I_{(\sigma_u = \sigma_y = \sigma_z = 0)} = 0.7392$

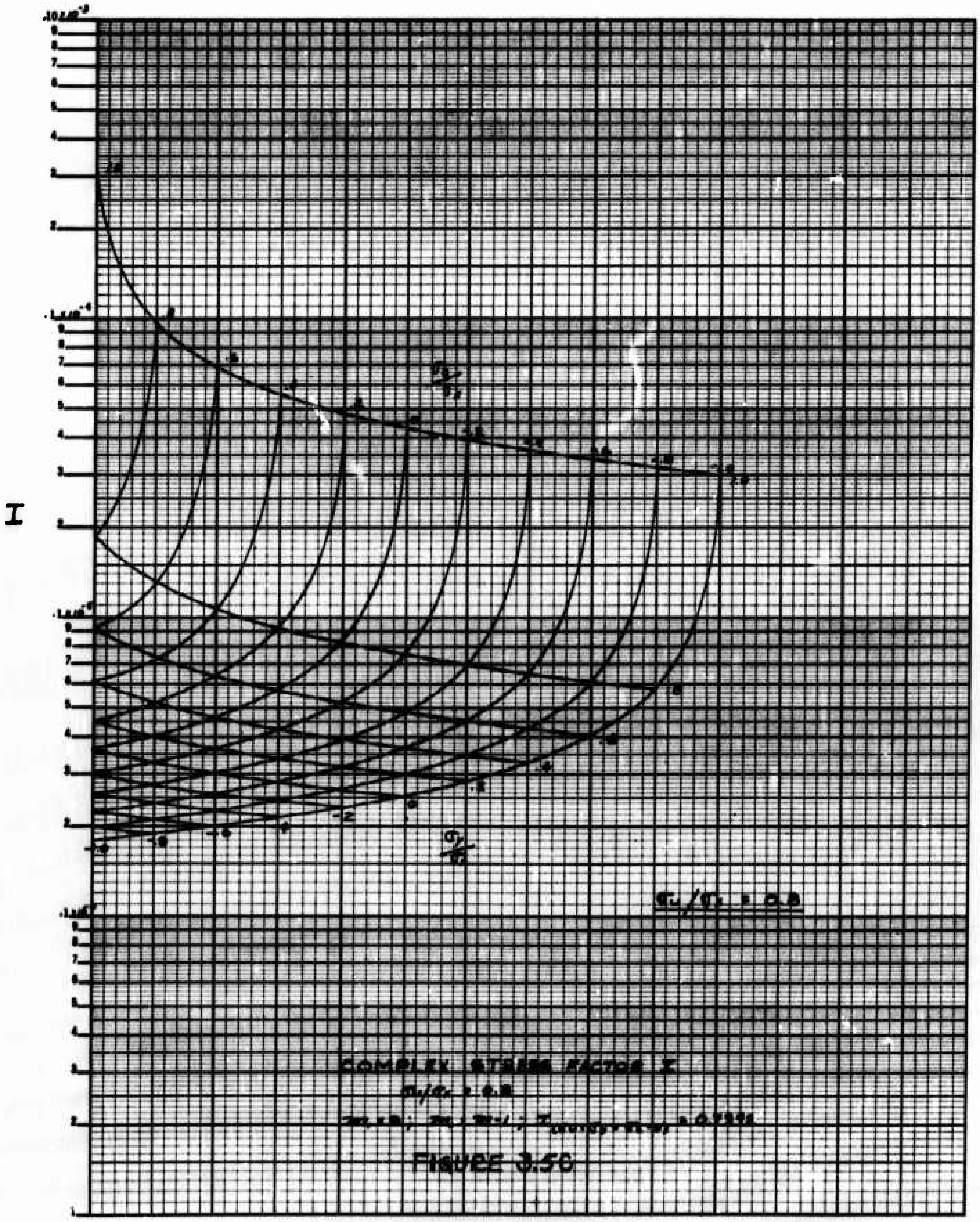


Fig.3.5(c) Complex stress factor I

σ_u/σ_x = 0.8

m_1 = 8; m_1 = m − 1; $I_{(\sigma_u=\sigma_y=\sigma_z=0)}$ = 0.7392

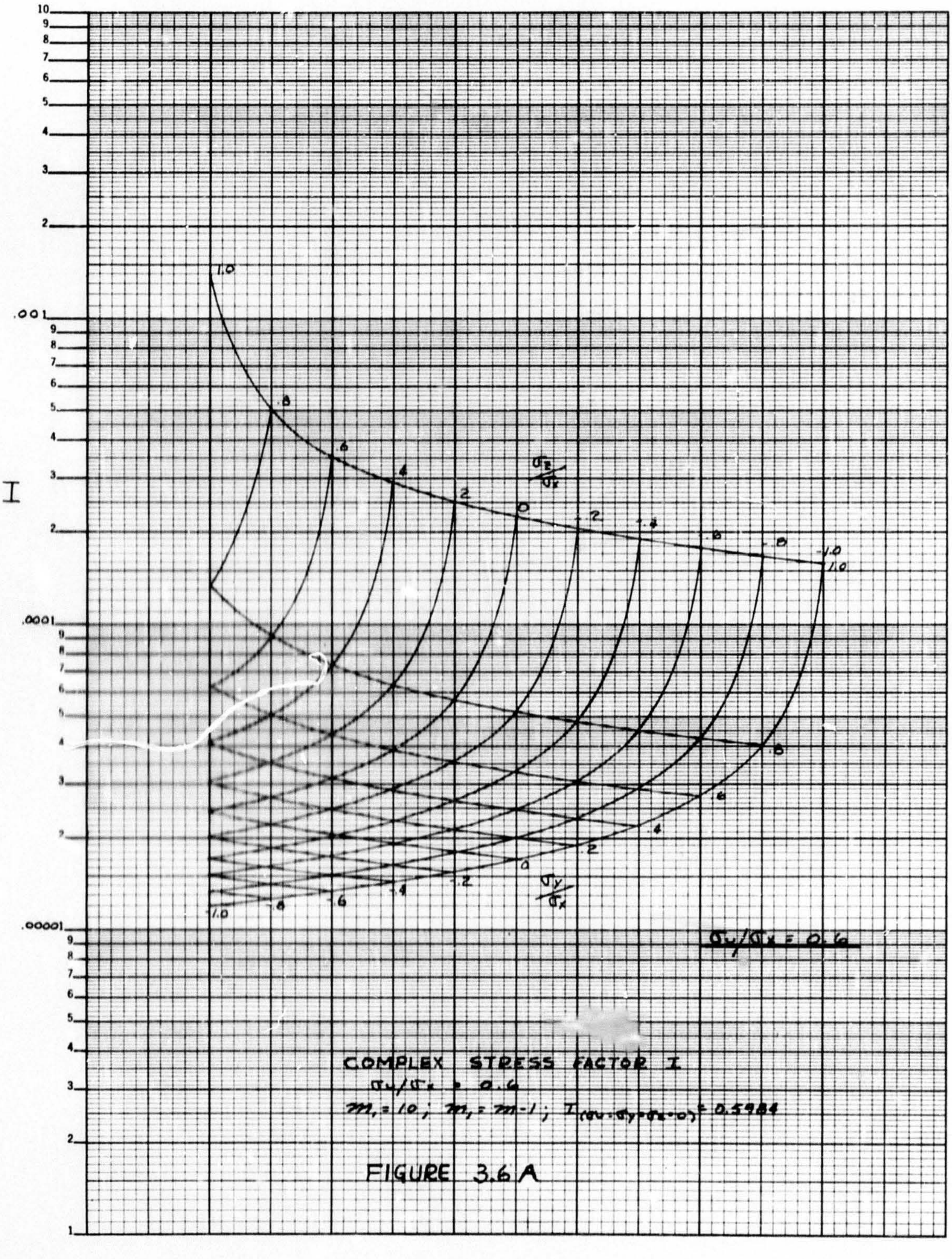


Fig.3.6(a) Complex stress factor I
$\sigma_u/\sigma_x = 0.6$
$m_1 = 10$; $m_1 = m - 1$; $I_{(\sigma_u=\sigma_y=\sigma_z=0)} = 0.5984$

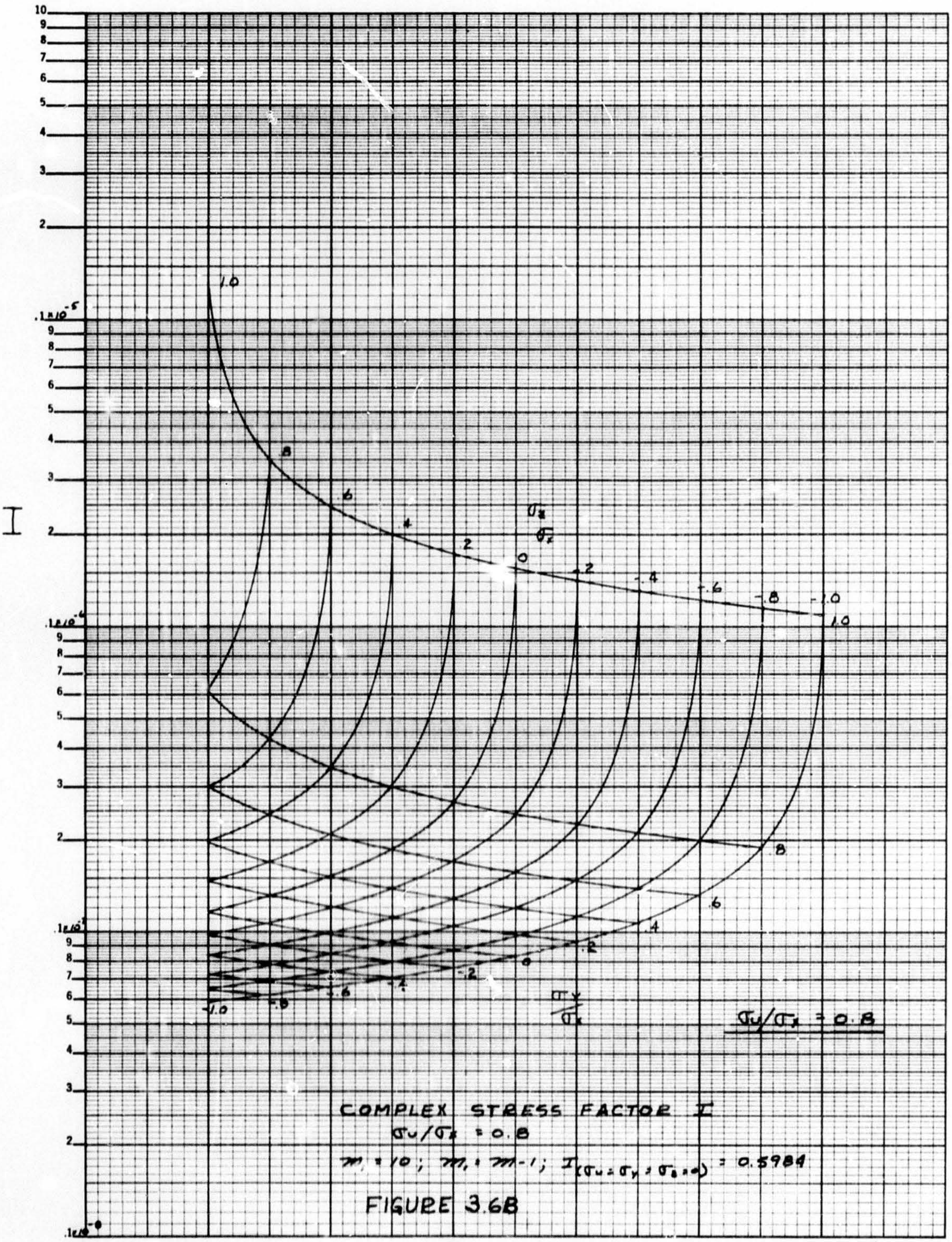


Fig.3.6(b) Complex stress factor I

$\sigma_u/\sigma_x = 0.8$

$m_1 = 10;\ m_1 = m - 1;\ I_{(\sigma_u = \sigma_y = \sigma_z = 0)} = 0.5984$

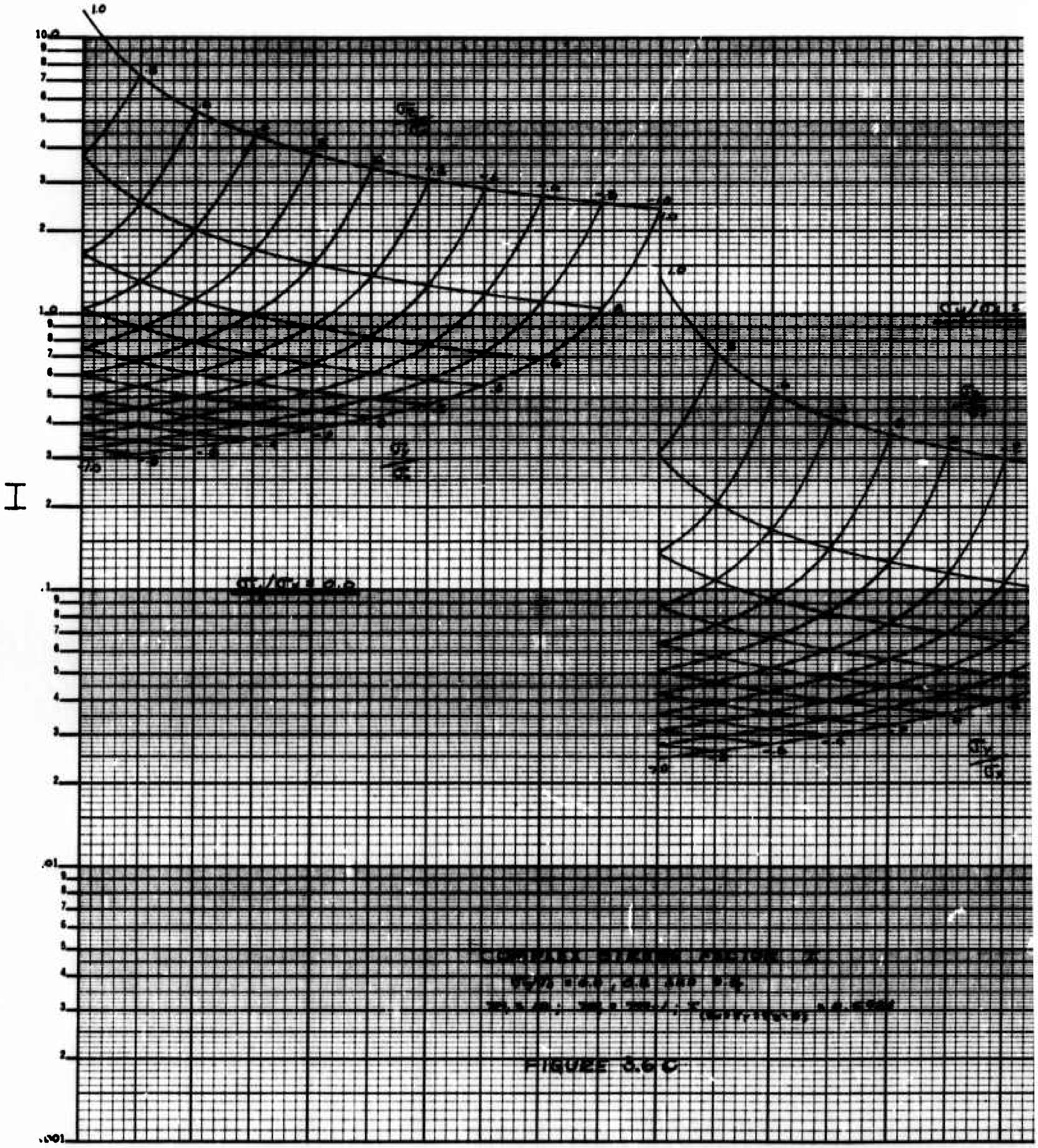
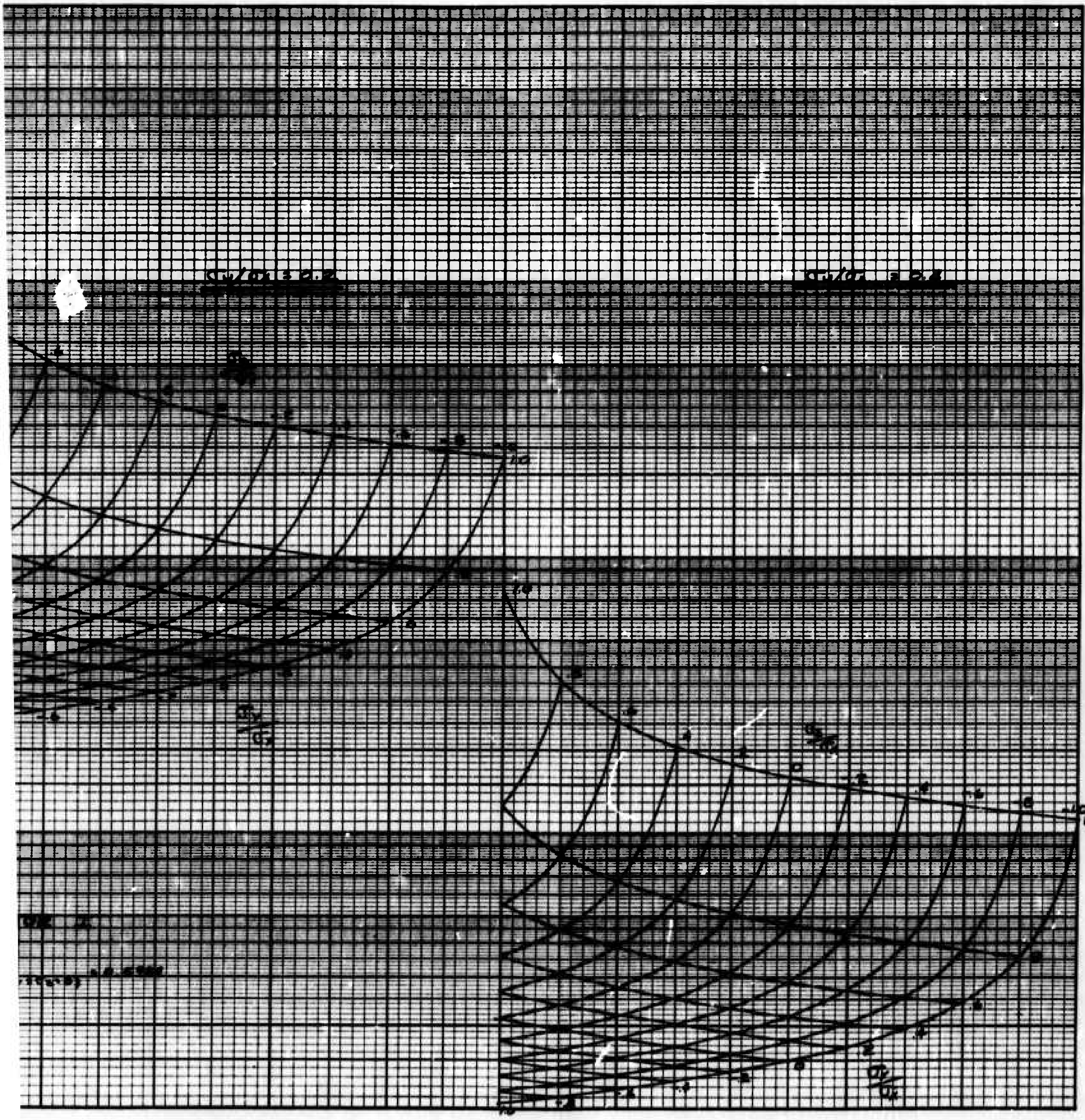


Fig.3.6(c) Complex stress factor I
σ_u/σ_x = 0.0, 0.2 and 0.4
m_1 = 10; m_1 = m – 1; $I_{(\sigma_u=}$



Complex stress factor I

σ_u/σ_x = 0.0, 0.2 and 0.4

$m_1 = 10$; $m_1 = m - 1$; $I_{(\sigma_u = \sigma_y = \sigma_z = 0)} = 0.5984$

B

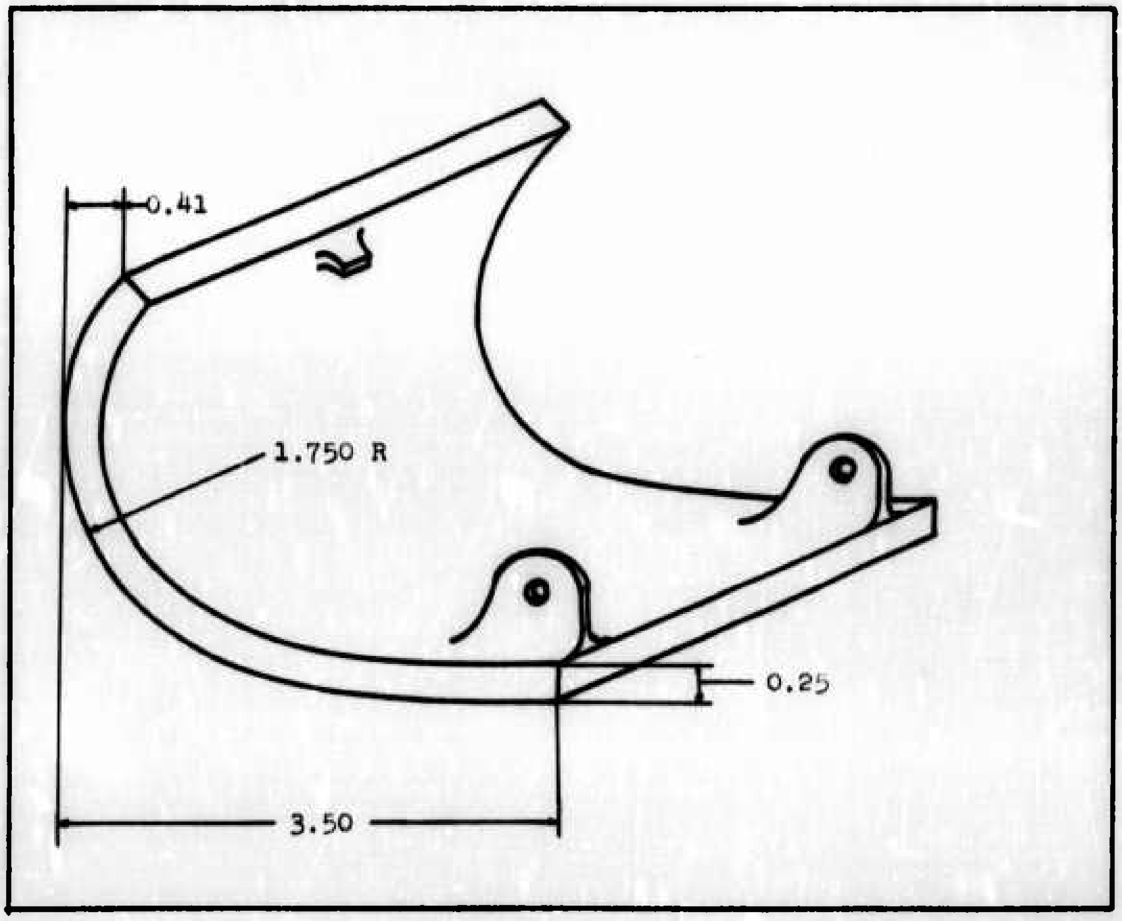


Fig.3.7 Sample problem. Wing leading edge segment

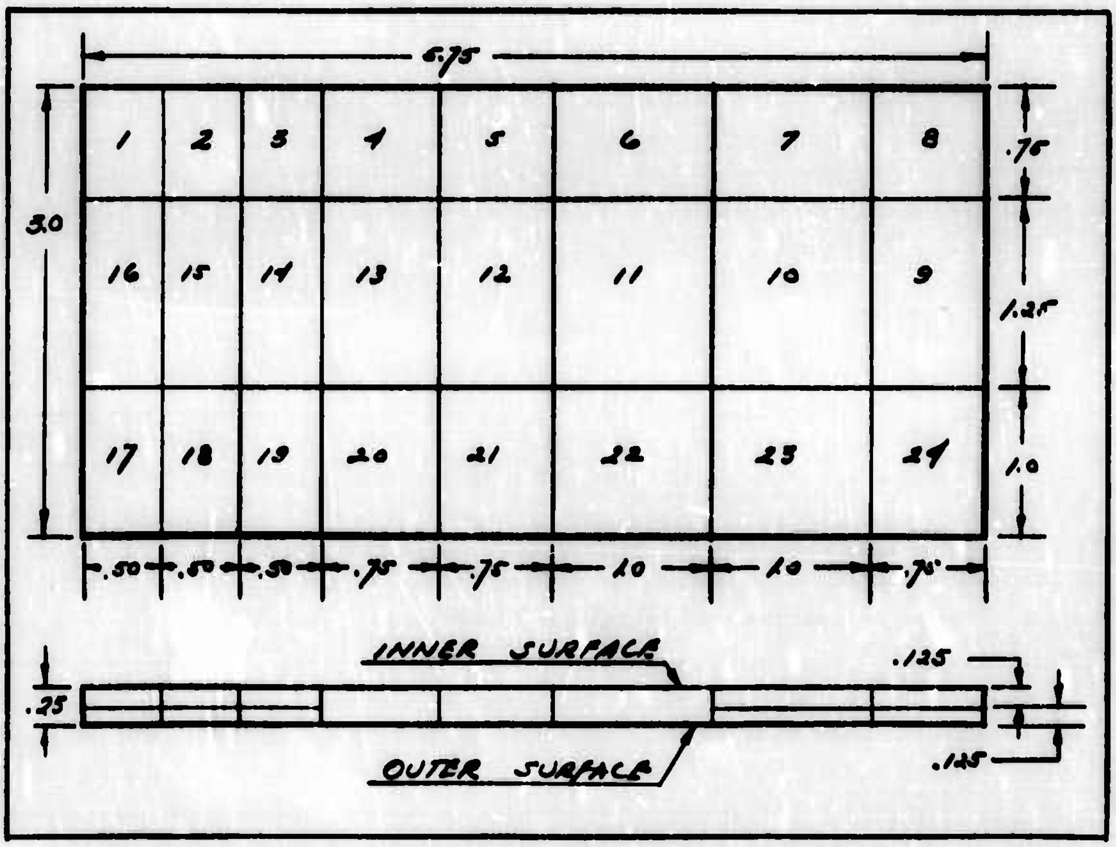


Fig.3.8 Sample problem. Distribution of elemental section pattern

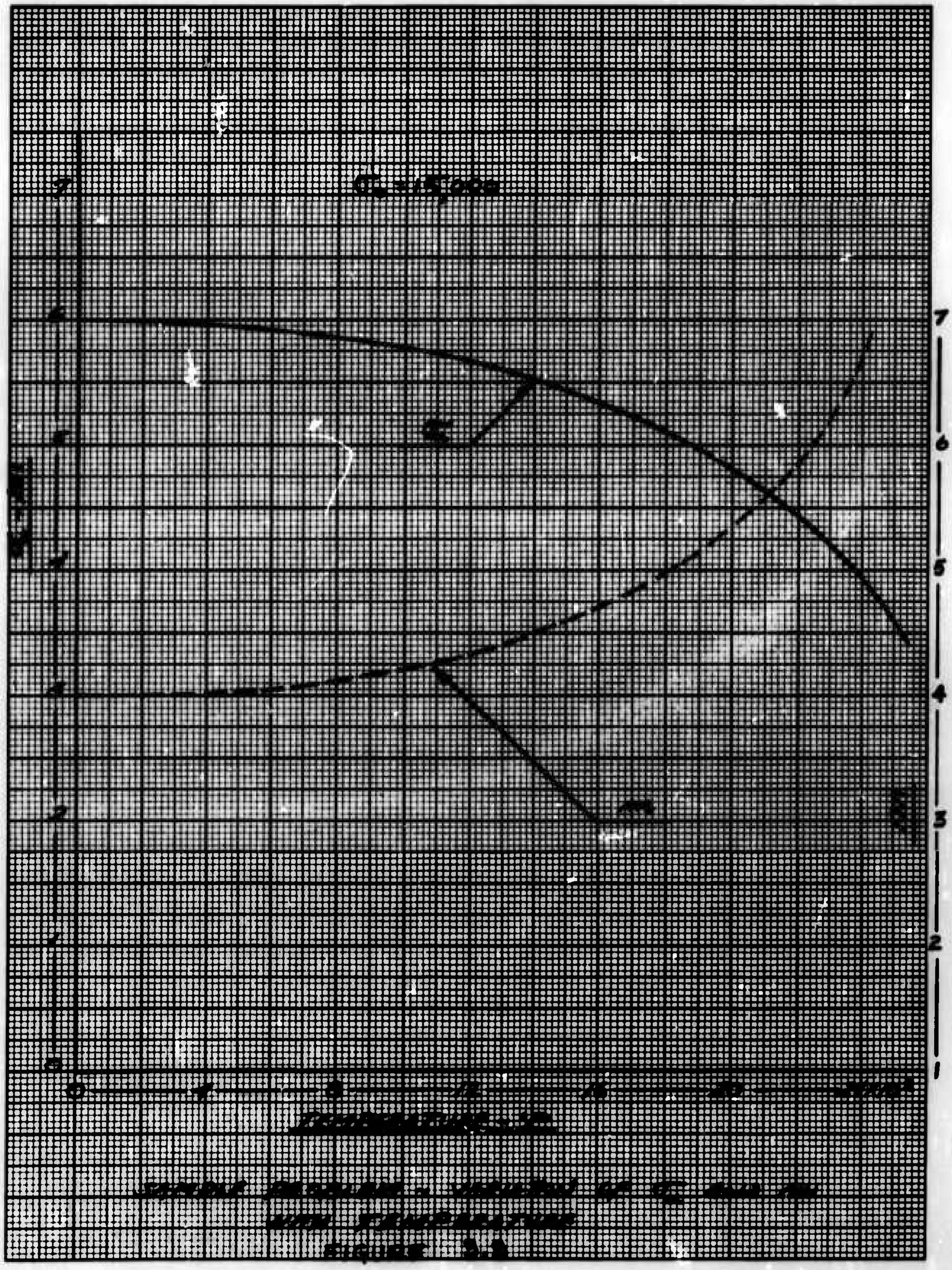


Fig.3.9 Sample problem. Variation of σ_u and m with temperature

4. STRESS ANALYSIS METHODS

4.1 INTRODUCTION

The importance, in brittle material design, of detecting the maximum stresses in a component, regardless of how localized they might be, has already been emphasized, as has the corresponding need for a sophisticated stress analysis. The limitations of conventional analysis methods, as used by stress analysts and structural designers, has also been reviewed briefly in Section 2. In the past the practice was to supplement conventional theory with local correction factors determined by application of the Theory of Elasticity. The latter is very powerful where it can be used, but unfortunately its applications are limited, by mathematical complexity, to applications of simple geometry. Fortunately, the errors in the application of simple theory are local effects, removed from the general complexity of the structure, and consequently elastic theory is, in fact, very useful.

In recent years a very powerful stress analysis method, the finite element method, has been evolved. This technique divides the structure into large numbers of simple elements, each of which can be analyzed quite easily. Coupling these elements together, with the requirements of equilibrium and strain compatibility, produces very large numbers of equations which must be solved simultaneously for the unknown stresses or displacements. The modern computer has made the solution of such equations a practical proposition. The finite element method offers the ability to handle very complex structures and complex loading situations, but it is not convenient for general studies, and it does require elaborate computational equipment. Fortunately, the latter is now available at most organizations conducting sophisticated engineering work.

In this Section useful material for the designer is presented on both methods and in both cases the data is oriented towards the particular situations expected to occur in designing with brittle materials. The 'corrected" simple theory approach is included for the benefit of organizations that do not possess computational equipment, and also because the method is particularly useful for preliminary design, optimization studies, etc. The finite element approach is included because it is a necessity for good design with brittle materials.

4.2 CONVENTIONAL STRESS ANALYSIS THEORY

The engineer's theory of bending, which is still an important practical tool of structural analysis and design, is concerned with the prediction of stresses in a structure, but under a number of very simplifying assumptions. The principal assumptions are:

a) Sections through the beam which are plane before application of the bending moment remain plane as the beam strains.

b) The structure is of constant cross section.

c) The material is isotropic and elastic, with direct proportionality between stress and strain.

d) The structure is subjected to a constant bending moment along its length.

In applications where these assumptions are seriously violated, the only recourse, prior to the advent of the finite element methods of analysis which are to be discussed later, was to use the theory of elasticity. While this theory produces accurate results it is limited, by mathematical complexity, to situations involving relatively simple geometry. Fortunately, the assumptions made in the engineer's theory generally result in localized errors in the stress distribution. Practical engineering practice therefore uses elastic theory to examine these local effects, which are removed from the general geometric complexity of the structure, and to produce correction factors to be applied to the stresses predicted by simple theory. The usefulness of this practice varies with application, and in this sub-section as much assistance as possible will be given in following the practice as it applies to anticipated brittle material airframe components.

As has been mentioned elsewhere in this Handbook, brittle materials will probably be used in airframe structures in relatively small elements as a result of limitations imposed by manufacturing methods. Consequently, the components are likely to be relatively short compared with their cross-sectional dimensions. The assumption that plane sections remain plane is frequently violated at free ends and a situation of particular significance in components intended for elevated temperature applications results from thermal stress effects. Thermal stresses produced by temperature gradients through the cross section of a beam produce a self-balancing system and their magnitudes are easily calculated from conditions of equilibrium and the assumption of plane sections. However, these stresses must reduce the zero at a free end of the beam, even though the temperature gradients do not. As a result warpage of the beam cross section occurs and the local stresses are not predicted by simple theory.

During high speed flight aerodynamic heating conditions vary as velocity, altitude, and vehicle attitude vary. In a typical beam-like structural component temperature gradients will be produced throughout the cross section but, assuming a constant cross section, these gradients will be constant at each station along the beam. Thermal stresses can be easily determined as already discussed. However, brittle material components are likely to contain integral transverse ribs and stiffening members, and local attachment lugs,

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all of which represent heat sinks which disturb the general temperature distribution. Discontinuities in temperature may also be produced where adjacent components overlap, so that part of each component is shielded from direct exposure to aerodynamic heat. This situation occurs, for instance, in ceramic heat shields the edges of which overlap to produce a seal, while still accommodating thermal expansion.

These discontinuities in temperature may also produce cross-section warpage and hence violation of assumption (a), and local stresses not computed by simple theory.

Another important temperature discontinuity effect which can be anticipated in ceramic components, will occur where integral lugs are provided on surface structures, such as heat shields, for attachment to the sub-structure. Such lugs provide a local heat sink which produces radial temperature gradients in the surface structure during transient heating phases of flight.

One other important situation in which accurate stresses are not predicted by simple bending theory occurs in the region of concentrated loads. In ceramic airframe components these situations will occur where such components are attached to the sub-structure and where provision is made for attachment of adjacent structural elements. Provision for attachment of a ceramic component will generally be made by some type of lug, integral with the component. With this arrangement the load is introduced more or less at a point and the distribution does not match the normal distribution of shear across the section of the beam. Readjustment of the shear distribution occurs with local modification of the bending stresses.

A similar situation occurs in plate-like surface structures, subjected to aerodynamic pressure loadings and attached to the sub-structure at local points.

Appropriate methods of load introduction into a ceramic component include integral stiffeners across the beam, reinforced and unreinforced holes in the beam with pins, load pads on the beam caps, etc. Since none of these arrangements introduce shear in accordance with the requirements of the plane strain assumption, local "correction" stresses are created. Typical quantitative data is given later.

Similarly, the introduction of concentrated loads into surface structures will be made either by integral supports, or holes will be provided for a mechanical attachment.

The second assumption, of a constant cross-section, will be violated in many applications. Gradual changes in cross-section, such as are produced by the typical taper in wings, fuselages, etc., are not significant in disturbing the stress distribution predicted by simple theory, but significant effects will result from abrupt and local cross-sectional changes caused by integral ribs, stiffeners, load application members, holes, cutouts, etc. Such effects will require the use of fillets which will reduce but not eliminate the resulting stress peaks.

The effects resulting from an abrupt change of cross-section are generally very localized and can be treated reasonably well by studying stress concentration effects using the theory of elasticity. Data convenient for the design of brittle structures is given later.

Assumption (c) is not a cause for corrective factors in brittle material structures. The materials are likely to be very elastic and isotropic, perhaps to a greater degree than typical metallic materials, and so the assumption is valid.

Assumption (d) is generally valid in determining the stresses at a given cross-section. Neglect of the rate of change of bending moment implies neglect of shear and for ceramic materials the deflections and deformations due to shear are likely to be very small compared with deflections due to bending. An exception may occur in beams which are very short compared with cross-sectional dimensions; say less than twice the major cross-sectional dimension. Corrections for this effect are chiefly corrections to deflections; the stress modifications due to a short beam are primarily those due to end effects and concentrated load application, and are covered under assumption (a).

In the following sub-section all of the above "correction" effects are re-examined quantitatively and, where possible, useful numerical data, is given. This data is generally limited to simple geometric situations since it has its basis in elastic theory. Where, as a consequence, it cannot be used directly, however, it may still be useful as a guide. Much of the data presented is taken from the literature but with modifications and extension to make it more convenient for the needs of the designer working with brittle materials.

4.3 CORRECTIONS TO BENDING THEORY

a) End Effects from Thermal Stresses

This problem is one of a broad class of problems, often referred to as "shear lag" problems, which are concerned with the diffusion of stresses from material which ends, at some station, as a free edge. The shear lag problem is typically encountered in airframe shell structures where wing surfaces are cut for engines, landing gear or the fuselage, and where fuselage surfaces are cut for doors, windows, etc. In these cases the usual construction is thin sheet with longitudinal stiffeners. A useful general study can be made by treating the problem as a two-dimensional flat plate problem and studying the

diffusion of stress from the cut stiffeners and sheet into the continuous stiffeners and sheet. Typical examples are given in Reference 4.1, and useful generalized charts for the design of structures on this simplified basis, are given in Reference 4.2.

More generally the brittle material components of interest will not be flat (e.g. leading edge segments) and the modified thermal stress distribution due to free ends becomes a complex function of geometry. Such a case is not treatable in generalized design chart nor can such a problem even be solved by other than finite element analysis. Even the solution of the problem for very simple geometric shapes is extremely difficult and has only been accomplished for a few specific stress distributions.

Reference 4.3 studies the problem of rectangular strips subjected to a self-balancing axial stress distribution applied at the free ends. Two stress distributions across the width of the strip, are given. These results can be used to determine the stresses near the ends of a rectangular strip subjected to two particular temperature distributions. Since, despite their limited scope, such data may provide useful guidance in more complex situations, the information is given in Figures 4.1 and 4.2. Similar results are given for a solid cylinder (based on Reference 4.4) in Figure 4.3. Figure 4.4 gives the thermal stresses in a longitudinally stiffened plate with free ends, and is appropriate for small stiffened surface panels, which may form an application for refractory non-metallic materials in very high temperature structures. The case given involves the minimum of only three stiffeners. Figures 4.1 and 4.2 represent the other extreme since they can apply to the case of an infinite number of stiffeners.

It is important to note, for design purposes, that at no point, for the cases given, do the stresses exceed the values that would be calculated by simple theory, for stations distant from the free end. However, the generality of this statement for other temperature distributions and for much more complex shapes, is not known.

b) Temperature Discontinuities

Again this problem is very dependent on geometry so that the only general solutions available are those concerned with simple geometric shapes. Even so the analysis is very complex. Solutions for a rectangular strip have been developed in Reference 4.5 and some useful design curves are given in Figure 4.5. More complex temperature effects may be obtained by superposition. The stresses shown are for conditions of plane stress, appropriate to a very thin strip. For the plane strain case the values should be multiplied by $1/(1-\mu)$. Note also that the stresses are reduced as the temperature discontinuity approaches the free edge. Typically the levels are reduced about 30% when the origin of the x and y axes is at a distance of 0.4b from the free edge.

Figure 4.6 gives thermal stresses in a plate, such as might arise locally in plate-like ceramic surface structures due to attachment to a cool substructure. Such an attachment represents a heat sink which produces a reduced temperature in the plate at the point of attachment. Conduction in the plate then produces temperature gradients radially from the attachment point. Since the attachment represents a "cold" spot, these temperature gradients will produce tensile thermal stresses.

Figure 4.6 gives data for three cases of practical interest. Case 1 is a hole in a plate where the edge of the hole, of radius a, is maintained at temperature ΔT below the temperature of the plate. This case simulates the situation when the plate is attached by a metal bolt, or other mechanical fastening.

Case 3 assumes, on the other hand, that the attachment is made through a ceramic post, fabricated integrally with the panel. The post is of radius a, and is held at a temperature ΔT below the panel. In this case the integral post applies constraints to the thermal deformations which modify the thermal stresses.

Case 2 assumes a uniform thickness plate, without a hole or post and might represent a clamped or brazed attachment.

c) Concentrated Loads

At the point where concentrated loads are applied to a beam, disturbances in the "simple" stress distribution occur if the load is not applied in a manner which matches the internal distribution of shear in the beam. In general this will not be the case; the distribution of the applied load will be dependent on the geometry and stiffness of the local structure through which the load is transmitted and of the local geometry of the beam. This dependence on geometry again prevents simple or general solutions but results for some specific simple cases are given in Figures 4.7 through 4.10. These cases have been selected as representative of practical methods of introducing loads into beams or plates.

Figure 4.7 gives the local stresses in a simple rectangular cross-section beam near a concentrated load applied to the upper surface. These local stresses are to be added to the bending stresses calculated by simple theory. Figure 4.7 is based on results given in Reference 4.6. It should be noted that these local stresses are negligible at a distance from the point of load application equal to the depth of the beam.

Figure 4.7 assumes the load applied at an infinitesimally small point so that immediately under the load the stresses are infinite. In fact, such stresses are modified by local deformations of the beam surface and the surface of the load application device,

so that the contact area is, in fact, finite. Figure 4.8 gives local stresses immediately under the load and should be used instead of Figure 4.7 for distances of approximately 2δ. Figure 4.8 is based on data given in Reference 4.6.

Figure 4.9 gives the local stresses, to be superimposed on stressed calculated from simple theory, if the load is applied at a hole, typical of a mechanical attachment. Figure 4.9 is calculated using a stress function given in Reference 4.6 and assumes an infinite plate. Its application to a beam having finite boundaries will therefore introduce errors unless, as is typically the case, the hole is small compared with the beam depth.

Again Figure 4.9 gives infinite stresses immediately under the load because it assumes a zero area of contact. Corrections can be obtained from Figure 4.8 for the effect of a pin of finite size, used to apply the load. In the preparation of Figure 4.8, it is assumed that the pin will normally be a reasonably close fit, in the hole, although not a perfect fit. Accordingly, Figure 4.9 is conveniently expressed in terms of the tolerance between the hole and pin diameters.

In ceramic structures integral lugs may be provided for load application to a beam and Figure 4.10 gives local stresses to be superimposed on bending stresses calculated by simple theory, where this is the case. Again Figure 4.10, (based on relationships given in Reference 4.6) is given for an infinite plate and is therefore most appropriate when the lug is remote from the boundaries of the beam. The relationships given in Figure 4.10 are also determined for a point load and accordingly approach infinity as the radius r approaches zero. It is therefore recommended that the minimum value of r be the radius of the load application lug. Very local effects due to the sudden change of cross-section at such a lug are not known.

The use of ceramic materials in plate-like surface structures, as heat shields, has been mentioned. Such structural elements will be subject to normal pressure loadings, and will probably be attached to the substructure by integral lugs, located at discreet points. Such lugs introduce concentrated reactions normal to the plate surface. Analytical expressions for a large number of cases of plates under normal loading are given in Reference 4.7. A case which, it is believed, will have general application to ceramic surface structures has been evaluated numerically and the results are given in Figure 4.11.

Figure 4.11 gives bending moments for a circular plate under pressure on a central support. The material is assumed to have a μ of 0.3 and it is further assumed that the diameter of the support is small compared with the diameter of the plate. ($b < .05a$) It is believed, without proof, that this data should give approximate results for the bending moments near supports for a large plate on many supports.

d) Stress Concentrations at Changes of Cross-Section

Typical stress concentration factor data for notches and fillets in flat and round bars, subject to axial load and moment, are given in Figures 4.1 through 4.16. These figures are arranged with a basic chart (Figure 4.1) appropriate to small notches and fillets, supplemented by additional corrections factors for large and deep notches and fillets and also for elliptical fillets. The data have been assembled based on Reference 4.2.

Details of the stress distributions at fillets, such as are required for failure probability assessment, are not given, but appropriate data is given in Section 7.

Figures 4.16 through 4.19 give data on the effects of holes and the material is sufficiently general to cover the effect of hole reinforcement, the effect of hole shape and the effect of an adjacent boundary. The method of using the curves is self-evident. Data for these curves have been taken from Reference 4.2.

Figures 4.20 and 4.21 give data useful in the design of tension fittings, while Figure 4.22 give stress concentration data for the design of tension lugs. Such lugs are shown on Figure 4.22 in two forms; with a square end, which is form used for the analytical and experimental work on which the figure is based, and with a round end, which is the form generally encountered in practice. Reference 4.8 indicates little difference in maximum lug stresses, between these two forms. Note that Figure 4.22, which is based on Reference 4.8, includes some effect of clearance between pin and hole. Reference 4.8 indicates that clearance has little effect on the stress at the side of the hole for $d/D < 0.5$ but that the effects of clearance are important when $d/D > 0.5$ and for maximum stresses at all values of d/D. Data for $d/D > 0.5$ is limited and Figure 4.22 should be considered as indicating trends.

4.4 FINITE ELEMENT METHODS OF STRESS ANALYSIS

4.4.1 General Concepts

The finite element analysis technique is a sophisticated numerical method which permits the generation of approximate solutions for the deformation, internal loading and other behavioral characteristics of complex structures under a variety of external loading or stimuli. In employing finite element methods, the geometry of the actual physical structure under consideration, which may be either a continuous system or a combination of discrete components, is mathematically approximated by replacing it with a large number of interconnected structural elements with simple geometric forms. Examples of such

elements are flat or curved plates of quadrilateral or triangular shape, beams, bars, conical or cylindrical rings and solid parallelipipeds. For these elements energy principles are used to determine approximate numerical relationships between elemental boundary forces and specified nodal displacements. These relationships, which are usually derived through the use of assumed forms of stress and/or displacement distributions within the individual elements, are expressed as stiffness, flexibility, moss and loading matrices. Through imposition of interelement compatibility and equilibrium conditions, behavioral matrices for the total structural system are generated by simple linear combinative assembly of the individual elemental matrices. After specification of the structural boundary conditions and applied loads, the characteristic structural stiffness or flexibility matrix can be solved directly using matrix algebra for the unknown displacements or forces on the element boundaries. Once the element edge displacements or forces are obtained, use of the a sumed distribution polynomials for displacements or forces in the interior of the elements allows the calculation of stress and strain magnitudes through application of the material strain-displacement and stress-strain laws. Thus, the finite element technique allows complex structural analysis problems to be solved by providing an approximation to the actual structural geometry and utilizing piece-wise continuous approximations to the forces, displacements, stresses and strains in the model. The accuracy of the solutions obtained from a finite element analysis is a function of level of sophistication of the assumed element behavior and the degree to which interelement compatibility and equilibrium conditions are capable of satisfaction with this given element behavior. In theory as the element is reduced to infinitesimal size, the idealized model approaches the actual structure and solutions become exact.

In essence two parallel or related methods are currently used for finite element analysis--the force and displacement formulations. In the force or flexibility method, interelement forces are the unknowns, and the basic building blocks are element flexibility matrices. A master set of compatibility equations is assembled using equilibrium conditions. The displacement or stiffness method uses elemental stiffness matrices. These are assembled, using compatibility conditions, into a set of equilibrium equations which are then solved for the unknown displacements. Although both approaches have been used for the development of structural analysis programs, the range of elements available in force method programs is relatively limited and does not extend to the types of solid elements required for brittle material work.

The principal features of the finite element method of stress analysis, and particularly the displacement approach, include its versatility with respect to practical structures with complex geometry and complex, multiple loading conditions, and its amenability to development in the form of a highly automated computer program.

The input data to such a program consists simply of a geometric description of the structure, the material properties, specified loads, and a definition of the elements which constitute the analytical model. This information is directly referred to the portion of the program containing the library of finite element formulations, which must contain a sufficient number of different types of elements to properly represent the range of situations encountered in the structure to be analyzed.

Once the pertinent element formulations have been drawn from the library and numerically evaluated for each element of the structure under analysis, the next operation is to develop and solve the systems of equations which describe the behavior of the total structure. The element formulations are combined in an automatic procedure to yield the desired equations and solution is accomplished through the use of the more efficient of the many mathematical subroutines available. The displacements are solved for in this manner and then back substituted into the element formulations to obtain the internal stress distribution.

Typically the printout of the solution consists of a list of the predicted displacements of the joints which connect the elements, the state of stress within each element, and the reactions on the boundary of the structure. Various checks are also built into the program and are printed out, to assess the validity of the results. Depending on the sophistication of the program it is possible to include the determination of principal stresses and to develop appropriate graphical output.

4.4.2 General Purpose and Special Purpose Programs

Following the general lines described above finite element analysis programs have been developed both on a very large scale, involving very broad capabilities, and as special programs of limited capability but with the ability to solve particular problems in the most expeditious manner. There are many special purpose programs available or under development and a good review of these is given in Reference 4.9. None of these seem to be particularly suitable for structures constructed from brittle materials however, because the latter require "three-dimensional" elements. Typical ceramic structural elements will have relatively thick walls, requiring consideration of stresses through the wall thickness and requiring in turn, finite elements with this capability.

Since a number of excellent general purpose programs, with capabilities appropriate for the analysis of brittle material structures, are available any organization requiring the capability would be advised to use one of these, rather than to attempt to develop new, special purpose programs.

The most important advantages of a general purpose program, if an existing, fully developed program is used, lie in the many special features which are available, and

which cannot be justified in a special purpose program. For example, preprinted input data forms, extensive internal libraries of material property data, options for such things as transforming material axes to accommodate arbitrary axes of orthotropy, grid point axis transformations to account for irregular boundary conditions, eccentric connection transformations for realistic modeling of frame joints and shell stiffeners, etc. Additionally, special purpose programs frequently encounter difficulties such as machine storage capacity that must be avoided rather than summounted in view of the limited applicability of the program.

General purpose programs also justify adequate documentation, suitable for general usage, thus facilitating the training of personnel in program use. Efficiencies also often result from the avoidance of manual transfer of data, with its potential for errors, between special purpose or single step programs. The integration of heat conduction and thermal stress analyses within a single program is an example.

On the other hand, special purpose programs which are developed for the analysis of a very restricted class of problems will generally be much more efficient in operation than the immensely sophisticated general purpose programs. This increase in operational efficiency, which results from the elimination of logic and programming associated with the many options of the general purpose program, is only gained at the cost of the development effort of the superfast special program. In deciding whether to use an existing program with its higher operational cost or to develop a special purpose program consideration must be given to the tradeoff between the factors of cost and time. If a great number of analyses of a limited class of structures is desired, the reduction in operating costs over a period will probably offset the expense of developing a relatively simple special program. If a more general capability is required, the most economical procedure would be to use one of the available existing general purpose programs with suitable finite elements in the library.

4.4.3 Capabilities of Available General Purpose Finite Element Computer Programs

In addition to special purpose finite element programs, there are presently available a number of "general purpose" programs as well. General purpose being loosely defined as follows:

a) A versatile finite element library.

b) A complete set of element matrices to support each finite element representation.

c) Large scale problem solving capability.

d) Applicable to a wide variety of structural classes.

e) Machine independent.

f) Available computational procedures to support required analyses, i.e. displacement, stress, stability, vibration, etc.

g) User oriented, as reflected by ease of input and straight-forward interpretation of output.

h) Powerful matrix abstraction capability.

A convenient summary of programs meeting some or all of these requirements is presented in Reference 4.10.

The availability of general purpose programs suitable for the analysis of brittle material structures, which inherently exhibit three-dimensional states of stress, is limited. Three of the programs which are available (either free upon request or by purchase) are as follows:

NASTRAN	-	NASA General Purpose Program for Structural Analysis
MAGIC	-	Matrix Analysis via Generative and Interpretive Computations
ASKA	-	Automatic System for Kinematic Analysis

These three programs are described briefly below. The selection of a program for implementation as a permanent structural analysis capability requires a detailed examination of the characteristics and features of the available programs with respect to the total analysis requirements and the available computer facilities. It is not appropriate to include in a handbook sufficient information for this purpose, but appropriate references are given.

NASTRAN

NASTRAN is described in detail in Reference 4.11. It is a large scale structural analysis program developed for the NASA, with great versatility and flexibility. It is capable of carrying out analyses of the static response to concentrated and distributed loads, to thermal expansion, and enforced deformation. It will also carry out an elastic stability analysis, and analysis of dynamic response to transient loads, steady state sinusoidal loads and random excitation. It will determine real and complex eigenvalues for use in vibration analysis and flutter, and it will conduct a dynamic stability analysis.

Limitations on the size of problem are only those imposed by running time and by the ultimate capacity of auxiliary storage devices. It is not bounded by core size.

The finite elements included in the program are rods, beams, shear panels, triangular and quadrilateral plates with both membrane and bending stiffness and anisotropic material properties, conical shell elements, doubly curved axisymmetric shell elements and triangular and trapezoidal cross-section ring elements. Apart from the triangular and trapezoidal ring elements, these elements are not particularly suitable for analysis of thick walled, ceramic structures. However, NASTRAN is designed so that other elements can be incorporated and element derivations for elements more suited to brittle material structures are available in the literature. They will be discussed later. Actual experience with NASTRAN has shown that the incorporation of new elements is, in fact, quite difficult.

NASTRAN is designed for use with essentially any computer. It is a modular program so that it may be updated by revamping within a module without modifying external appearance. It is, in fact, intended by NASA to update the capability periodically.

The program is designed with simple input deck preparation and to minimize chances for human error in problem preparation. For the same reasons the need for manual intervention during program execution has been minimized.

The program contains a curve plotter routine that will generate graphs of response quantities as functions of frequency or time, as appropriate. The plotter includes logarithmic scales and will provide titles, axis labeling, etc.

Structure plotting is also available as an aid to visualizing the shapes of geometrically complex structures, the buckling and vibration modes, deflected structural shapes, etc. These plots can be presented in orthographic, perspective or stereoscopic projection.

The program also has restart capability, essential for the economic running of large problems, and all aspects are well documented for maximum visibility.

NASTRAN is coded completely in double precision with the exception of element stress calculations. This feature, although designed to minimize errors associated with round-off, etc., has in some cases led to unnecessarily long execution times, especially on computers where double precision is not required.

In addition, the lack of a complete finite element library (no solid elements) as well as the lack of a complete library of element matrices (work equivalent pressure loads, flexural thermal load matrices) have limited the programs use in some cases.

NASTRAN is available for purchase, along with appropriate documentation from Computer Software Management and Information Center (COSMIC), University of Georgia.

MAGIC

The MAGIC System for Structural Analysis is described in detail in References 4.12, 4.13 and 4.14. The MAGIC System and NASTRAN are similar in many respects so that this discussion will be concerned primarily with the differences. MAGIC is generally similar in size and scope to NASTRAN but with, on the one hand, a more sophisticated set of finite element representations and on the other somewhat less versatility in the area of forced dynamic response calculations. As with NASTRAN, MAGIC is under continuing development.

The elements presently included in MAGIC II are the frame element, axial force member, quadrilateral shear panel, companion interelement compatible quadrilateral and triangular thin shell elements of zero curvature, triangular cross-sectioning, trapezoidal cross-section ring and core element, doubly curved toroidal thin shell ring and shell cap element. In addition, a quadrilateral and triangular plate element along with an incremental frame element are included primarily for the performance of instability analyses.

Of most importance, however, are the solid elements available in advanced versions of the MAGIC System. These include the tetrahedron, rectangular prism, triangular prism and symmetric triangular prism. A triangular cross-section ring which accommodates asymmetric loading on axisymmetric thick walled structures is also available.

Included with each finite element representation are element matrices for stiffness, stress, thermal stress, prestrain load, distributed mechanical load (pressure), incremental stiffness and consistent mass.

The computational procedures available from the MAGIC System include linearly elastic displacement, stress, thermal stress, (with and without condensation), prescribed displacements, stability and vibration analyses (with and without condensation).

For very large scale analyses involving many thousands of degrees-of-freedom, the powerful matrix abstraction capability available with the system provides for static and dynamic substructuring as well. However, problem size is limited only by computer running time and the capacity of auxiliary storage devices.

MAGIC was designed to be machine independent (coded exclusively in FORTRAN IV) and is modular in nature. Additional elements and new computational procedures are easily incorporated as required.

The system is also designed to be user oriented. Preprinted input data forms are an ingegral part of the system. Output is designed to be easily interpretable by the analyst. Included as output are displacements, element forces, element stress, and system reactions.

Many supplementary convenience features are incorporated. Grid points may be input in rectangular, cylindrical or spherical coordinate systems. Internally generated transformations are included for grid point axes, material axes, stress axes, eccentric connections for shell stiffeners, element matrix repeat, etc.

Selected double precision is used as a basis for coding the MAGIC System. The program is available free upon request from the Air Force Flight Dynamics Laboratory, Wright-Patterson AFB, Ohio.

ASKA

ASKA (Reference 4.15) is again similar to NASTRAN in its scope and versatiblity but has the advantage of a very large number of available elements and a capability for substructuring. It is based on the matrix displacement method, is modularized by systematic growth, and in addition to the usual static stress and deformation analysis under loads and temperatures it will analyze for instability, plasticity, large displacement effects, modes and frequencies and transient response due to arbitrary time dependent loads, both for damped and undamped structures.

As with the other programs mentioned ASKA is problem oriented to relieve the engineer of unnecessary detail and in particular built in checking procedures are provided for input data. The program does not stop at the first error found but tries to clarify as much as possible per run, with automatic termination if the remaining calculations would be meaningless.

The substructuring capability permits very large problems to be attacked by solving portions of the structure individually, under boundary loads and coupling stiffnesses, and iterating until the boundary conditions of adjacent substructures are compatible. Problems with 9000 unknowns have been solved.

The element library of ASKA involves, in addition to the usual simple elements, families of ring elements, membrane shell elements for axisymmetric bodies, three-dimensional sector elements for analysis of problems with axisymmetric geometry but unsymmetric loading, and three-dimensional elements with straight and curved edges, etc. The term families refers to the fact that the elements are available with both linear and higher order strain assumptions, the latter leading to improved accuracy (with an increase in complexity) and to a much better smoothing of stresses from element to element.

4.4.4 Elements for Analysis of Brittle Material Structures

Since brittle material structures are expected to involve relatively thick-walled components, in contrast to the thin shells of metallic construction, it will be necessary to consider three-dimensional stress states and to do this with finite element computer programs requires "solid elements". It must be noted, however, that analysis using three-dimensional finite elements is relatively costly in comparison with analysis using the more customary one and two-dimensional finite elements. The preparation of input data and the interpretation of results are substantially more complex for three-dimensional problems and, because the number of degrees of freedom is generally greater, so is the computational effort. A number of solid elements suitable for this purpose are available, and are, or shortly will be, operational with the MAGIC and ASKA general purpose programs. Typical of these are the rectangular prism, tetrahedron, triangular prism and the triangular cross-section ring. These are shown in Figure 4.23 and derivations can be found in References 4.16, 4.17 and 4.18, respectively.

The rectangular prism element can be used in conjunction with the tetrahedron and triangular prism elements for the analysis of arbitrary solid geometries or it can be used with plate elements for built-up regions. In the reference given the prism is mathematically discretized into a finite number of displacement degrees of freedom by the assumption of displacement mode shapes. For the rectangular prism element the assumed displacement functions satisfy the requirements of displacement continuity along interelement boundaries and they require that the edges of the prism remain linear in deformation. As a consequence the element cannot bend under any conditions. The assumed functions also lead to a total of 24 displacement degrees of freedom for the element representation. Linear elastic material behavior is assumed, with a capability for orthotropic material behavior. Stress behavior is defined by three direct stresses and three shear stresses calculated at the centroid of the element.

The tetrahedron element is used with the rectangular prism for the idealization of solids of arbitrary configuration, in regions of irregularities. Capabilities also exist to assemble three tetrahedrons into a triangular prism element, automatically in some programs. A considerable reduction in input is realized, leading to a corresponding reduction in the possibility of input error when large scale analyses are performed. A

linear polynomial mode shape is assumed for each of the three displacement functions leading to 12 coefficients for the element, corresponding to three translational degrees of freedom at each of the four vertices. Interelement continuity in displacement is satisfied and due to the assumption of linear edge displacements the edges of the tetrahedron remain linear in deformation. Linear elastic orthotropic material behavior is assumed. Due to the displacement assumptions a state of constant strain exists throughout the element. Three direct stresses and three shear stresses, calculated at the element centroid, define the stress behavior.

Generally similar comments apply to the axisymmetric triangular cross-section ring, which is presently available with axisymmetric loading.

All of the above element formulations assume linear edge displacement. Greater accuracy and smoother stress-distributions can be obtained with solid elements involving higher order strain assumptions, and better matching of geometry with fewer elements can be obtained with elements having curved edges. Such sophisticated elements are described in Reference 4.16.

At the present time a further extension of the above concept is underway, although so far as is known, is not yet generally available. It can be demonstrated that generally, for a given total number of degrees of freedom within a structure, accuracy is increased for larger elements with a greater number of degrees of freedom. In order to take advantage of this feature without losing an important advantage of finite element methods, their ability to model complex geometry, isoparametric elements are being developed. These are elements with curved sides which can be adjusted to match geometric boundaries. While such elements are complex mathematically, and require numerical integration to develop the element properties, the complexities are not apparent to the user. In fact the data preparation task is reduced by the fewer number of elements required for a given problem.

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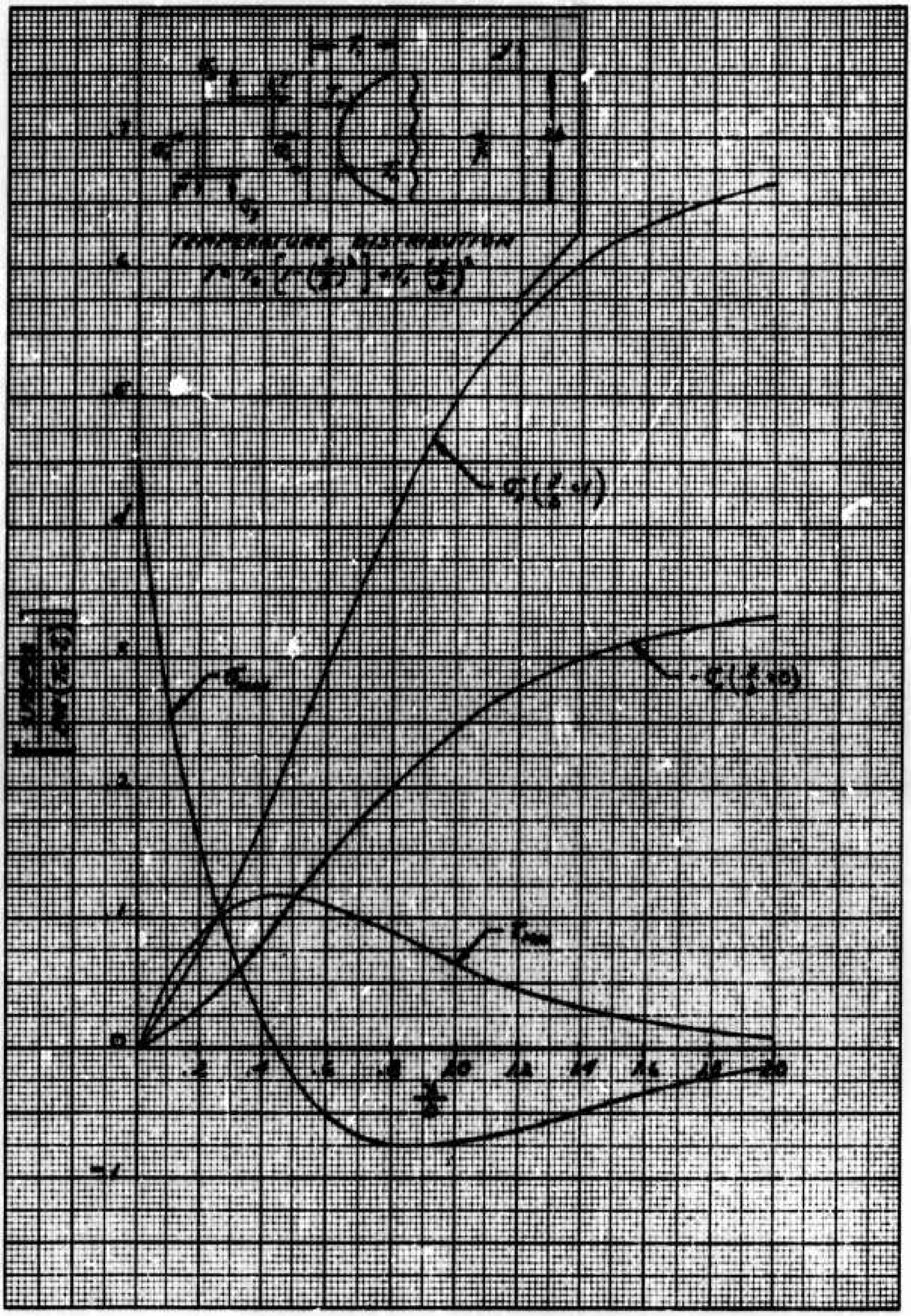


Fig.4.1 Thermal stresses near the end of a rectangular strip

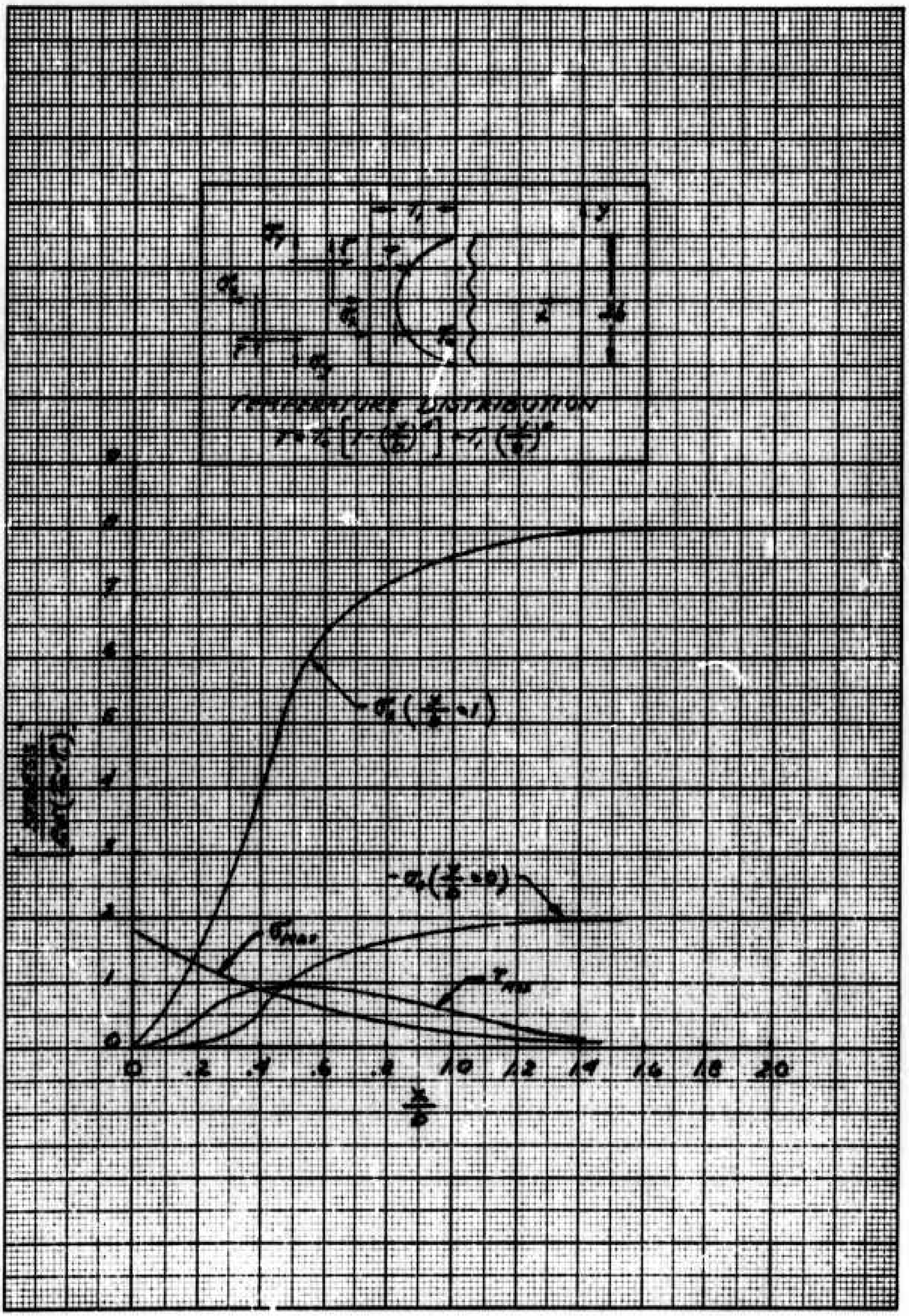


Fig.4.2 Thermal stresses near the end of a rectangular strip

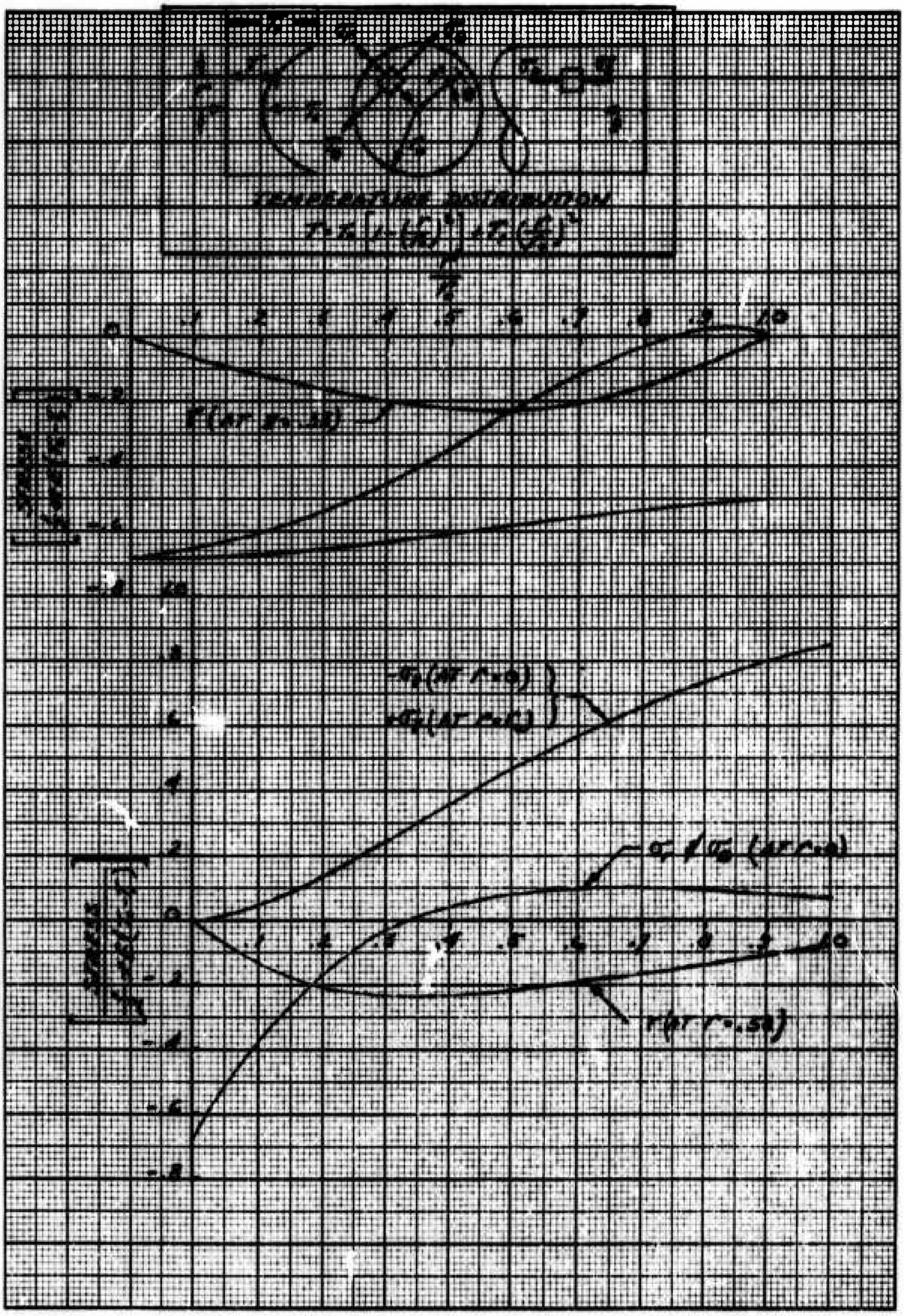


Fig.4.3 Thermal stresses near the end of a solid cylinder

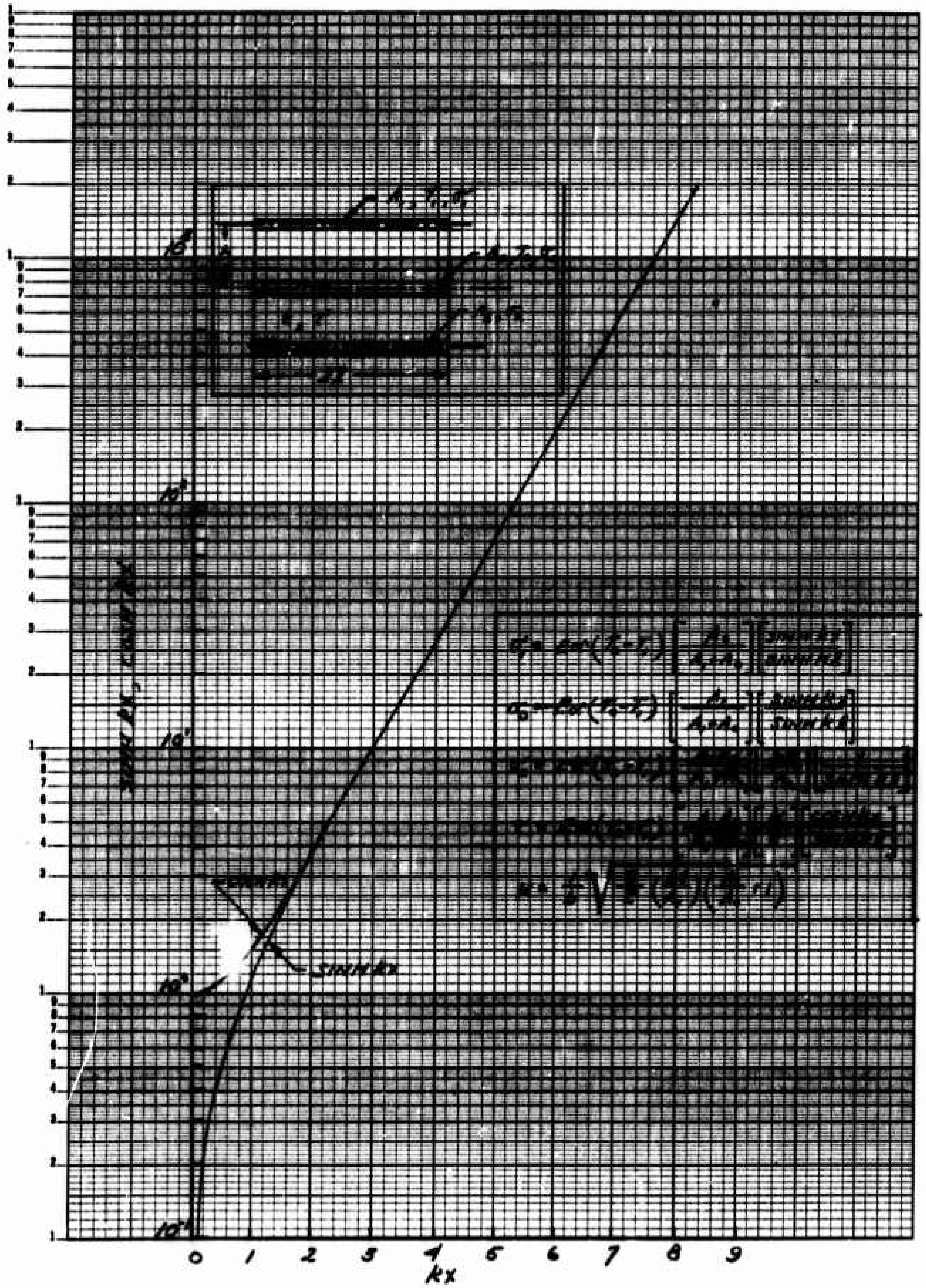


Fig.4.4 Thermal stresses in a stiffened plate

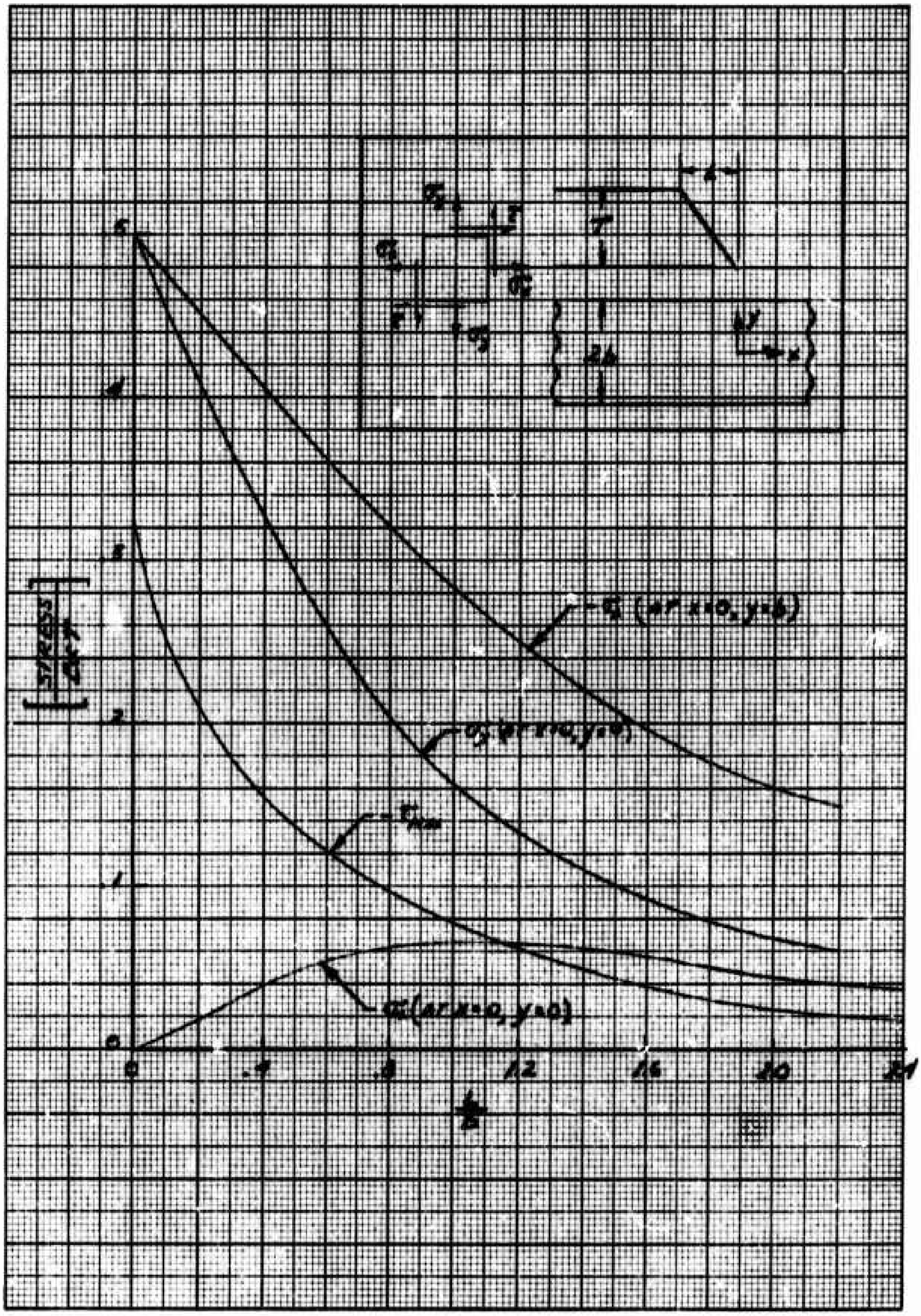


Fig.4.5 Thermal stresses due to a temperature discontinuity – rectangular strip

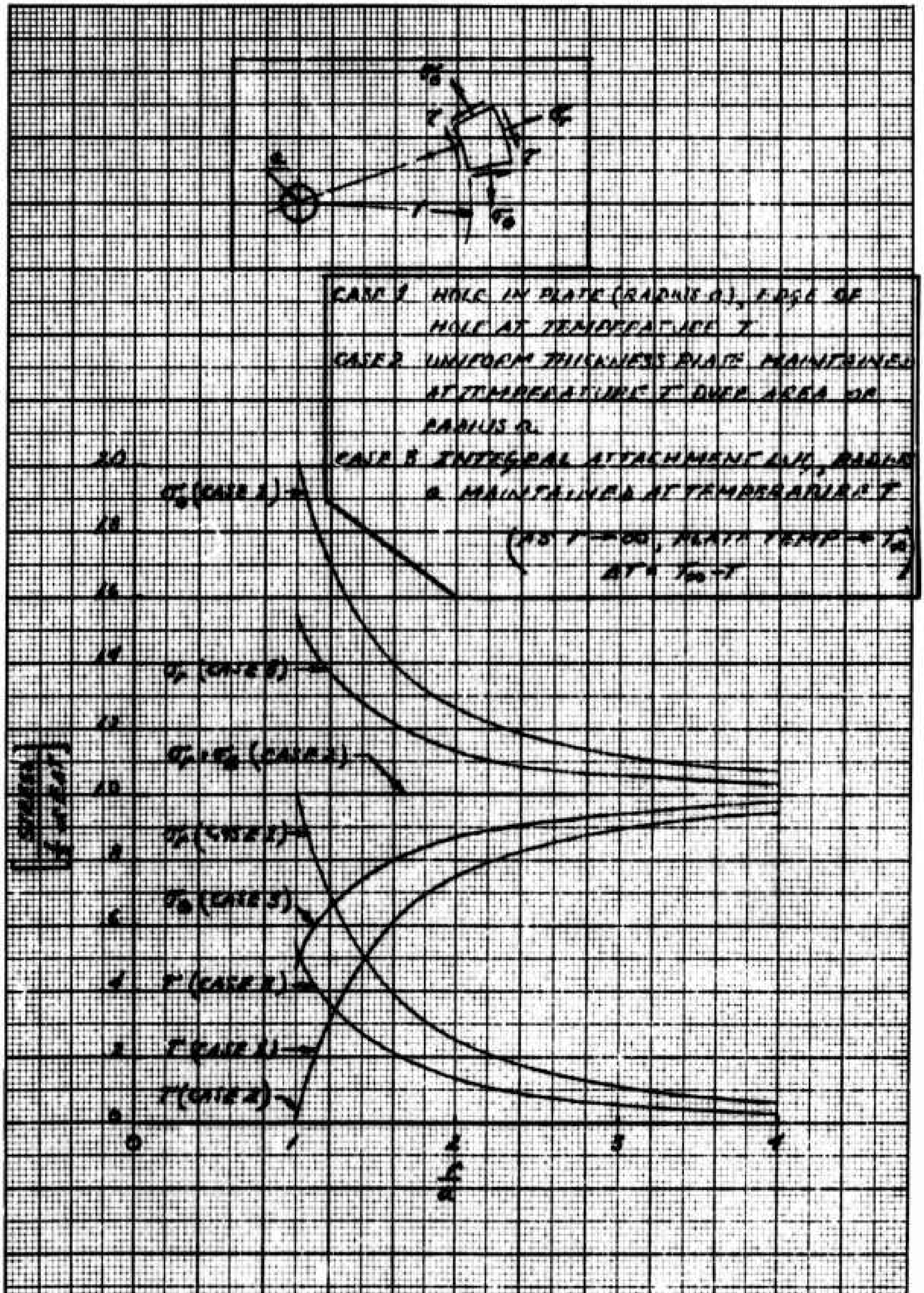


Fig.4.6 Stress due to a cold spot in a plate

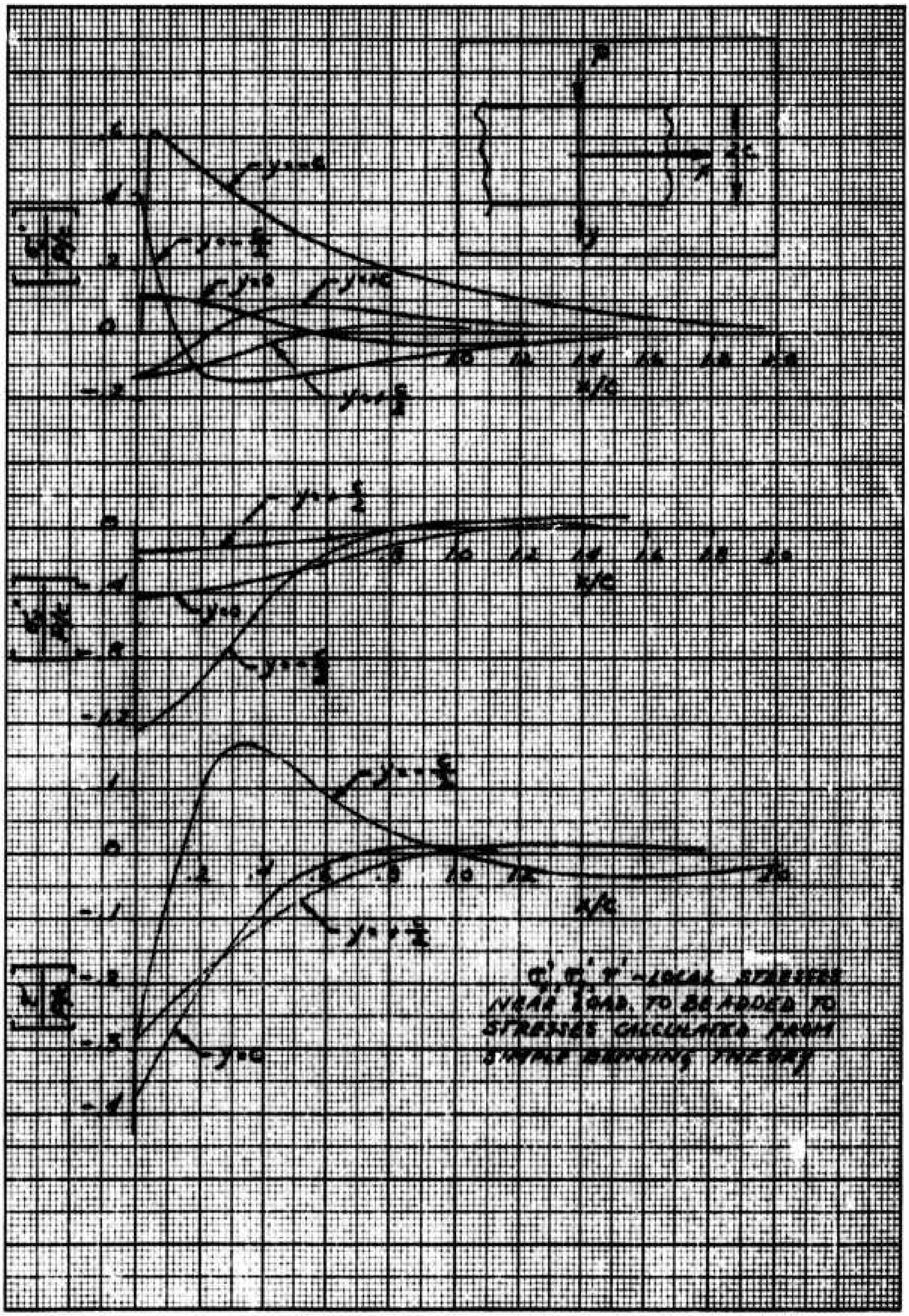


Fig.4.7 Local stresses due to a load applied at the surface of a beam

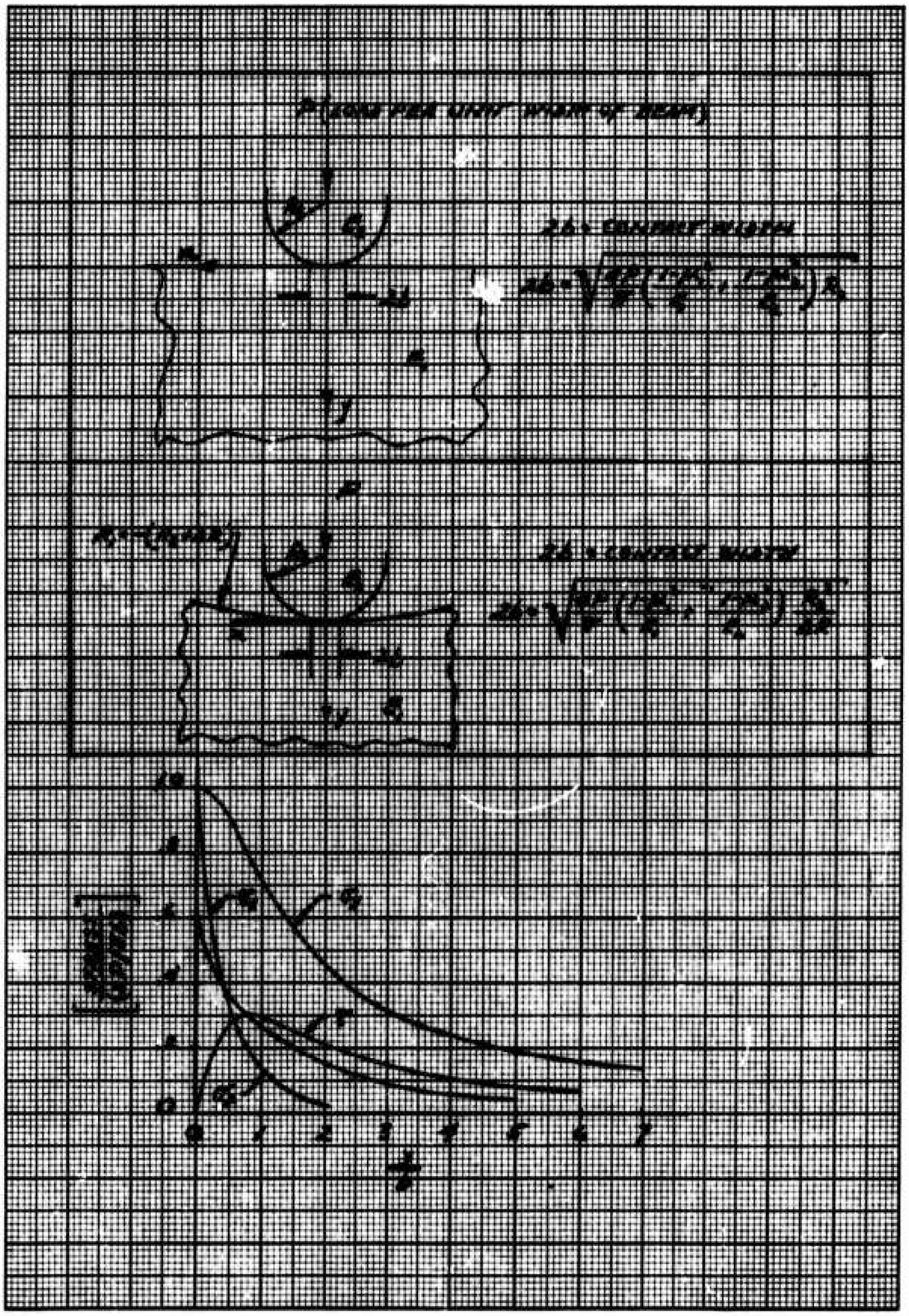


Fig.4.8 Local stresses at point of load application

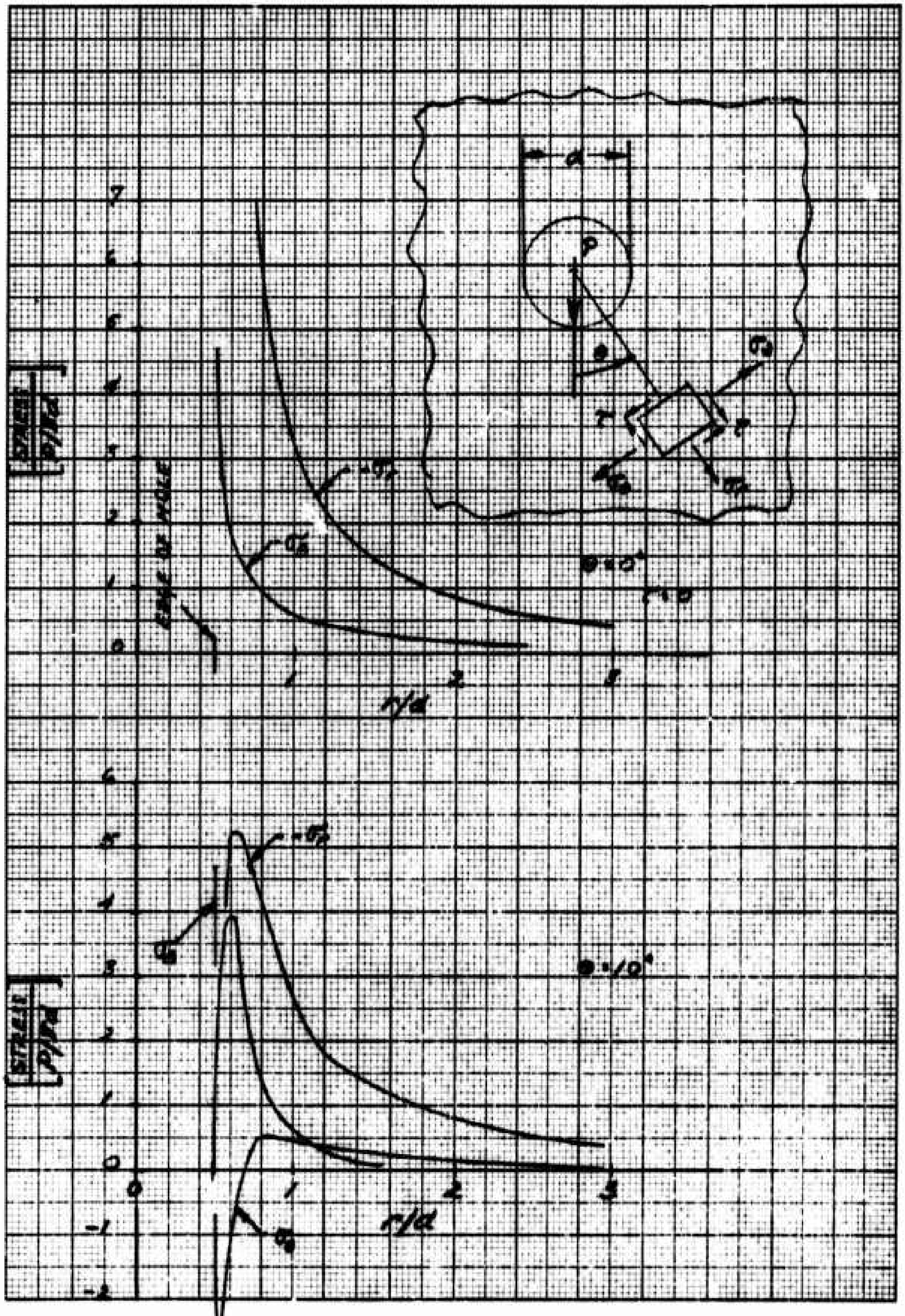


Fig.4.9(a) Local stresses due to a load applied at a hole

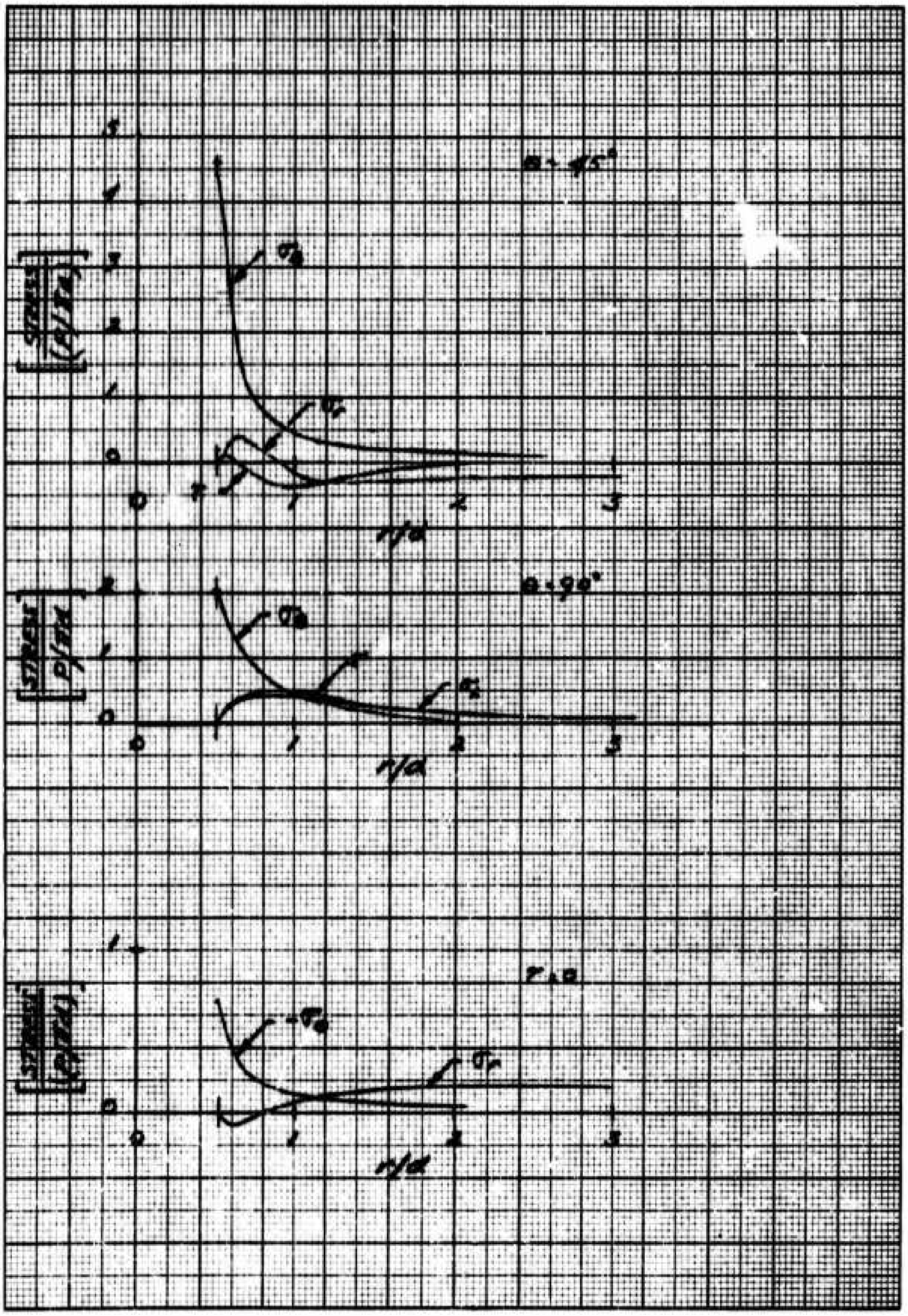
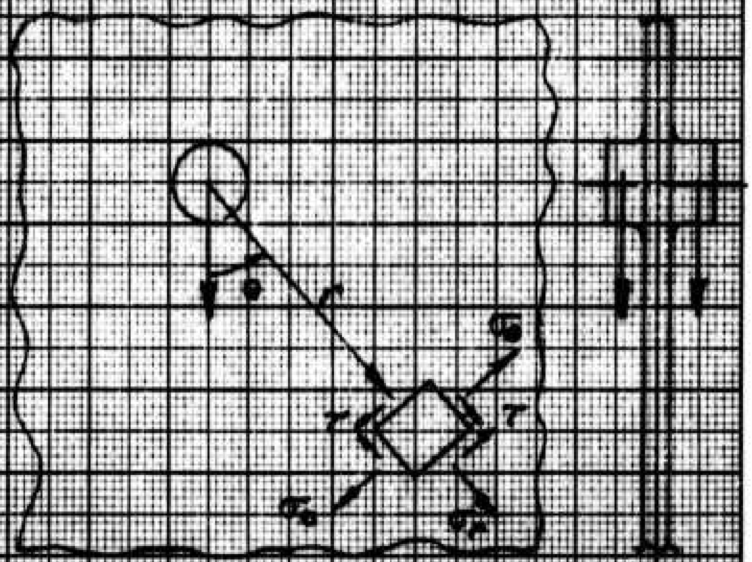


Fig.4.9(b) Local stresses due to a load applied at a hole



$$\frac{\sigma_r}{P/\pi} = \left(\frac{3+\mu}{4}\right)\left(\frac{\cos\theta}{r}\right) = -0.825\left(\frac{\cos\theta}{r}\right) \quad (\text{FOR } \mu = 0.3)$$

$$\frac{\sigma_\theta}{P/\pi} = \left(\frac{1-\mu}{4}\right)\left(\frac{\cos\theta}{r}\right) = 0.175\left(\frac{\cos\theta}{r}\right) \quad (\text{FOR } \mu = 0.3)$$

$$\frac{\tau}{P/\pi} = \left(\frac{1-\mu}{4}\right)\left(\frac{\sin\theta}{r}\right) = 0.175\left(\frac{\sin\theta}{r}\right) \quad (\text{FOR } \mu = 0.3)$$

Fig.4.10 Local stress due to a load applied at an integral lug

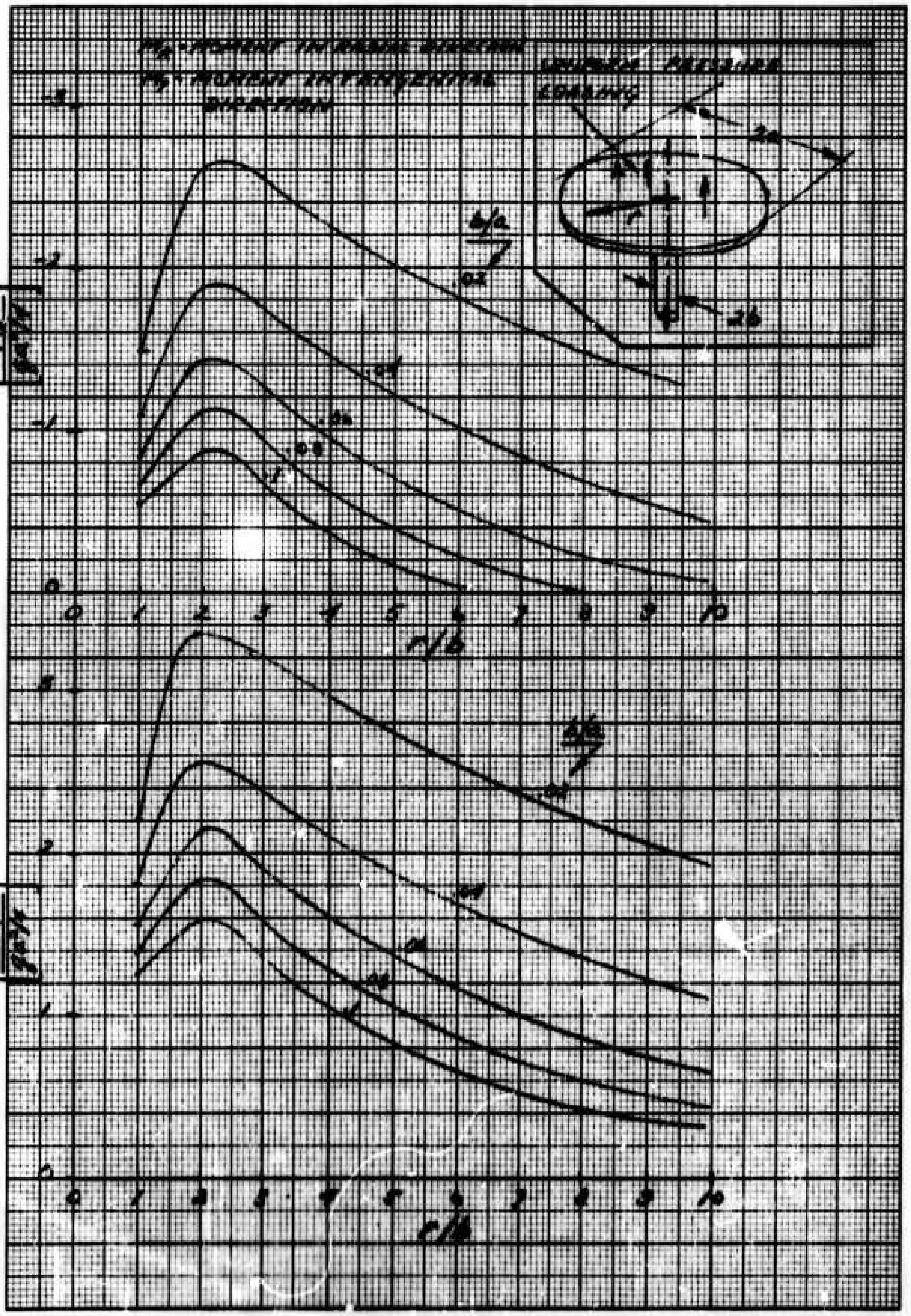


Fig.4.11 Bending moments in pressure loaded plate with concentrated reaction

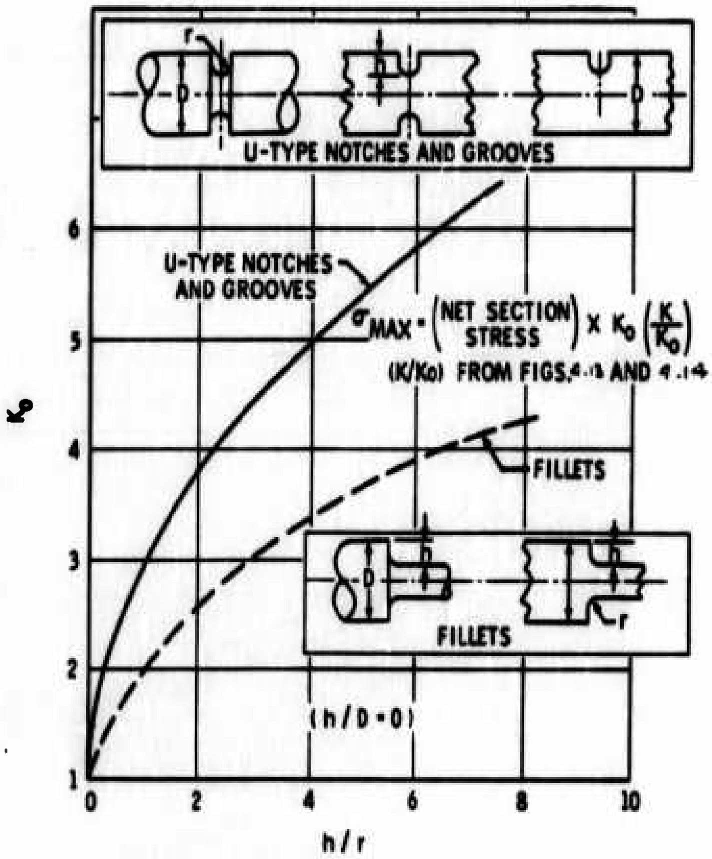


Fig.4.12 Stress concentration factors for bars and plates with small notches and fillets

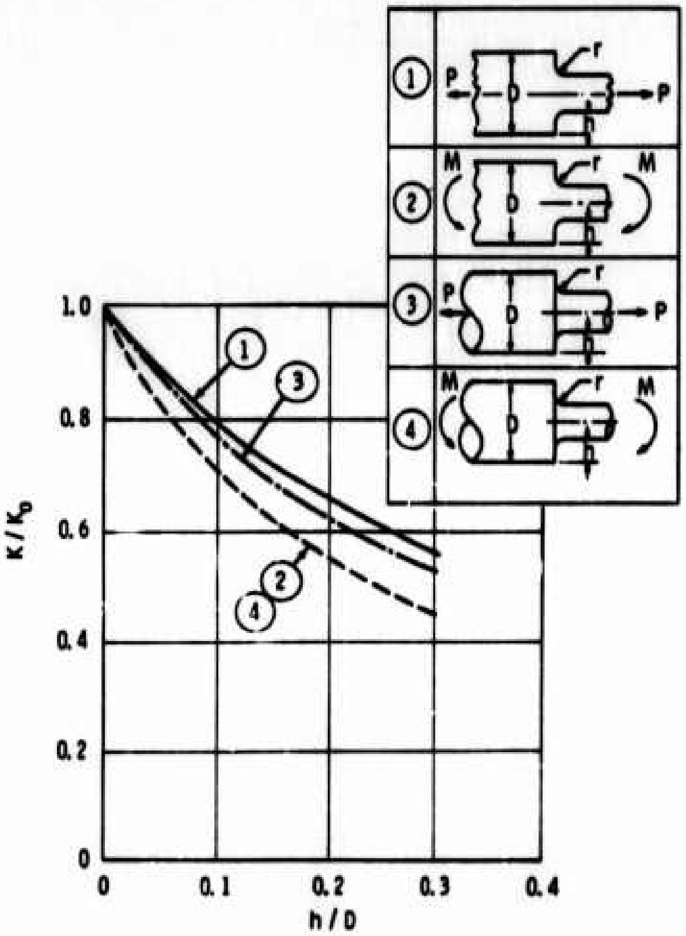


Fig.4.13 Stress concentration factor corrections for large fillets

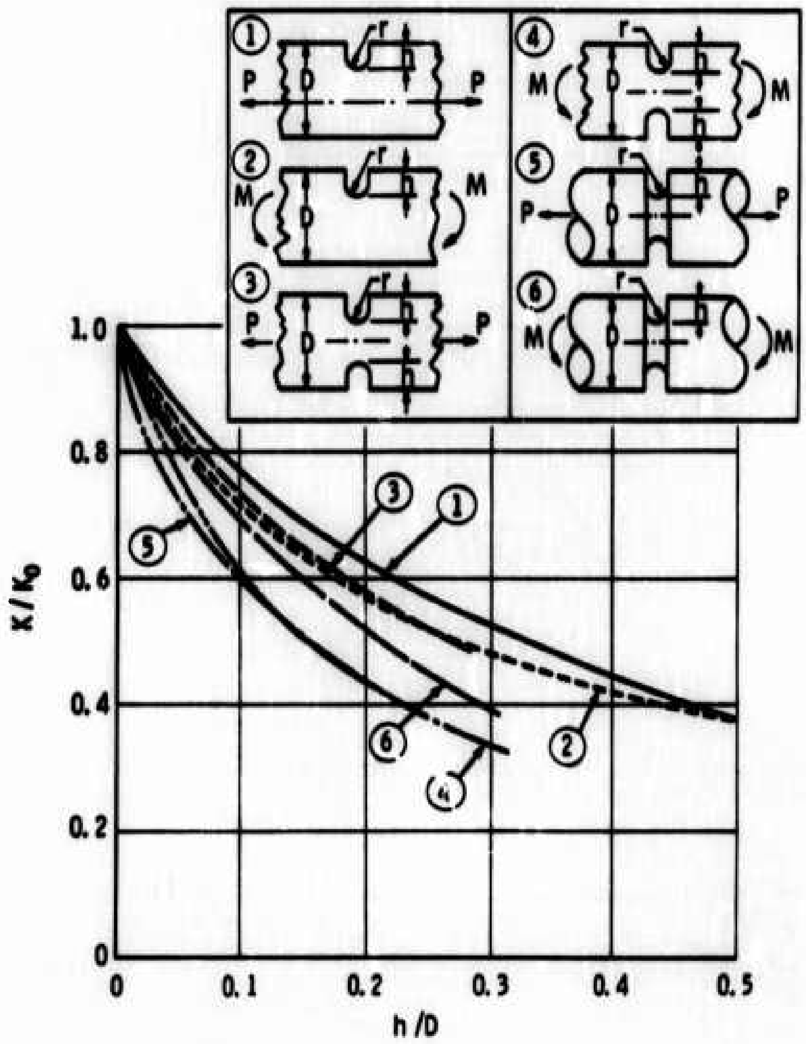


Fig.4.14 Stress concentration factor corrections for large grooves

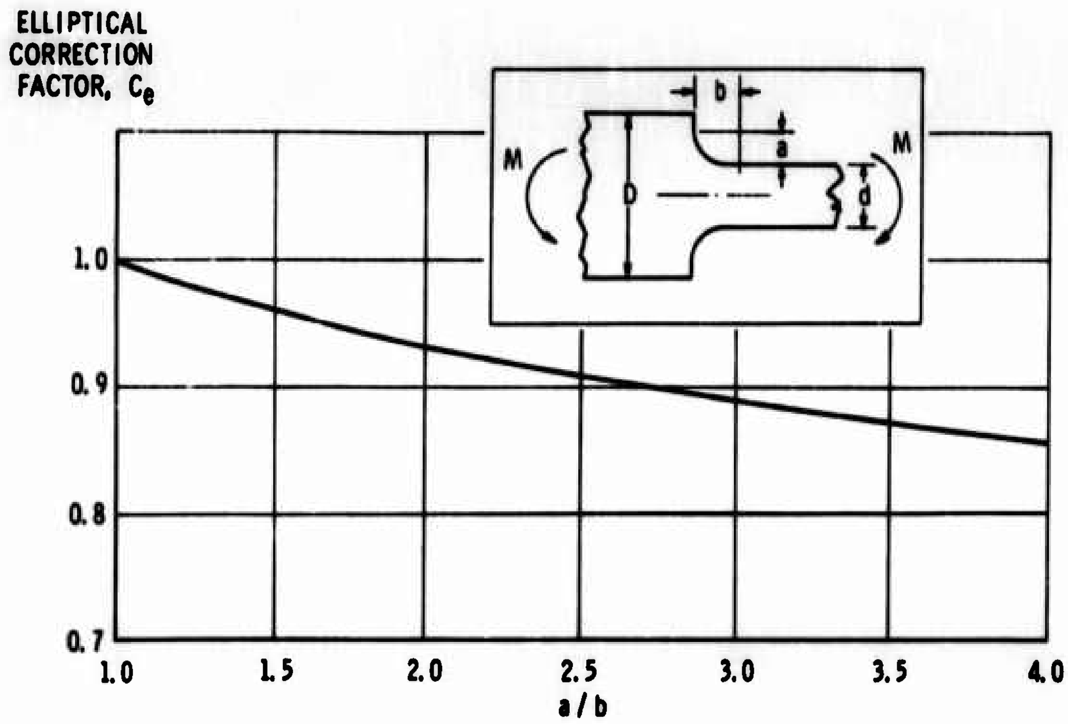


Fig.4.15 Stress concentration factor corrections for elliptical fillets

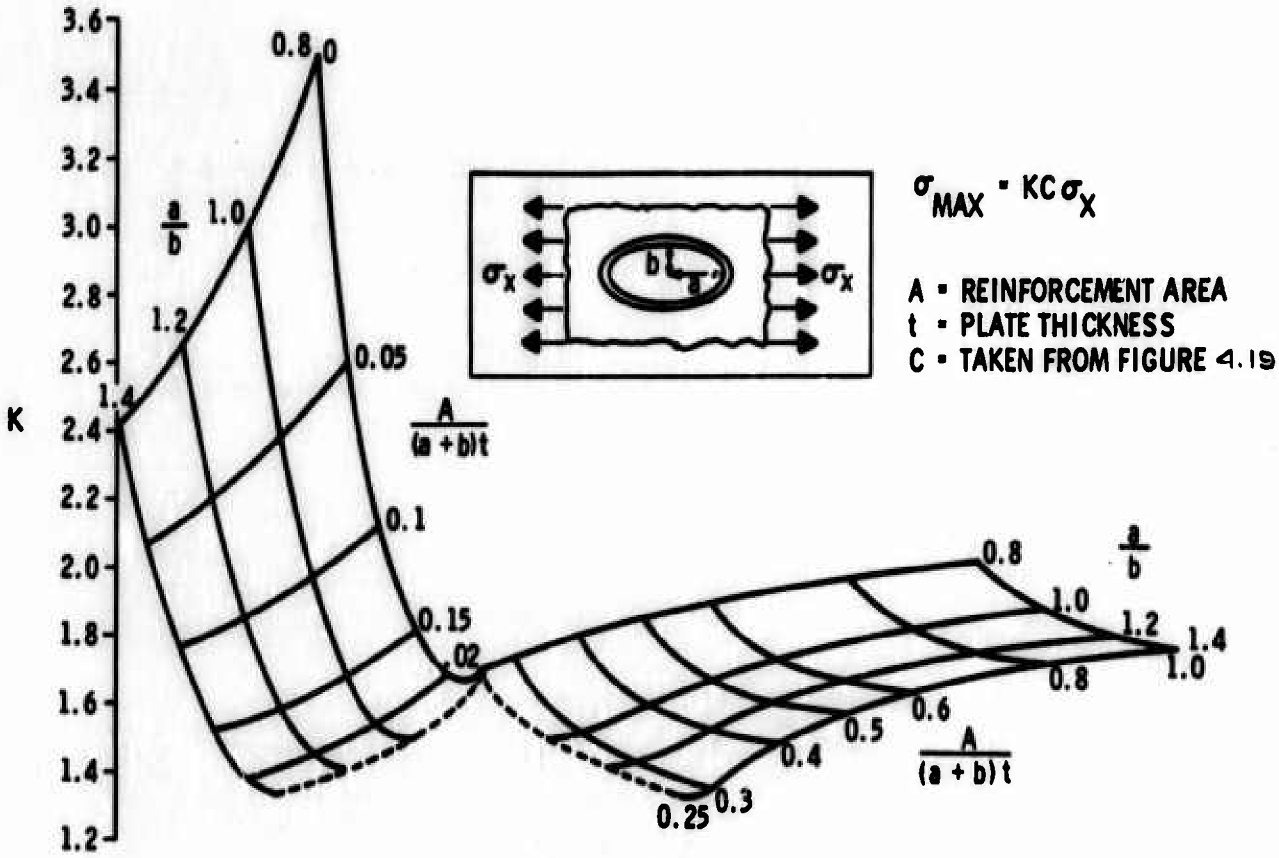


Fig.4.16 Stress concentration factors for reinfo.ced holes – uniaxial stress

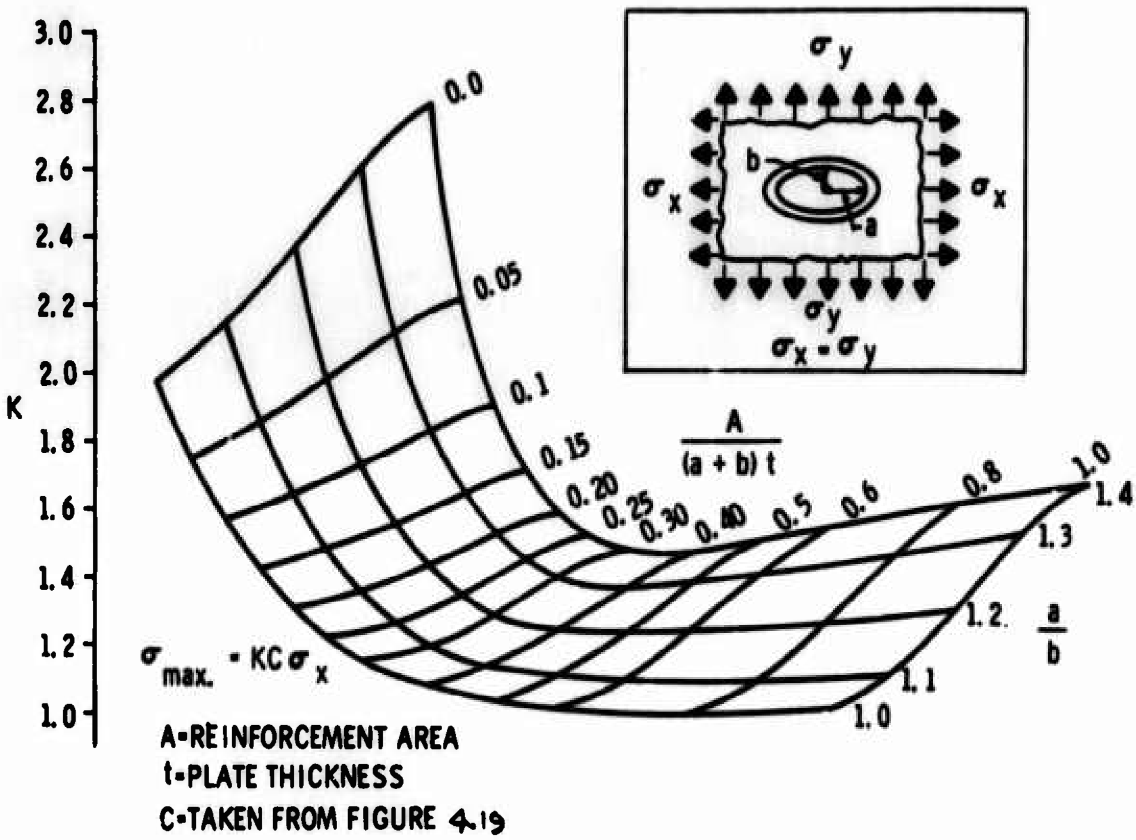


Fig.4.17 Stress concentration factors for reinforced holes – 1:1 biaxial stress

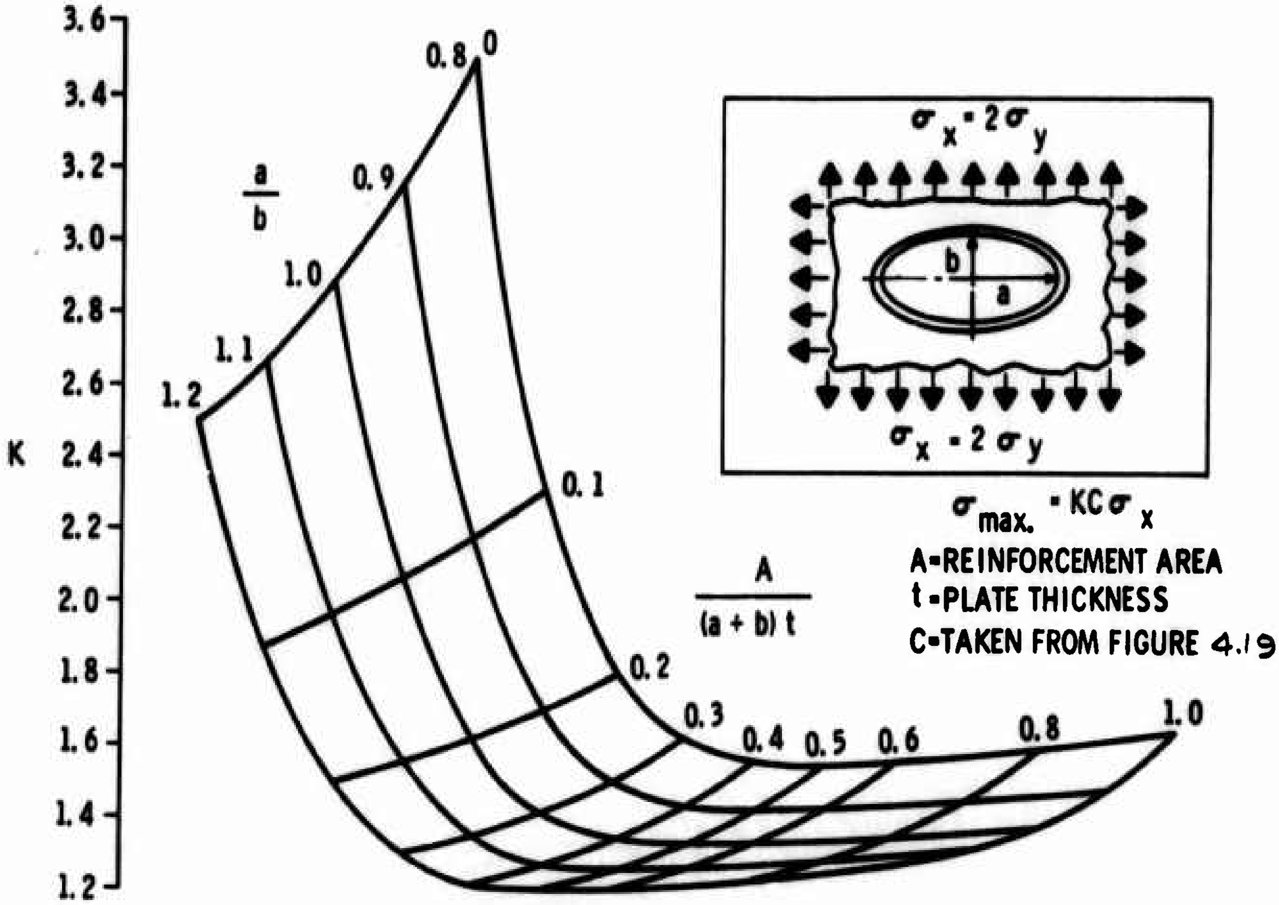


Fig.4.18 Stress concentration factors for reinforced holes – 2:1 biaxial stress

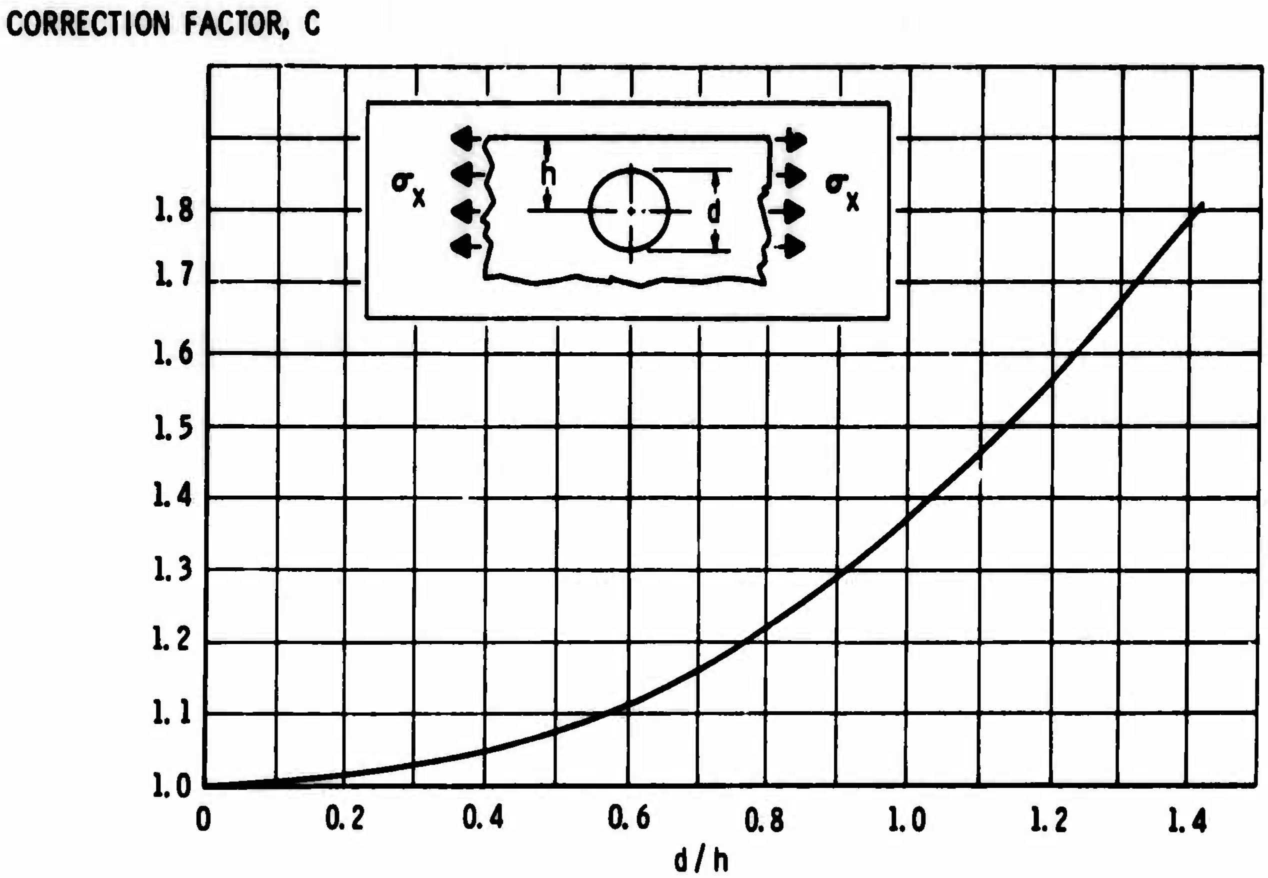


Fig.4.19 Stress concentration correction factor for holes near a boundary

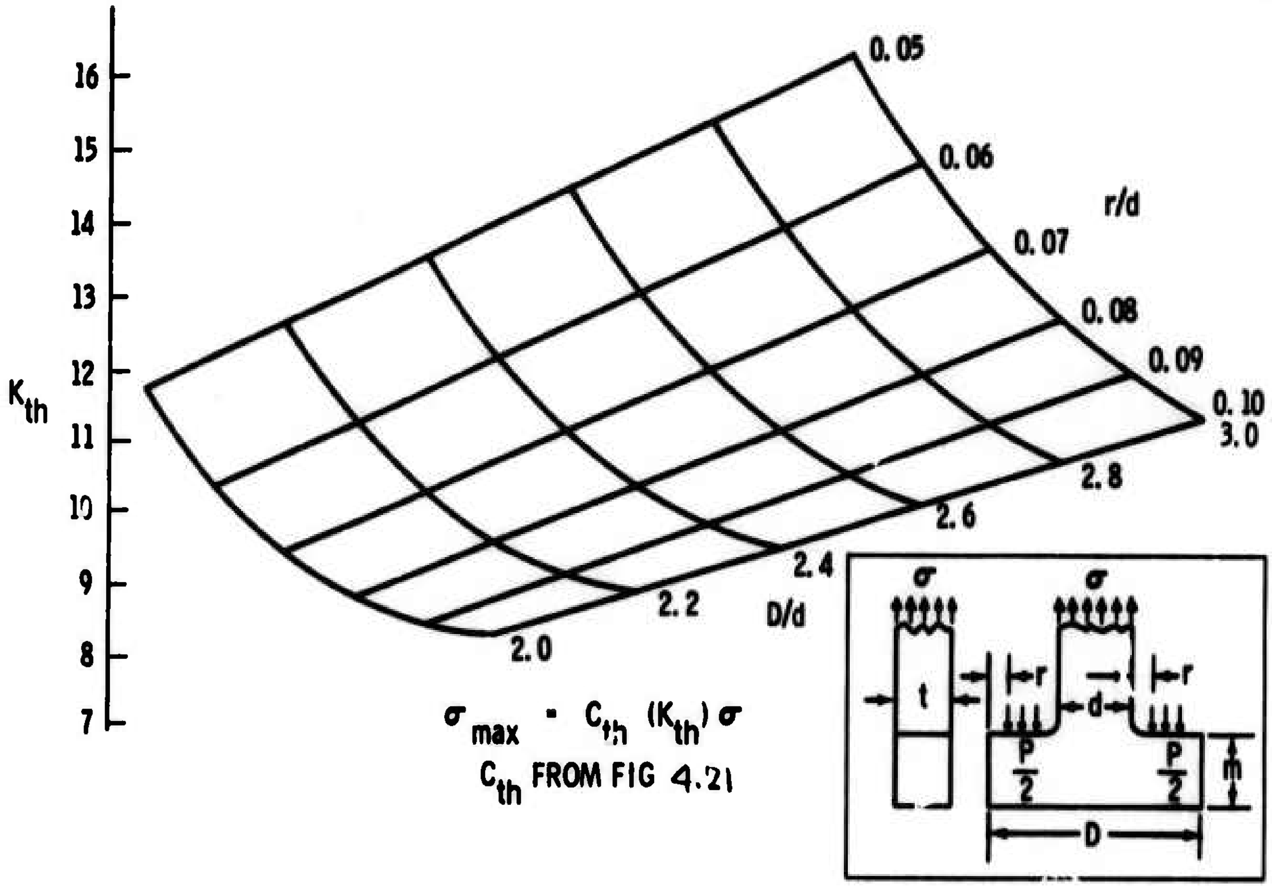


Fig.4.20 Stress concentration factor for "T" fitting

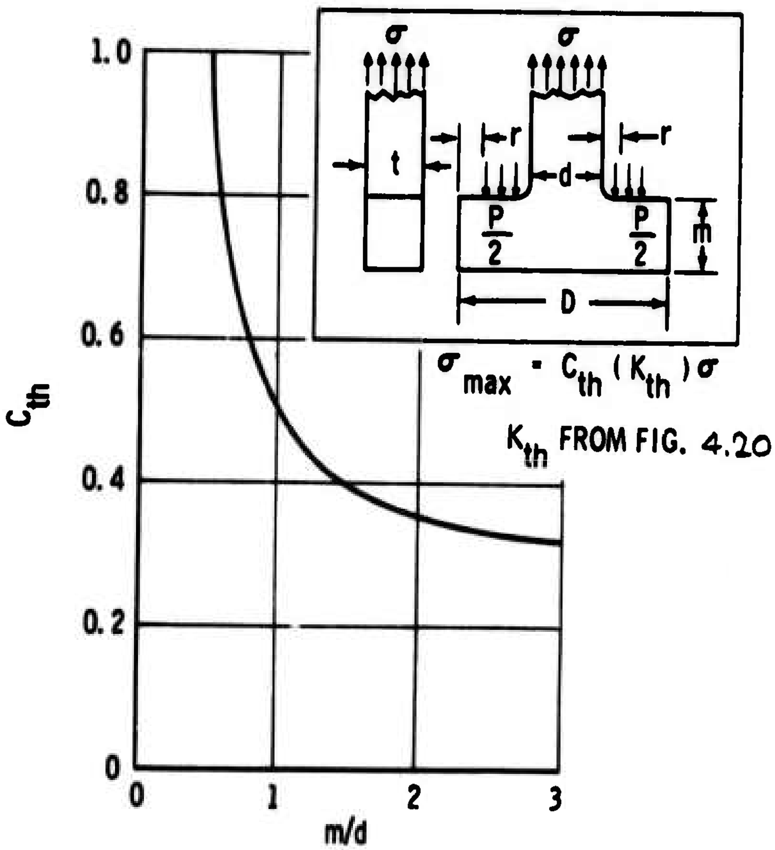


Fig.4.21 Stress concentration factor for "T" fitting

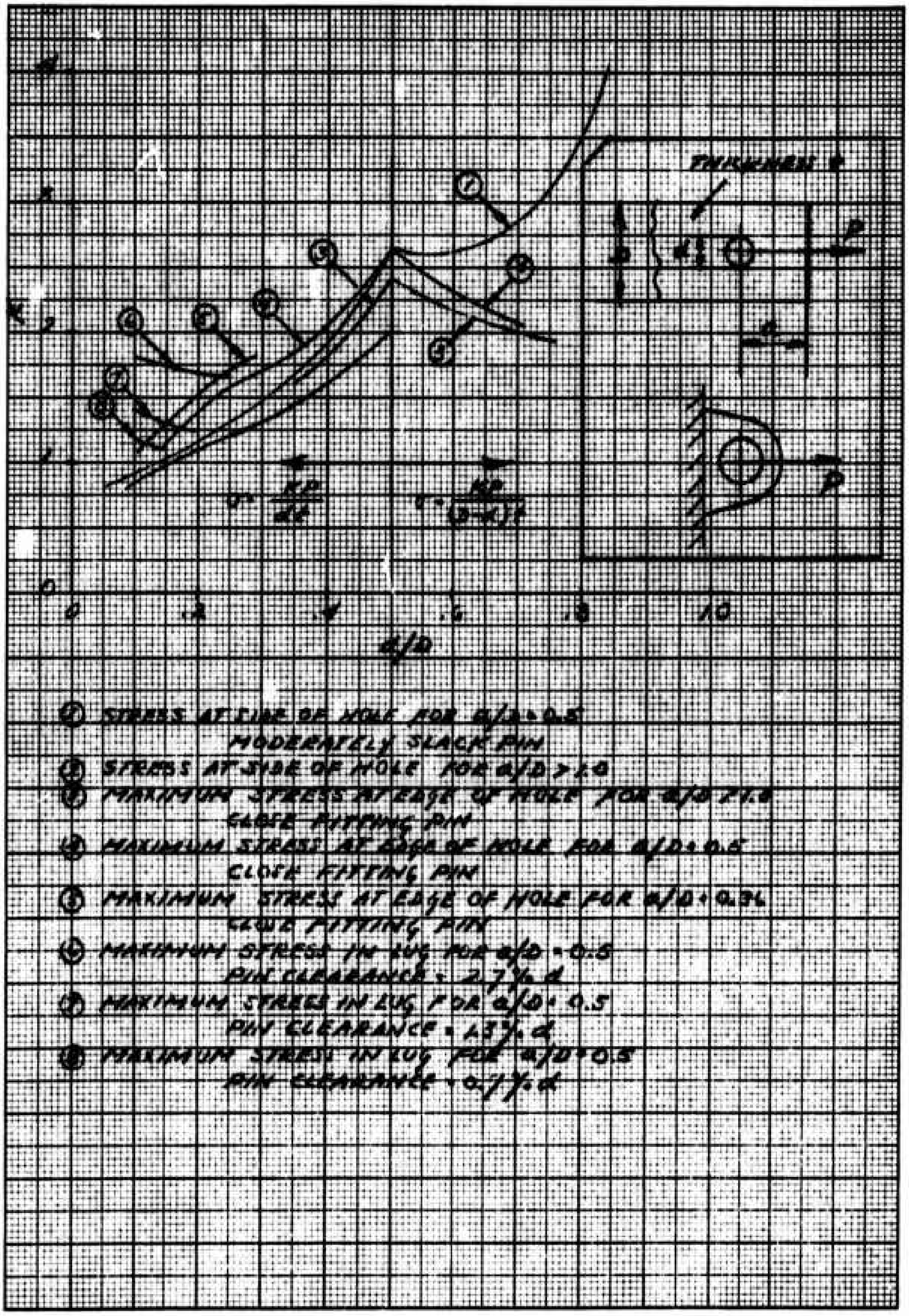


Fig.4.22 Stress concentration factors for lugs

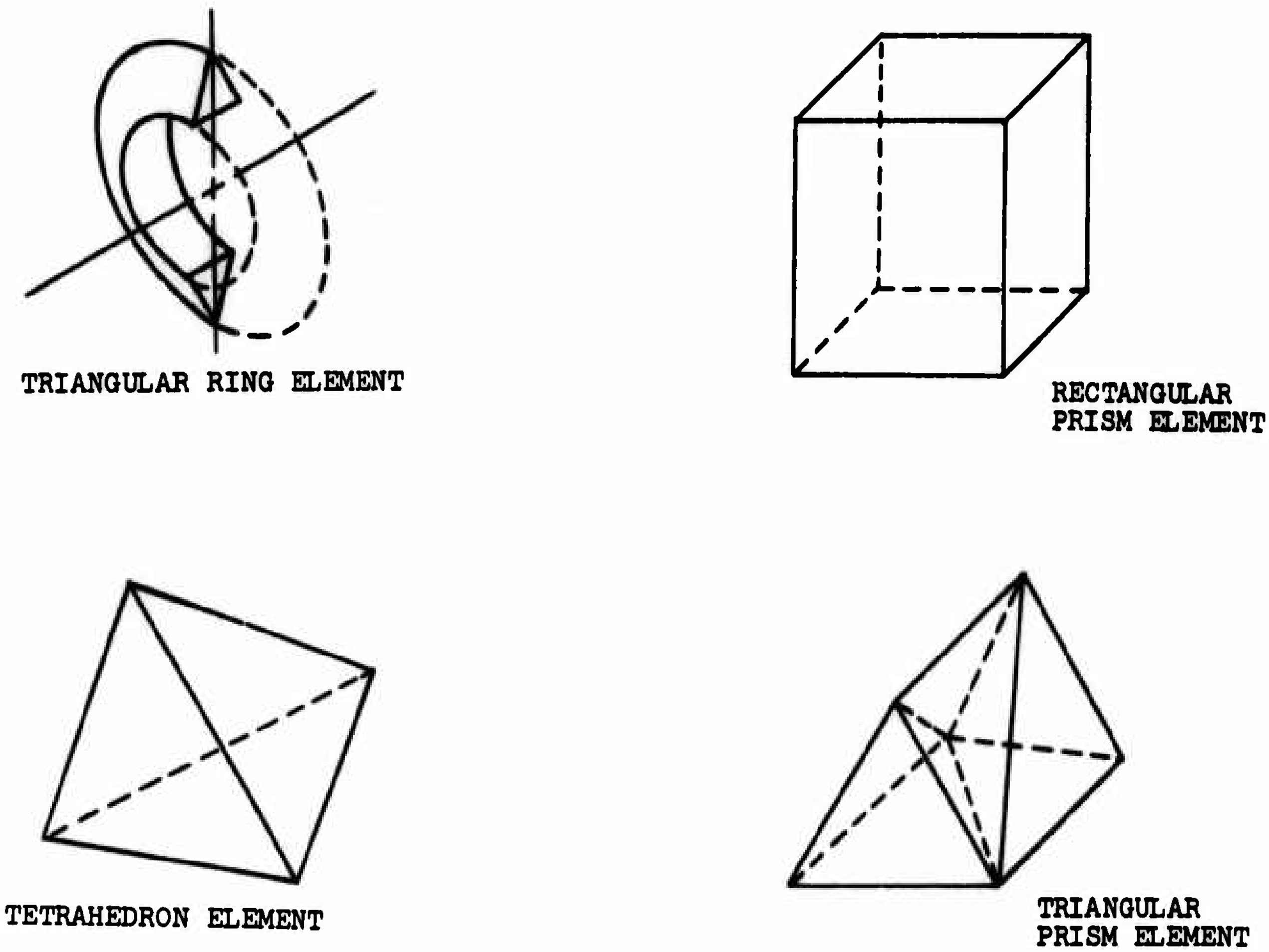


Fig.4.23 "Solid" finite elements

5. MATERIAL CHARACTERIZATION

5.1 GENERAL

In structural design with metallic materials, the designer has little difficulty in characterizing his materials or in determining mechanical and physical property data for design and analysis purposes. The situation is quite different with nonmetallic refractory materials; the lack of ductility makes these materials very sensitive to flaws, defects, structure, etc., which in turn makes them sensitive to every detail of the processing. As a result detailed standard specifications for ceramic materials are lacking, as are tables of mechanical and physical property data. In both cases the designer must, in general, develop his own.

To assist in this task, this Section presents general, but quantitative data on the effect of various material characteristics and features on mechanical properties, such as for instance, the effects of porosity and grain size on strength. This information should be useful in establishing the material characteristics required to achieve the most desirable properties. Information is also given on the subject of characterization to assist the designer in preparing material specifications and in arranging process and quality controls. The subject of strength under complex stress states is also covered, again without reference to specific materials, in view of its importance in stress analysis.

While mechanical and physical property data on specific materials have been avoided, because of their dependence on the material manufacturing process, data can be found in the literature if very approximate values are needed for preliminary design. A good source of such data is Reference 5.1. Even in the earliest stages of design, however, consideration should be given to the statistical aspects of strength, see Sections 3 and 6, and data for this purpose can rarely be found in the existing literature nor has it been possible to include such information in this Section.

5.2 MECHANICAL PROPERTIES

5.2.1 Fracture Mechanisms

The subject of fracture mechanisms in typical polycrystalline ceramic materials was discussed briefly in Section 2 where it was stated that current fracture theories provide little quantitative data that will help the designer select or develop the best material for a particular application. A certain amount of qualitative information exists, however, together with a number of semi-empirical relationships which show trends. This data should help the designer, and accordingly a more detailed discussion of the subject is presented here.

It is well for the designer and structures engineer to recognize at this point that the physical understanding and the theoretical treatment of fracture in brittle ceramic materials is not, at this time, in a very satisfactory state of development. A lack of complete understanding and analytical treatment of all of the phenomena associated with fracture is not surprising considering that interest in the application of ceramic materials for structural purposes has been very limited. What is discouraging, however, to the practicing engineer, is the lack, within the existing literature, of any attempt to relate the various concepts and incorporate them into a single rational theory. There is, for instance, the Griffith theory which assumes cracks and flaws, and predicts failure on the basis of energy unbalance. Another approach assumes dislocations and other defects in the atomic structure which move under applied stress until stopped by obstructions where the defects collect to form microcracks. In either case it is not clear what is happening at the crack tip, to cause fracture. Presumably, the only action of a crack is to concentrate the stresses, but except for magnitude they are identical to the stresses present in an uncracked piece of material acted upon by external loads.

Elsewhere within the literature there are indications that crack growth and fracture proceed when atomic bond strength is reached at the crack tips but the relationship between this concept and the dislocation theory is obscure. Similarly, there is no indication whether the atomic bond theory applies at grain boundaries in practical polycrystalline materials. The literature also discusses many sources of microcracks such as thermal expansion differences between different material phases, anisotropic thermal expansion within the crystals of a polycrystalline material, surface damage due to abrasion, etc., but again the real mechanism of fracture which is implicit in the presence of a crack is not explained.

In order to provide a framework for subsequent presentation of quantitative data on fracture, some clarification of this situation is desirable and accordingly the following is offered. Assume that the failure mechanism is rupture of atomic bonds, either within the crystal structure or at grain boundaries, in a polycrystalline material. Actual rupture is probably preceded by the movement of dislocations which in turn are caused by an excess or deficiency of atoms in the crystal structure, or by the collection of point defects. These defects move, under stress, until stopped by some obstruction such as a grain boundary, or another slip plane when they pile up until a crack is formed. Cracks can also be formed by mechanical twinning, at least in Al_2O_3 (Reference 5.2). The crack, in turn, concentrates the stresses until atomic bond breakage occurs and the crack propagates. As the crack lengthens elastic strain energy in the surrounding material is released, and is absorbed by the creation of new surfaces. When the energy released by

crack lengthening is greater than the surface energy of the new crack surfaces formed, the crack will propagate rapidly through the material (Reference 5.3) leading to complete failure. Presumably either or both of the mechanisms of dislocation movement and atomic bond breakage may be acting at the crack tip since both are the consequence of stress and both will lead to crack growth. Reference 5.2 states, in contrast to the above, that dislocations cannot move in a completely brittle material so that whether dislocation movement or atomic bond rupture is eventually responsible for crack growth probably depends upon whether the material has a slight degree of plasticity; another unresolved subject within the ceramic literature.

Almost certainly, the mechanisms described above occur first in areas of stress concentration within the material. Among such areas of stress concentration are pores and voids, flaws such as lack of bond between adjacent grain faces, the presence of impurities and inclusions which in effect create pores and lack of bonding, and surface irregularities causing geometric stress concentrations.

Sources of stress to cause crack growth and fracture, include externally applied loads, internal temperature gradients, localized thermal stresses due to thermal expansion differences between different phases in a multiphase material (References 5.4 and 5.2), anisotropic thermal expansion with respect to crystallographic direction in a polycrystalline material (Reference 5.4), volumetric expansion due to phase change (References 5.4 and 5.2), and stresses at the surface due to machining operations such as grinding (Reference 5.2).

Superimposed on the above situation is a stress corrosion effect which is important in some ceramics (References 5.5 and 5.4). Water vapor within the atmosphere certainly causes crack growth and premature failure in aluminum oxide under sustained stress, (References 5.6, 5.7 and 5.8). Some references (5.4) indicate that the effect of water vapor on aluminum oxide does not occur in polycrystalline material while, on the other hand, experimental evidence of the effect on polycrystalline alumina is given in Reference 5.8. The actual mechanism does not seem to be understood except that liquids may lower surface energy and hence lower strength (Reference 5.2). Liquids may also, however, have beneficial effects by rounding crack tips and hence increasing strength (Reference 5.2).

5.2.2 Strength and Stiffness of Ceramics

Porosity, grain size and the presence of flaws and microcracks are perhaps the most important factors affecting the strength of ceramic materials of a particular chemical composition. Trace impurities are also important, relative to their volume, since they tend to concentrate in grain boundaries where they create both defects and crack stopping mechanisms. Stiffness is affected by porosity but not by grain size, flaws, microcracks or grain boundary composition.

5.2.2.1 Porosity, Grain Size and Flaws

Below 10% porosity, which is the practical range of interest, approximations for elastic moduli are given in Reference 5.2, as follows:

$$G/G_o = 1 - 15\,(1 - \nu_o)\,P/(7 - 5\,\nu_o)$$

$$K/K_o = 1 - 3\,(1 - \nu_o)\,P/2\,(1 - 2\,\nu_o)$$

where G = shear modulus, K = bulk modulus, G_o and K_o are values at zero porosity, ν_o = Poisson's ratio and P = volume fraction of pores.

The above relationships are good for spherical pores; the reduction in stiffness could be increased by 50% for nonspherical pores and uneven distribution of porosity.

Other expressions for the effect of porosity on elastic modulus are given in Reference 5.4, as follows:

$E = E_o e^{-K_1 P}$ (Reference 5.9 gives a value of 3.95 for K_1 for alumina)

$$E = E_o \left[1 + \frac{K_2\,P}{1 - (K_2 + 1)\,P} \right]$$

$$E = E_o\,(1 - K_3\,P)$$

where E_o is the modulus for a nonporous body, P is the volume fraction porosity and K is a material constant.

Reference 5.2 also gives an expression for the effect of temperature on Young's modulus as follows:

$$E = E_1 - BT_e^{(-T_o/T)} \quad E_1 - BT_e^{(-T_o/T)}$$

where E_1 is modulus at absolute zero; T = absolute temperature; B and T_o are constants for each material. This expression is for a single crystal but is good for polycrystalline

materials below the temperature at which grain boundary sliding occurs. This is typically at about one half of absolute melting temperature.

The effect of grain size and porosity on polycrystalline ceramic strength is given by an equation proposed by Knudsen and reported in Reference 5.2 as follows:

$$S = S_o\ (GS)^{-a}\ \exp\ (-bp)$$

(GS) = average grain size, p is volume percent pores, S_o a and b are empirical constants. No typical values are given for these constants.

Reference 5.4 gives a similar expression for the effect of grain size on strength, as follows:

$$\text{Bond strength} = S_o + K\ G^{-1/2}$$

Where G = grain size, K is a material constant, S_o is frictional stress on an unlocked dislocation as it glides along a slip plane. S_o is frequently near zero.

Reference 5.2 also suggests that the effect of porosity on strength depends on the ratio of pore size to the size of flaw causing failure. This subject is treated more extensively in Reference 5.10 from which the following statements are drawn:

(a) If the pore size is large compared with the critical flaw size, so that the flaw lies entirely within the high stressed material, then the effect of the pore can be treated by the engineering stress concentration theory. On this basis the introduction of the first pore will produce a precipitous reduction in strength, typically a reduction of 50% as would be expected from a spherical or circular pore.

(b) If the pore size is small compared with a critical flaw then the flaw will control strength and will be unaffected by the pore. In this case strength should show a monotonic decrease with increasing porosity. Typically, the effect of porosity on high strength polycrystalline ceramics falls in this range and the Knudsen equation, presented previously, applies.

(c) If the flaw size is of the order of the pore size so that a segment of the flaw is subjected to stress concentration, then porosity will cause a precipitous decrease in strength with the first pore, but not to a value corresponding to the appropriate stress concentration factor. The Knudsen equation will be unconservative for this case, but the engineering stress concentration theory will be overconservative.

Reference 5.10 also gives a relationship for the effect on strength of small additions of a second phase material. On the other hand, a second phase material effectively creates a pore, but since there is some continuity of load carrying capability across the pore as a result of the strength and stiffness of the second phase material, the effective stress concentration factor is reduced. The expression given, for a biaxial stress state, is:

$$S = \frac{S_o}{K}\ (1 - \phi)^{-1/2}$$

where ϕ = volume fraction displacement of second phase and K is the stress concentration factor which includes the effect of the second phase material. Expressions for K can be found in Reference 5.11.

When a porous ceramic is subjected to a temperature gradient the pores have a disturbing effect on the temperature field. Local thermal stresses are produced, in the vicinity of the pore, in addition to the overall thermal stresses produced by the applied gradient.

From Reference 5.12 this micromechanical thermal stress is given by:

$$\sigma_m = K\ \alpha\ E\ \tau\ a$$

where α = thermal expansion coefficient

τ = temperature gradient

a is a measure of cavity size

K is a constant depending on cavity shape and is 1 for a cylindrical hole perpendicular to the heat flow.

This local thermal stress must be added to the macromechanical thermal stress produced by the temperature gradient. Reference 5.12 points out that this micromechanical thermal stress may have a beneficial effect by initiating fracture at many pores. Further propagation will then occur by a large number of cracks limiting the extent of each due to a fixed available elastic energy for a self-contained thermal stress situation.

In Reference 5.9 Gitzen states that for alumina, brittle fracture appears to be controlled by the Griffith Orowan mechanism in which pre-existing surface defects produced by grinding

propagate at a critical stress given by

$$\sigma_{cr} \approx \sqrt{\frac{E\gamma}{C_o}}$$

C_o is the critical crack length, which is approximately the grain size, typically 15 microns.

E is Young's Modulus.

γ is surface energy. Data on values for surface energy are given in References 5.13 and 5.28.

5.2.2.2 Environmental Effects

The literature contains a number of references to the effect of atmosphere on the propagation of cracks in a body under sustained stress, (static fatigue) but little useful quantitative data is given.

Reference 5.8 reports results for an aluminum oxide tested in an uncontrolled atmosphere, under constant static stress. At stresses above 75% of the short time ultimate the life of the material was only a few minutes. To avoid damage by chemical attack, it is commented that sustained stresses should not exceed 60% of the short time ultimate.

References 5.6 and 5.7 study crack growth in glass and sapphire as a function of moisture content in the atmosphere and theoretical relationships are given for glass. In the absence of data on other ceramics, these relationships might serve as a guide but they must be used very cautiously since glass is not considered typical of other ceramics in this respect.

Reference 5.6 examines the effect of moisture on crack velocity and shows, for glass, some very large effects. Orders of magnitude differences occur in crack velocity, for a given load or stress, as the relative humidity varies from near dry to 100%. At any given atmospheric moisture content the curves of crack velocity plotted against load, have a characteristic shape involving three regions.

In region I there is an exponential increase of crack velocity with applied load. The environmental effect is an activation process dependent on the chemical potential of the water vapor in the environment and is reaction rate limited.

From 5.7, for region I,

$$\text{crack velocity} = (.0275a \ \mathrm{x}_o^{\,n} \ \ell^{bP})/n$$

a & b are constants

n is the order of the chemical reaction with respect to water.

P is applied force

x_o is mole fraction of water in the gas

In region II the fracture mechanism changes to a transport rate limited process in which the stress activation process at the crack tip is faster than the rate at which water vapor can diffuse to the tip. Hence crack velocity will be stress independent.

From 5.7, for region II, $\text{crack velocity} = .0275 \ c \ D_{H_2O} \ \mathrm{x}_o/\delta_n$

D_{H_2O} is the diffusivity of water vapor in the gas

c is a constant

δ is the thickness of the gas boundary layer in the crack.

In region III the crack velocity is again exponentially dependent on the applied force but the curve has a much steeper slope than region I. There is presently no physical explanation available for this region.

5.2.2.3 Strength Under Thermal Stresses

Typically the strength of a structural element subjected to temperature gradients would be checked in the conventional manner by calculating the resulting thermal stresses and comparing with the material fracture strength. However, a system of thermal stresses is self-contained within the structural element and represents a fixed amount of stored elastic energy. Consequently, crack nucleation, once initiated, may or may not proceed to complete fracture depending whether the body contains sufficient elastic energy to create enough crack surface for the crack to propagate completely through the body.

This idea is examined in Reference 5.13 where the concept of degree of damage, resulting from thermal stress, is introduced. In this reference the stored elastic energy due to thermal stress is equated with surface energy to determine the degree of crack propagation relative to the size of the element. The following results are obtained:

To prevent crack nucleation due to thermal stresses the material should have high strength and thermal conductivity but low elastic modulus and coefficient of thermal expansion.

To minimize material damage, once cracks have been created, requires low strength and high elastic modulus.

Porosity lowers strength, modulus and thermal conductivity and hence lowers the thermal stress fracture resistance of a material, but it also reduces the degree of damage if cracking begins. The effect of pores in reducing stored elastic energy is believed to be more important than their effect as crack arrestors (Reference 5.12).

Pores and dispersed second phase material also reduce thermal stress damage by introducing stress raisers and initiating early cracking while the average stress, which is the driving force for crack propagation, is low (Reference 5.14).

The above ideas are carried further in Reference 5.14 where it is recognized that the residual strength after crack propagation by thermal stresses, is the important consideration. From the resulting analysis it is determined that the extent of crack propagation is a function only of the number of cracks and the initial flaw size and is independent of material properties. Thus the only way to minimize crack propagation (under thermal stresses), and to maximize the resulting load carrying capability of a given material is to increase the length of the critical flaw.

5.2.2.4 Fatigue Strength

In ceramic materials both fatigue under cyclic loads and the so-called static fatigue are recognized. The latter is generally believed to be caused by environmental effects and has already been discussed. It must be recognized, however, that during repeated loading testing both effects will usually be present.

Few references are available on the subject of cyclic fatigue in ceramics. Reference 5.8 reports tests on alumina conducted in an uncontrolled atmosphere, with the stress cycled in tension only. From this work the allowable stress at 10^5 cycles is approximately one half of the single load ultimate. The curves given do not flatten very much with number of cycles, to 10^6, so that there is no indication of an endurance limit. Furthermore, the effect of stress ratio is not particularly significant.

Reference 5.5 gives a cyclic endurance limit for alumina of 50% of the short time static ultimate.

5.2.2.5 Strengthening Mechanisms

Grain Size

The effect of grain size on strength has been indicated above in various theoretical relationships from which it will be evident that a decrease in grain size will increase strength. This conclusion is generally drawn from bend test data, which is very sensitive to surface defects, and presumably the defect size is related to grain size. Fracture testing, however, leads to opposite conclusions. Fracture energy increases with increased grain size, Reference 5.15.

Practical ceramics are polycrystalline materials with grain boundaries which are laden with other than the basic material. Fracture energy is a composite quantity derived from the energies of the grains and the intergranular material and is therefore dependent on the ratio of transgranular to intergranular fracture which in turn is dependent on temperature, grain size, intergranular material, and grain boundary thickness. The crack path is also affected by the geometric arrangement of the material since a large hard grain will make the crack go through high fracture energy material or take a long path around. Therefore, large grain size favors high fracture energy. It is concluded, in Reference 5.15, that optimum strength is obtained with fine grain surface texture and a coarse grain interior.

Surface Conditions

Rupture strength tends to be sensitive to surface conditions so that surfaces may be coated or glazed to increase strength. Coating provides protection for the surface or it acts as a barrier to the exit of dislocations. Glazing removes surface damage but it may develop surface crazing (Reference 5.4). Quenching has also been used as a strengthening mechanism (Reference 5.16). It is assumed that quenching leads to compressive stresses in the surface, but some investigators have found the mechanism to be a volume rather than a surface effect. Strength increases, at room temperature by a factor of 2.0, are reported (5.16) from a combination of quenching and glazing in conjunction with a prior high temperature treatment in a fluorine containing atmosphere.

Reference 5.16 also indicates, as would be expected, that too rapid quenching can produce thermal shock.

The chemical addition of ions or compounds to the surface is also reported to increase strength by the introduction of surface compressive stresses.

Composition Effects

Single element additions, for example C in TiC or Titanium in ZrO_2, are mentioned in Reference 5.4 as having a limited effect on strength but marked improvements in thermal shock resistance.

Additives are often used to increase density or rate of densification in sintering and strength may be increased through increased density and reduced porosity, prevention of grain growth, chemical reactions increasing bond strength, and grain boundary segregation (Reference 5.4).

Some additives to ceramics will cause precipitation hardening similar to that found in metals. The strength increase is due to blocking of slip or dislocation motion or in very brittle materials to crack stopping. The addition of a fine uniform dispersion of a second ceramic phase can also increase rupture strength by impeding dislocation motion and limiting crack and flaw size (Reference 5.4).

If a second metallic ion is added uniformly to a matrix (i.e. tantalum ions added to tantalum carbide) so that the ion is in solid solution the rupture strength may be increased (Reference 5.4).

Work Hardening

Ceramics may be work hardened by various deformation methods significantly raising rupture strength. Deformation techniques vary from slow tension to explosive shock. Powders can be explosively shocked, and after pressing and sintering the work hardening is maintained.

5.3 FAILURE UNDER COMPLEX STRESS STATES

In Sections 2.5 and 2.6 the subject of failure and fracture of brittle materials is discussed briefly, and it is explained that present knowledge of fracture does not permit the prediction of mechanical properties on a theoretical basis. In Section 5.2 a number of empirical relationships have been given to show the characteristic effects of certain parameters such as porosity, on strength; the only other type of general strength data which is available to the designer are a number of theories which define strength levels under complex stress states. These theories are established on phenomenalogical grounds; none are completely satisfactory and considerable differences of opinion exist about their relative merits.

Some of the theories proposed for brittle failure are discussed briefly in Section 2, and others have evolved very recently since Section 2 was prepared. All of the theories to be discussed are summarized in Figure 5.1, for biaxial stress states, as envelopes of combinations of principal stresses which can be sustained without failure. No data applicable to brittle materials has been found in the literature on triaxial stress states.

5.3.1 Maximum Normal Stress Theory

The most commonly used criteria for describing the biaxial fracture strength of brittle materials is the maximum normal stress theory, which assumes that the material will fracture when one of the principal stresses becomes equal to the uniaxial strength. With this theory the strength is not affected by principal stresses other than the maximum. The criterion makes no prediction of compression strength in terms of tensile strength and both values must be determined experimentally from uniaxial tests. Reference 5.17 presents early test data for relatively brittle materials which shows good confirmation of the theory in both the tension-tension and tension-compression quadrants. However, the range of tension compression values included is very limited and Reference 5.17 also quotes more recent test data to show that the theory is not valid in the tension compression quadrants. Reference 5.18, on page 208, refers to a modified form of the maximum normal stress theory which takes some account of the principal stresses other than the maximum, but there is no method for applying this modification in the absence of extensive test data.

5.3.2 Mohr's Failure Theory

Another important failure theory is Mohr's theory of strength which formulates the condition of material failure in a general manner and can be used for either ductile or brittle materials. It accommodates failure either by fracture, when the largest tensile normal stress has reached a limiting value depending on the properties of the material, or by slip, when the shearing stress in the plane of slip has reached a limiting value. The theory provides also, in the latter case, that the limiting shearing stress can depend on the normal stress acting across the same plane.

Mohr's theory is developed in detail in Reference 5.18 considering first its application to ductile materials in which failure is represented by slip. Based on observations of the orientation of slip lines, which are detectable on the surface of deformed metals, Mohr neglects the intermediate principal stress σ_2. Then a failure condition is represented by the major principal stress circle corresponding to σ_1 and σ_3 and plotted on $\tau - \sigma$ axes. A number of major principal stress circles corresponding to failure determined experimentally for various combinations of σ_1 and σ_3 can be drawn and Mohr postulates that the envelope of all of these maximum principal stress circles is the limiting curve describing failure. This is illustrated in Figure 5.2 in which the points P denote the planes on which failure occurred for the various principal stress combinations. In other words the theory assumes that failure conditions are described by the envelope of any and all principal Mohr circles representing stress states on the verge of failure. Clearly, failure can be used to describe either yielding or fracture.

In general, sufficient experimental data is not available to construct the Mohr envelope and a simplification is made by assuming that the envelope curves consist of two straight lines. These can be determined from only uniaxial tension and compression data. In this form the Mohr theory is the same as a modification of the maximum shear stress theory in which sliding along the slip planes is assumed to be inhibited by friction associated with compression stresses acting on these planes. The maximum shear stress theory and its modifications are not of direct interest for brittle materials but details can be found in Reference 5.18 where it is attributed to Coulomb, and Reference 5.19 where it is attributed to Reyto.

The simplified Mohr theory is shown in Figure 5.3 Reference 5.17 which refers to this theory as the Coulomb-Mohr theory, defines the straight lines by the equation:

$$\frac{\sigma_1}{\sigma_{tu}} \quad \frac{\sigma_2}{\sigma_{cu}} = 1$$

where σ_{tu} and σ_{cu} are the uniaxial tensile and compressive strengths, respectively.

In order to apply the simplified Mohr theory to brittle materials a number of considerations must be made. Referring to Figure 5.3 extension of the straight line envelope to point A implies tensile strengths greater than those demonstrated in the uniaxial test used to define σ_{tu}. In fact, if the material could not fail in shear (as is often assumed for brittle nonmetallic materials) and if the presence of a shear stress had no effect on the tensile strength, then the envelope would become a vertical line through point D. If an experimentally determined compression ultimate strength is available, however, it probably implies that the material can also fail in shear. In this case another envelope boundary would be a horizontal line through point F. If there is an effect of shear stress on the tensile strength, then the shear boundary will have some slope such as CB. In the absence of test data other than tensile and compression ultimate, it would clearly be unwise to use any other straight line boundary than CB. Similarly it is conservative, in the absence of additional test data, to assume that at point B the type of failure becomes cleavage and that the boundary follows the circular arc BD. Any other assumption implies circles through point D with diameters greater than OD. Such circles would exceed the boundary CB and would therefore be inconsistent. Similar reasoning requires that the compression boundary be the circular arc CE.

The Mohr envelope is shown for biaxial stress combinations as a $\sigma_1 - \sigma_2$ plot in Figure 5.1. The use of the circular arcs BD and CE in Fig. 5.3 produces the same results as the maximum normal stress theory in the tension-tension and compression-compression quadrants. In the tension-compression quadrant the use of the simplified Mohr theory produces a linear variation between the uniaxial tensile strength and the uniaxial compression strength. Since the Mohr criterion uses experimental values for uniaxial tension and compression it obviously predicts correctly the ratio of compressive to tensile strength.

5.3.3 Griffith Failure Theory

The Griffith failure theory, which was developed for brittle materials, also provides a basis for biaxial failure criterion by assuming that failure occurs when a critical tensile stress is reached on the boundary of the crack.

In Reference 5.3 Griffith develops relationships between applied principal stresses σ_1 and σ_2 and the critical tensile strength of the material. By using these relationships for a uniaxial tensile case, the critical strength can be expressed in terms of the conventional uniaxial ultimate strength σ_{tu}. The result can be substituted back into the biaxial stress expressions giving the following:

If $(3\sigma_1 + \sigma_2)$ is positive: $\quad \sigma_1 = \sigma_{tu}$

If $(3\sigma_1 + \sigma_2)$ is negative:

$$\left(\frac{\sigma_1}{\sigma_{tu}} - \frac{\sigma_2}{\sigma_{tu}}\right)^2 + 8\left(\frac{\sigma_1}{\sigma_{tu}} \quad \frac{\sigma_2}{\sigma_{tu}}\right) = 0$$

These expressions apply only if $\sigma_1 > \sigma_2$.

From the above expressions the uniaxial compression strength is predicted as eight times the uniaxial tensile strength, which is in good agreement with some test results on brittle materials, but not all. The expressions also show that $\frac{\sigma_1}{\sigma_{tu}} = 1$ for any biaxial stress situation where σ_2 is tensile or if compressive, is numerically less than three times σ_1. If σ_2 is compressive and numerically greater than three times σ_1 the quadratic equation above must be used. Actual values are given in Figure 5.1.

From Figure 5.1 the Griffith criterion coincides with the maximum normal stress criterion in the tension-tension quadrant. It also predicts an increase in the compression strength in the maximum principal stress direction due to the presence of a compression stress in the direction normal to the maximum principal stress. This conclusion is questionable, although there is very little experimental data to provide any basis for verification.

5.3.4 Weibull Failure Theory

The Weibull theory, which is a statistical theory described at length in other sections of this handbook, also provides a basis for predicting material failure under biaxial stress. The derivation of the relationship between the applied principal stresses, σ_1 and σ_2, and the material strength is given in Reference 5.20. The resultant relationships are complex and difficult to use and are not reproduced here. A typical result, for an m of 12, is shown in Figure 1. In the tension-tension quadrant, it predicts failure stresses which are generally less than those predicted by all other theories but which are consistent with some limited test data (Reference 5.21). The Weibull theory does not predict a compression failure, which is inconsistent with observation, and it predicts that in the tension-compression quadrant the tensile strength is increased beyond the uniaxial value by the presence of a normal compression stress. Again, some very limited test data is available which can be used to substantiate this prediction, but there is also much which disputes it.

5.3.5 Babel-Sines Failure Theory

Among the more recent activities intended to develop improved biaxial failure criteria is work by Babel and Sines (Reference 5.22) which generalizes the work by Griffith. Griffith considered only the limiting case of sharp strength controlling discontinuities. Many brittle materials, particularly porous materials, show discontinuities which are not sharp, and the work of Babel and Sines therefore included the influence of discontinuities of all degrees of severity. This was done by studying the effect of flaws of ellipsoidal shape on the fracture strength of brittle materials and using the assumption made by Griffith that the flaw will extend to failure when a critical tensile stress is reached on the boundary of the discontinuity.

In practice neither the shape of the flaw nor the critical tensile stress are known but these "microscopic" parameters can be replaced by "macroscopic" ones, the uniaxial compression strength σ_{cu} and the uniaxial tensile strength σ_{tu}, which can be determined experimentally. The resulting equations expressing σ_1 and σ_2 in terms of σ_{tu} and σ_{cu} are as follows:

$$\frac{\sigma_1}{\sigma_{tu}} = 1 - \frac{\sigma_2}{\sigma_{tu}} \left(\frac{\left(\frac{\sigma_{cu}}{\sigma_{tu}} - 8\right)}{\frac{\sigma_{cu}}{\sigma_{tu}} + 4 + 4\sqrt{\frac{\sigma_{cu}}{\sigma_{tu}} + 1}} \right)$$

$$\frac{\sigma_1}{\sigma_{tu}} = \frac{\sigma_2}{\sigma_{tu}} - \frac{1}{2}\left[\frac{\sigma_{cu}}{\sigma_{tu}} - \left(\left(\frac{\sigma_{cu}}{\sigma_{tu}}\right)^2 - 8\,\frac{\sigma_{cu}\,\sigma_2}{\sigma_{tu}^2} \right)^{\frac{1}{2}} \right]$$

The first of the above equations applies in the tension-tension quadrant and partly into the tension-compression quadrant while for high ratios of compressive to tensile stress the second equation applies.

The result of this work is a criteria which predicts a uniaxial compression strength between three and eight times uniaxial tensile strength depending on the shape of the discontinuity. The extreme values of three and eight are obtained from the limiting cases of a sphere and a sharp crack, respectively. Accordingly, in the tension-compression quadrant the Sines criteria gives envelopes which lie between those given by the Mohr and Griffith criteria. Consequently, it is also inconsistent with some test results which show compression strengths greater than eight times uniaxial tension. This is thought to be due to the fact that under high compression loads some cracks will close and stresses can exist on the boundary of the discontinuity.

In the tension-tension quadrant the Sines criteria predicts an increase in strength over uniaxial values as a result of the biaxial stress state which is also questionable on the basis of experimental data.

5.3.6 Maximum Strain Energy Theory

Reference 5.18 dismisses this theory, which assumes that the total elastic energy stored in the material is significant as a limiting condition, because it is not applicable to the hydrostatic pressure case. Under this latter condition large amounts of elastic energy may be stored without causing fracture. Nevertheless, it is presented in Reference 5.23 to correlate with experimental data determined for brittle case iron and it is used in Reference 5.24, in modified form, to represent the results of tests on magnesium silicate and graphite. This modified maximum strain energy theory has the following form: (Reference 5.24)

In the tension-tension quadrant

$$\left(\frac{\sigma_1}{\sigma_{tu}}\right)^2 - 2\mu\left(\frac{\sigma_1}{\sigma_{tu}}\right)\left(\frac{\sigma_2}{\sigma_{tu}}\right) + \left(\frac{\sigma_2}{\sigma_{tu}}\right)^2 = 1$$

In the tension-compression quadrant

$$\left(\frac{\sigma_1}{\sigma_{cu}}\right)^2 - 2\mu\left(\frac{\sigma_1}{\sigma_{cu}}\right)\left(\frac{\sigma_2}{\sigma_{tu}}\right) + \left(\frac{\sigma_2}{\sigma_{tu}}\right)^2 = 1$$

A representative curve is plotted in Figure 5.1. Note that these relationships give the correct ratio of ultimate compression strength to ultimate tensile strength since they are based on measured values. Also, in the tension-tension quadrant the above relationship is one of very few theories that are more conservative than the maximum normal stress theory.

5.3.7 Octahedral Shear Stress Theory

This theory gives the same results as the maximum distortion energy theory and a typical development is given in Reference 5.19. The theory was developed for ductile materials and predicts yielding under combined stresses, if only the uniaxial tensile yield strength of the material is known. It is limited to materials having similar strengths in tension and compression.

For the case of biaxial stresses the resulting relationship takes the form:

$$(\sigma_1^2 + \sigma_2^2 - \sigma_1 \sigma_2)^{\frac{1}{2}} = \sigma_{ty}$$

Since it was developed for materials with equal tensile and compressive strengths it should not be directly applicable to brittle materials. It has been used to represent the fracture of brittle cast iron, however, under biaxial tension and compression stress combinations by applying a stress concentration factor to the tensile stress σ_1. (References 5.25, 5.26). This concept was derived particularly for cast iron since this material contains graphite platelets which are assumed to create crack-like shapes in the iron matrix. Stress concentrations exist at the tips of these cracks under tension, and the situation can be well represented by introducing the appropriate stress concentration factor and assuming that the crack propagates to failure when the yield strength of the ductile matrix is exceeded locally. Under compression stresses, on the other hand, it is assumed that the graphite flakes transmit load and no stress concentration is involved.

In view of the basis for the derivation of this failure theory, it would be unwise to assume that it can be applied to any brittle material, and nothing has been found in the literature to support such an assumption.

5.3.8 Experimental Correlation of Failure Theories

Experimental data upon which to base a material failure theory is very limited in the literature. Most investigators have conducted a small number of tests which generally substantiate the individual theories which are being proposed, and these test results can be found in the references given above. The total body of data, however, is still small. This is particularly true if data for nonmetallic materials is sought. In Reference 5.22, for instance, a considerable amount of data for the evaluation of various theories is actually concerned with cast iron and brass. Probably the principal sources of experimental data for ceramic materials are References 5.27, 5.21, 5.17. In addition there is information on glass, but since this material is particularly sensitive to surface conditions care must be taken in applying the results to nonmetallic refractory materials.

Another serious limitation of most available test data is that variability in the fracture strength of the materials is generally not considered. The subject of material variability is discussed elsewhere in this handbook. Although the causes of this variability are not specifically known, there are many factors which can contribute and there is ample evidence from uniaxial tests that the variability exists. In Reference 5.22 Babel indicates that there are no biaxial test data which are based on both careful test technique, to minimize extraneous variables, and a sufficient number of specimens to obtain statistically reliable results. Without these considerations, it is not possible to distinguish between the various criteria. In his work Babel makes some allowance for variability by assuming standard deviations based on flexure tests, but there is no assurance that this is an adequate treatment of the problem.

5.3.9 Recommended Practice

Reference has been made elsewhere in this handbook to a conference of specialists that was held in London in Sept. 1967 under AGARD sponsorship, (Ref. 5.28) with the objective of obtaining opinions from the specialists on a number of basic questions involved in brittle material design technology. In view of the many theories available to describe failure under biaxial stresses and the great difficulty in obtaining adequate experimental data as a basis for selecting a theory, the subject was discussed extensively at the London conference. The only unanimous conclusion that could be obtained on this subject was an acceptance of the maximum stress theory in the tension-tension quadrant. Since this conclusion represents the best opinion from a number of specialists, who were selected carefully in an attempt to obtain a viewpoint representative of essentially all investigators in the technology, it is offered as a basis for design practice. Some caution must be observed, however, since there is test data which shows biaxial stress levels in the tension-tension quadrant which are less than those given by the maximum stress theory. As already mentioned, however, it is not clear whether this is the result of material variability or whether it truly represents an inadequacy of the maximum stress theory.

One other agreement which was achieved at the London meeting was that there are no shear failures in brittle materials of the class being considered here. However, this conclusion cannot be accepted without further consideration since it leaves experimentally observed compression failures unexplained.

From the above discussion the suggested design practice for establishing material failure properties under biaxial stresses is as follows:

(a) In the tension-tension and compression-compression quadrants use maximum stress theory based on uniaxial test data for the particular material of interest.

(b) In the tension-compression quadrant use a straight line connecting the uniaxial tension test data to the uniaxial compression test data. So far as can be determined from the limited experimental data available this practice should be conservative. It gives the same results as the Mohr criteria which in turn accepts the possibility of material failure in shear under combined tension and compression. This recommendation is made, however, simply on the basis of drawing a conservative envelope around the test data with no implication of shear failure.

(c) The above procedure should be combined with the statistical description of material strength. The material failure diagram, therefore, becomes a separate diagram for each level of failure probability. No difficulty is involved if uniaxial testing is conducted as it should be, with a sufficient number of specimens to provide statistically satisfactory data.

5.4 MATERIAL CHARACTERIZATION

Typically, in the design of metallic structures, the designer is hardly concerned with material characterization. He defines his material requirements by referring to specifications, and he is essentially assured of the structural properties of the material, the similarity of properties between laboratory samples and finished hardware, and the reproducibility of properties over large numbers of components. This situation results from a combination of refined process and fabrication control, which has been developed for the widely used metals, together with extensive work on the generation of mechanical property data, and the tolerance of metals to minute defects. This latter characteristic is very important in ensuring reproducibility of mechanical properties.

With ceramic materials, at their present stage of use and development, none of the above conditions hold, and in order to have control over the mechanical properties of his structure the designer must give considerable attention to material characterization. He must specify and control many material characteristics in addition to chemical composition and mechanical properties, if the material is to perform efficiently and reliably, and if the material in the structural component is to perform similarly to the laboratory specimens from which his material design data were determined. However, it is not sufficient to ensure similarity of processing conditions from laboratory specimens to full scale hardware, or from one batch of components to another. When all practical precautions are taken to control processing there is no positive assurance that the material produced at various times and in various sizes is truly similar. There have been many instances where differences in material are apparent despite similarity of processing. This is particularly true with ceramic materials where stringent process controls are difficult to exercise and the effects of slight process variations not well established. For the same reasons it is not possible, except for very preliminary design, to select material properties from a handbook and expect that these will be experienced in a structure.

The precise description of an object as complex as a ceramic body is very difficult. There are various levels of characterization, atomic, micro and macro. The aim is to describe a material by composition and microstructure so that its processing does not have to be described. Total characterization in this sense is not presently possible however, and may never be. The relationships between such characteristics as grain size and shape, surface roughness, etc., and the mechanical properties and their variability are not well understood, and in many cases are not known. The importance of many characteristics and the properties they influence are not known, and conversely it cannot be stated, for a particular mechanical property, exactly which characteristics should be controlled. Nor is the relationship between many of the processing parameters and the resulting mechanical properties well understood or, in many cases, is it understood at all. Accordingly, a compromise is made with a description of the material, in both its raw and finished state, a description of its processing history, and a description of its mechanical and physical properties.

The problem is further complicated by the difficulty of measuring some of the characteristics, particularly in finished components. Grain boundary thickness is an example. One other complication is the fact that some factors may obscure the influence of others at some times but not at others. For example, grain size is important to strength and strength variability, but this effect may be entirely obscured by a rough surface. As the surface finish is improved or the grain size increased,however, the grain size effect will eventually become apparent. Thus before the emphasis of one factor is judged, all other potentially limiting factors must be specified.

In the above situation a precise list of characteristics and factors which should be specified to control material properties cannot be given. Instead, reliance must be placed on the opinions and judgment of experienced individuals and References 5.29 and 5.28 give what are probably the best available opinions at this time. Both are the results of committee activity involving groups of individuals selected for their knowledge of ceramic materials, including specifically the subject of material characterization. Reference 5.29 presents an extensive list of characteristics which should be identified to ensure reproducibility. This list, which is reproduced in simplified form, in Figure 5.4, includes

not only parameters known to be important, but also those which are suspected of having an effect on mechanical and physical properties.

The first two categories in Figure 5.4 provide processing data, since at present an exact reproduction of the processing is necessary, but not sufficient, to obtain reproducibility. The remaining categories cover composition, grain structure, flaws, and surface condition, all presently considered important for complete characterization. Many of the factors included in Figure 5.4 cannot be measured during a production run of material, or in completed components, hence the need for careful process control. They may be useful, however, in the early phases of a program when the material processing is being established. During this period destructive inspection methods can be used to characterize the material fully, and the process can be adjusted until all of the desirable characteristics are being obtained.

Some clarification of the terms used in Figure 5.4 is necessary and the following is taken from Reference 5.29. Grain boundaries are the interfaces between grains or phases. They do not include the bulk of identifiable phases segregated between grains. Thickness of a grain boundary is that region which differs structurally and identifiably from either bordering bulk phase.

The distribution of pores and microcracks, the size and size distribution, and the shapes are all imporant to mechanical properties. Relative orientation is included to indicate whether cracks and pores follow grain boundaries or are oriented along crystallographic planes in grains.

With respect to surface characterization, topography includes microfussures, notches and blebs as well as gross corners and curvatures. Chemistry of the surface includes any deviations from bulk compositions that may occur as a result of volatilization, contamination or leaching during firing, finishing or use. Extrinsic defects include such factors as blebs, inclusions and isolated massive grains.

The approach to characterization which was adopted at the meeting reported in Reference 5.28 was different from that described above. The London meeting was arranged to examine a number of fundamental questions relating to brittle material design, specifically as a basis for this handbook. In approaching the subject of material characterization the objective was to establish, as a guide to designers, the minimum level of characterization necessary. This is specified, in Reference 5.28, as follows:

Chemical composition (including all impurities of 1% concentration or greater),

Sizes, shapes and distribution of grains and pores,

Identity of phases other than the principal phase,

Surface finish,

Details of the processing method.

Two other related points were established in the conclusions of Reference 5.28. The first was that considerable improvement in material reproducibility can be obtained by advising the material producer of the above characteristics and particularly bringing to his attention any changes that occur in these characteristics. The second point was the difficulty of making large components with exactly the same characteristics as small test specimens and it was suggested that consideration should be given to obtaining the small test specimens required for material property determination, from large pieces of material.

Neither of the above references indicates a requirement for characterizing the starting powders from which the ceramic materials are made although there are opinions that this is important, too. Characterization should include chemistry, including elemental composition, impurities and trace elements, particle size distribution, and particle shape. Furthermore, density is not specifically mentioned in either reference, except that it is related to porosity, but there is no doubt that density and density distribution are important parameters with respect to stiffness and strength.

The view of characterization presented above does not distinguish between the different functions of a) establishing the material composition and processing to develop the required properties in laboratory specimens and samples, b) scaling up the processing technique so that the same material characteristics and properties can be achieved in full sized hardware of complex practical shapes and c) ensuring that the characteristics and properties are reproduced in each and every piece of hardware.

Item a) is limited by insufficient knowledge of the relationships between material characteristics and the resulting material properties. Much of the available information has been presented in Section 5.2 and in view of its limitations it must be combined with considerable judgment and experience, and trial and error experimentation to establish the required characteristics. Since destructive testing is possible, however, it is possible to make measurements of most of the characteristics specified in Figure 5.4 and hence to verify that the required values or ranges of values have been attained. Methods of measuring these characteristics are of interest to the designer chiefly with respect to the accuracies obtainable, since these control the tolerance limits to be used in the specification of values. A good summary of measurement and inspection methods together with accuracy limits is given in Reference 5.29, and further discussion of measurement methods from the point of view of material characterization, is given in References 5.30 and 5.31.

Scaling up of material processing to produce structural components with the same characteristics and properties (recognizing the significance of increased volume on the variability of properties) as were obtained in laboratory specimens, (item b) is also very much a matter of experience and trial and error experimentation, but again the resulting characteristics can be readily checked because destructive inspection is possible. The discussion above, with respect to item a, applies.

Ensuring that the required characteristics, and hence properties are reproduced, within specified limits, in each structural component is much more difficult because it must depend heavily on nondestructive inspection. These techniques are quite limited, particularly with respect to the measurement of material structure and defects, important characteristics affecting strength.

Three important nondestructive techniques for defect examination are ultrasonics, X-ray, and dye penetrant.

The detection of flaws by ultrasonics involves the propagation of acoustic energy within the material and the detection of either the reflected or attenuated wave after it has encountered material anomalies. Fractures, voids and cracks present solid to gas interfaces with grossly different acoustic properties and acoustic energy impinging on such surfaces is reflected almost entirely. Similar effects occur at inclusions of foreign material. Ultrasonic testing is a compromise, based on the acoustic properties of the material since although resolution increases with acoustic frequency so also does the scatter and attenuation of the signal.

There is very little data in the literature to indicate the type and size of flaws that can be detected in ceramic materials. Reference 5.32 is concerned with graphite and indicates rather large discontinuities of 3/8 to 1/2 sq. inches in area, as the limit of resolution. Ultrasonic inspection of graphite and zirconium carbide is reported in Reference 5.29 but no indication is given of the ability of the techniques to detect flaws.

Radiographic techniques including X-ray, gamma-ray and neutron sources can be used with radiation counters, fluorescent screens and photographic film to examine the internal features of ceramic materials. Density variations can be detected and voids as small as 1 to 2% of the product thickness (Reference 5.29) can be seen. Unfortunately, the identification of cracks by radiographic techniques is difficult. If the X-ray beam passes across the crack from one face to the other, there is virtually no attenuation of the beam intensity and therefore no indication. If the beam is aligned perfectly parallel with the crack sides, an indication is possible but in most cases cracks will be too tight for a very obvious indication. In these cases the detection of cracks will require very careful examination of X-rays taken from many different angles and even then in most cases it will not be clear whether in fact the indications are cracks. Nothing has been found in the literature to indicate the minimum size of cracks which it is possible to detect in ceramic materials, with radiographic techniques.

Another approach to the detection of defects is the use of liquid penetrants employing visual dye, fluorescent dye or filtered particles. These techniques are generally good for high density materials, porous materials producing indications which are difficult to interpret. The techniques are quite sensitive and cracks not otherwise detectable visually are made evident, however these techniques are restricted to surface defects.

These nondestructive inspection techniques will not measure material structure and will give only relatively gross indications of defects. Accordingly, two other methods of assessing the characteristics of the material in finished products have been attempted. The first of these is reported in References 5.30 and 5.31 in which attempts were made to correlate mechanical properties with the results of ultrasonic and radiographic inspection without specifically identifying defects or attempting to correlate defects with their effect on strength. Radiographics were examined, for instance, with respect to density variations. The local density variations appeared to be related to the severity of internal microcracking which was controlling the strength of the material. The presence of the microcracks also tended to scatter ultrasonic energy thus providing a record which could be related to strength. Some correlation was obtained, by these methods, between material strength and the ultrasonic and radiographic records, but the correlation was not very strong and there is no indication that the procedure could be used effectively with every ceramic material.

The second method, which is reported in References 5.32 and 5.33 and described in References 5.29, attempted to obtain direct correlation between modulus of elasticity and strength, and a number of nondestructive inspection observations. The principal measurements were longitudinal wave velocity, which was measured acoustically, and electrical resistivity. Both measurements can be made nondestructively and locally at any point on a completed component. The references quoted showed good correlation between the measurements and modulus of elasticity, density, and ultimate tensile strength.

None of the methods described above are particularly satisfactory for the purpose of ensuring that the mechanical properties of ceramic materials are consistently reproduced from one component to the next and in all parts of each component. However, the desired results can be obtained reasonably well by a combination of approaches. Assuming that the material can be well characterized and the process well defined during development, using destructive inspection methods where necessary, the following procedure for ensuring reproducibility of finished components is recommended:

a) Characterize starting powders.

b) Control the processing very closely.

c) During the material and process development evolve characteristic NDT responses using radiographic and ultrasonic methods, and do this for each major step in the processing. For instance, typical records should be obtained for important regions of the component in the green state before firing, after firing, and after finish machining. During development these responses can be correlated, to some degree, with the mechanical performance of the finished product. To do this effectively, however, a sufficient number of components must be fabricated that "worst cases", as determined by NDT, can be selected for mechanical testing.

d) Use the characteristic NDT responses developed above in conjunction with NDT measurements made on each component at each stage of the process, to reject unacceptable components as early as possible before expensive subsequent processing steps are undertaken.

e) Supplement (d) above with random sampling and destructive testing and characterization by destructive inspection methods.

f) Use destructive inspection methods and destructive testing under representative loading conditions on samples selected from the finished components.

g) Conduct proof tests on all finished parts. Such proof tests need not precisely simulate the loading and stress distributions developed during operation since the methods given in Section 3 can be used to assess the reliability of a component under operating conditions from a proof test conducted under some convenient loading condition.

To use all of the above methods in combination will frequently be expensive, and some trade-off will be required between a) inspection costs, proof testing costs and fabrication costs, and b) the weight benefits which will result from the increased working stress levels which can be reliably used as a more comprehensive inspection is made.

All of the above is given in general terms with very little quantitative information to aid the practical designer. This situation reflects the current state of the art and the almost complete lack of experience with the production and use in quantity, of high strength ceramic structural components.

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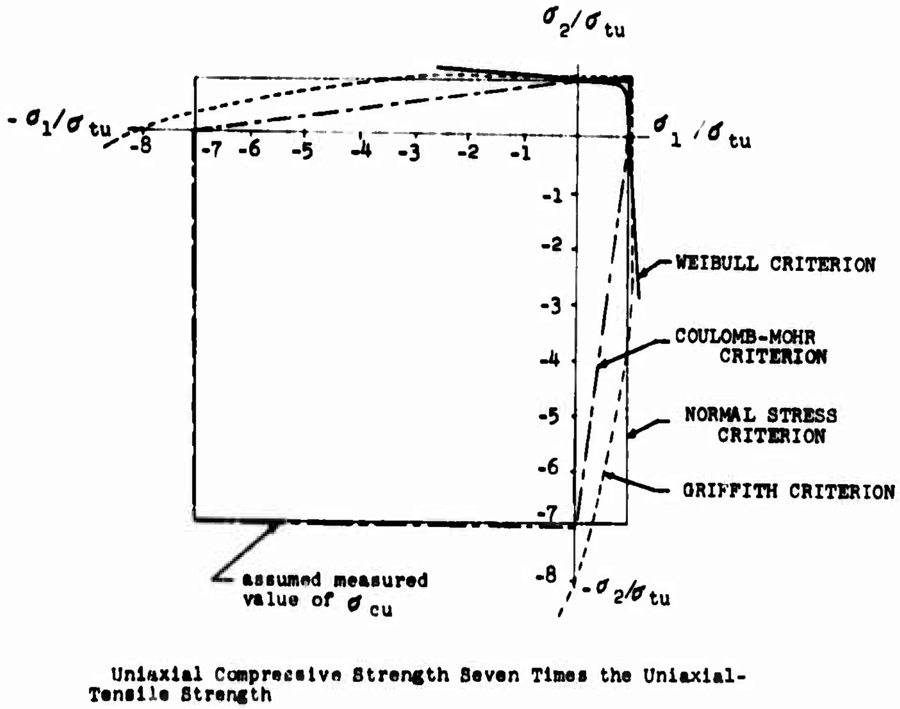

Fig.5.1 Comparison of failure criteria for brittle materials

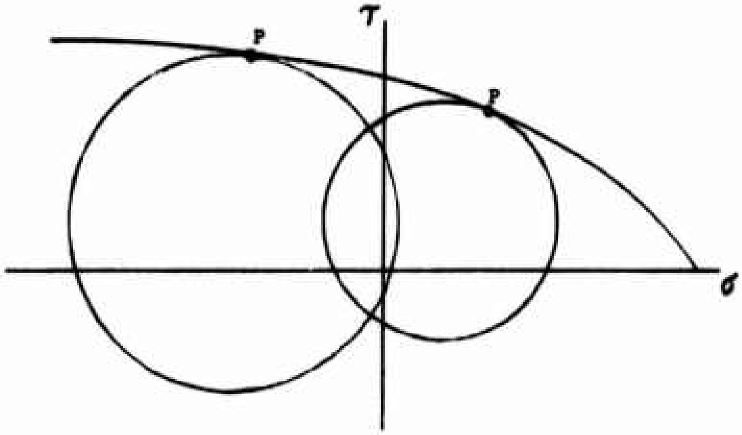

Fig.5.2 Mohr strength envelope

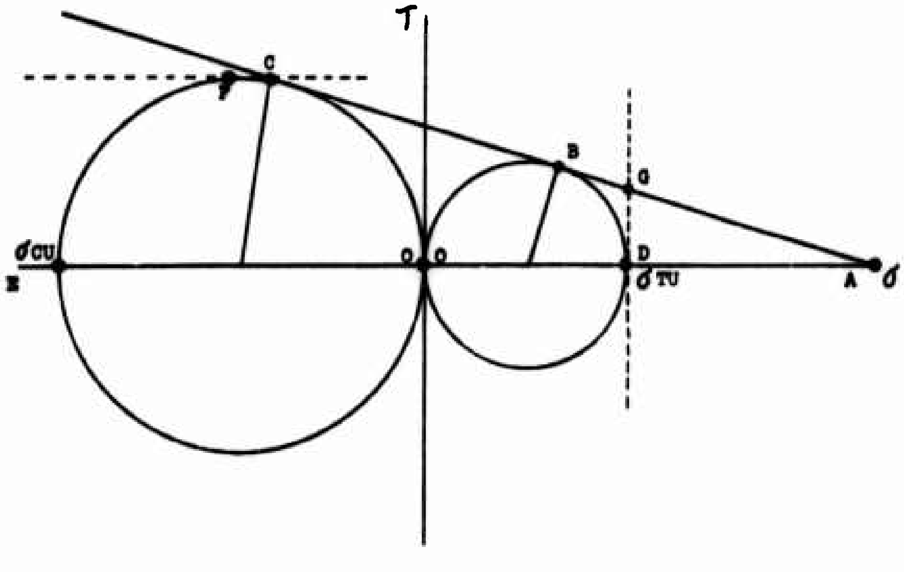

Fig.5.3 Simplified Mohr strength envelope

FIGURE 5.4 CHARACTERIZATION OF CERAMICS

(Taken from Reference 5.29)

DESCRIPTION OF MATERIAL	- Nominal composition
SAMPLE HISTORY	- Method of preparation Potential sources of contamination Special treatment - thermal, chemical, etc. Shaping technique Specimen size and shape
AVERAGE COMPOSITION	
GRAINS (Repeat for each phase)	- Phase identification Quantity Size and size distribution Shape and shape distribution Preferred orientation Internal strain Composition - Stoicheometry Impurity clustering Impurity concentration distribution Dislocations - Density Type Substructure
GRAIN BOUNDARIES	- Composition and composition distribution Structure Relative orientation of grains Grain boundary strain
PORES, MICROCRACKS, CRACKS	- Quantity Distribution in structure Size and size distribution Shape and shape distribution Relative orientation Gas composition
SURFACES	- Topography State of stress Absorbed gases Chemistry
EXTRINSIC DEFECTS	- Type Position

6. DESIGN CRITERIA

6.1 INTRODUCTION

The term criteria is used with various meanings throughout the aerospace community so that some definition of the term, as used in this Handbook, is first necessary. It is used here to include (a) the specification of the conditions which must be considered in the design of a structural component, (b) the required levels of strength and stiffness under these conditions, and (c) the requirements for demonstrating that the criteria have been met.

Ideally it should be possible to limit criteria to only these considerations, so that the manner in which the structure is designed, the methods used, the sources of data, etc., should not be considered in the criteria. In practice, however, it is not economically feasible, with complex vehicles, to wait until ground and flight testing of the complete vehicle reveals poor design practice. Everything must be done, at the earliest possible stage of design, to ensure that the best current practice is being followed. Accordingly, it is quite common to include in criteria those aspects of design philosophy which are significant with respect to structural integrity and in some cases to also specify design practices and analysis methods. For instance, the level of factor of safety selected may be dependent on the methods of analysis used, in which case the criteria must consider both. The objective is usually to ensure that the criteria reflects all previous experience with respect to the recognition of potential modes of failure, and this frequently requires more than the items mentioned in the first paragraph. At the same time, designers should not be unnecessarily constrained and innovation should not be stifled.

It is necessary to consider the subject of design criteria in conjunction with brittle material design since there are some significant changes necessary from airframe and space vehicle criteria as typically developed for metallic structures. The principal reason for this change is the need to recognize the probabilistic nature of material strength properties, which makes it necessary to at least consider whether all of the criteria should be treated on a probabilistic basis.

With the conventional approach to criteria, material variability is, in fact, considered and typically 99% failure probability levels are defined as a basis for selecting material allowables. These allowables are then used with ultimate loads, which are limit loads multiplied by a factor of safety. Limit loads are the maximum loads expected in service. Thus to some degree at least, probabilistic methods are used in the conventional treatment of airframes and a finite probability of failure under ultimate loads is accepted. However, the characteristic strength distribution curve for metals is such that the probability of material failure under stresses from limit loads is essentially zero. The combination of this characteristic, together with ground testing to verify the design, flight testing to verify loads and load distributions, and operational restrictions based on the demonstrated strength, is intended to ensure a zero probability of failure, for the structure, in service. Only very infrequently, usually when some new vehicle characteristics lead to a new and unexpected structural failure mode, is this requirement not met.

However, if materials of wide strength variability, such as ceramics, are used with the conventional deterministic approach to criteria, the structure may not have a zero probability of failure under limit loads. To many individuals, and for some applications, this is not acceptable. Where it is acceptable, then the question of an acceptable value for failure probability arises, since there is nothing in current practice to establish a precedent.

Current criteria practice also begins to break down for vehicles where significant aerodynamic heating effects are present because it is no longer possible to define the airframe strength with a single number such as a load factor. Many parameters are involved and rational combinations of these parameters are necessary to produce meaningful design conditions. The introduction of arbitrary factors of safety on some or all of these parameters leads to irrational and inconsistent situations; and again the answer lies, ultimately, in the direction of probabilistic methods. While this is becoming a problem with all high speed vehicles, it is particularly important when brittle ceramic materials are used since the applications for these materials will almost always involve very high temperatures, with all of the associated effects. For these reasons, therefore, consideration must be given to criteria in this Handbook.

If the criteria is to be approached probabilistically, it is desirable to be able to specify the allowable overall failure probability of the structure, considering all of the various loads and load repetitions and their probability of occurrence, the associated temperature effects, and the variable material and structural response characteristics under the spatial and time varying stresses from these conditions. In addition the reliability of analysis and test methods and of experimentally determined data should be included and all of these considerations should be combined into an overall probability of failure value.

The ideal situation is never attained in current practice. The principal reason is the vast amount of statistical data and computational effort that would be required. This becomes economically impractical, particularly since much of the data requires extensive full scale ground and flight testing. Even if it were considered worth the

cost, the task cannot be completed until a number of vehicles have been flown extensively to gather data, and obviously, in this case, the results are available too late to influence the design.

At best the probabilistic method is applied only to some elements of the total assessment of structural integrity. The specification of material allowables, on this basis, has already been mentioned. Probabilistic loads are used for some of the environmental induced conditions for space vehicles. Probabilistic strength characteristics and probabilistic loads however, are never combined in a statement of structural failure probability, and analysis and test data (other than material properties) is never treated statistically.

Currently, the loads arising from the various design conditions are treated partly deterministically and partly probabilistically. Deterministic methods were used in the early days of airframe design when relatively simple descriptions of the overall loads and strength levels were sufficient. They have been retained because they are simple to use, they are particularly convenient for initial design work when extensive knowledge of the structure and the vehicle are not available, they provide a simple means for establishing contractual requirements during procurement, and they provide a simple framework for the establishment of operational limitations. They also permit responsibility to be assigned in the event of a catastrophy, since it can usually be established whether the airframe failed to meet its strength requirements (contractor responsibility) or whether it was operated beyond the established limits (operator responsibility).

Probabilistic methods of defining loads have none of these features. Such loads cannot be determined until a large amount of information is available on the structure, the vehicle and its mission. Even then the computational work is of enormous magnitude. Furthermore, it is impractical to demonstrate the structural reliability by test, or to establish probabilistic loads by flight testing, so that many of the features of the deterministic approach are lost.

The deterministic approach currently predominates for conventional aircraft while probabilistic methods are more extensively used in space vehicle design. However, deterministic methods become somewhat inconsistent and arbitrary when the thermal effects of high speed flight are important and no really good criteria have yet been evolved.

Deterministic methods of load specification also involve the use of safety factors, and although these factors are arbitrary and are often used inconsistently with respect to thermal effects, their use is part of the level of structural integrity which experience has shown to be satisfactory. Any proposed changes in criteria must therefore recognize existing safety factor practices. Perhaps of most importance with deterministic methods, however, is the large amount of successful experience which has been accumulated and the large numbers of structural engineers who are thoroughly familiar with their use.

In this Section the approach to be taken to criteria for structures or components fabricated from brittle materials is to emphasize probabilistic methods, but to continue with the separation of load and strength considerations. Any efforts to change existing load specification and determination practices is well beyond the scope of this Handbook although some discussion and description of these practices will be given since they are an important part of structural integrity.

Emphasis will then be placed on the strength aspects of criteria, such as the selection of material allowables, factors of safety, etc., and recommended criteria practices will be given. The objective will be to maintain the same strength and integrity levels as have been shown satisfactory in the past. In addition an attempt will be made to establish criteria with respect to thermal effects, in view of their probable importance to "ceramic" structures.

It must be emphasized that this Section presents only suggestions and recommendations for criteria for structures fabricated from brittle materials. There is currently a complete lack of information, in the literature, which can be used to verify or confirm these suggestions, nor was there any opportunity, during the preparation of this material, to conduct verification experiments. So far as possible the assumptions upon which the recommendations are based are clearly stated, and their experimental verification is the first step in establishing criteria.

6.2 CURRENT CRITERIA PRACTICE

6.2.1 Design Conditions

a) Manned Aircraft

The principal loading conditions which design the primary structure of conventional manned aircraft are maneuvers, gust, landing and take-off.

In all current aircraft criteria (References 6.1, 6.2 and 6.3), maneuvering loads are treated as deterministic by specifying an envelope of load factor against velocity. Where they are not limited by vehicle performance capability these load factors are, in fact, determined from statistical data accumulated over many years of actual flight operations with vehicles, not necessarily similar, but performing similar missions. Thus the deterministic method of load specification has its roots in statistics and,

although not stated explicitly, a certain non-exceedence probability level is implied. The specification of a load factor level instead of a non-exceedence probability level is a very simplified approach in which the only parameter considered is the broad classification of the airplane by its function. For large complex vehicles a more rational examination of design load factors is frequently made at a later stage in the design by conducting simulator studies. If the load factors upon which the design was originally based are found to be too low, the airframe will necessarily be modified, and quite frequently only localized detail changes will be found necessary. If the original load factors are too large, however, it will rarely be convenient to benefit from the pontential weight reductions because weight saving necessarily implies redesign over large areas of structure.

Generally the important maneuver design conditions will involve transient effects such as the pitching and rolling accelerations and velocities required to achieve maneuver attitude and to recover from the maneuver. Again the appropriate design parameters are selected from past statistical records, with a great deal of engineering judgment, and a minimum consideration of the characteristics of the vehicle being designed.

Permitting the pilot to fly to the boundary of the maneuver envelope implies a relatively high probability of limit load occurrence, and although limit loads are usually intended to be the maximum loads experienced in service this situation must be expected to lead to a finite probability of limit load exceedence. For example, in fighter aircraft the pilot is not likely to be considering the airframe Vn diagram during combat operations; in training aircraft inadvertent exceedence is likely, and in commercial aircraft there may be rare emergency situations in which the pilot is not able to remain within the design boundaries. These situations are covered, quite unscientifically, by safety factors, a subject to be discussed later. Safety factors, however, may cover other considerations than limit load exceedence.

For fatigue considerations the question of repeated maneuvers at different levels, which again should be treated on a probability basis, is reduced to a deterministic form by specifying "blocks" of loads at various levels. The number of load repetitions at various fractions of the design limit load factor, which is presently the way the repeated load criteria is formulated for military aircraft (Reference 6.4), is presumably based on statistical data. However, specifying the requirements as mentioned above and relating these requirements only to the class of aircraft, again represents a very approximate method of determining the design loads from statistical data.

For both military and commercial aircraft (Reference 6.2, 6.3), gusts are treated deterministically, in the establishment of limit loads, by considering discrete sharp edged gusts, with correction factors for aircraft response. In addition, Reference 6.3 specifies gust shape. Again, the specified gust velocities are selected from large amounts of statistical data and there is implied a non-exceedence probability level, although its value is not known. For repeated loads and fatigue considerations military criteria require a probabilistic treatment of gusts by specifying a power spectral density. This is in contrast with the treatment of repeated maneuver loads described above. Commercial aircraft criteria are not specific about the manner in which repeated gust loads should be treated, but current practice, for large vehicles, involves the probabilistic, power spectral density methods.

Landing loads are also probabilistic in nature, particularly since the pilot has no direct measure of sink speed at the moment of touchdown, but in current criteria the situation is treated deterministically by specifying maximum sink speed. Again these values have been determined from the accumulation of statistical data on many different aircraft so that the current criteria again represent a simplified but convenient reduction of probabilistic data to deterministic design conditions. Again, in many cases, similator studies will be made when the design has proceeded sufficiently and again if the initial design values have proven unsatisfactory the costs of changes can be large.

Take-off conditions are also treated deterministically, in current criteria, by specifying load factors but more recently the probabilistic treatment of the repeated loads produced by runway roughness has become the practice. The situation parallels the treatment of gusts where the repeated loads determined probabilistically are principally used for the fatigue analysis of the structure.

For supersonic aircraft present criteria requirements with respect to heating effects are quite general, with both military and commercial criteria requiring only that such effects be considered (References 6.1, 6.10). Revisions to the military specifications are under consideration, and although these have not yet been published there are indications that discrete deterministic design conditions will be retained in order to utilize existing maneuver load and gust criteria. Design trajectories will probably be used to establish discrete design conditions in which load and thermal effects are combined. Trajectories most likely to result in (a) material strength reductions, (b) development of substantial thermal stresses and (c) development of substantial mechanical stresses will be required, but it will be recognized that the maximums of each of these effects will not always occur along the same trajectory and that the worst combination may be some other trajectory. No consideration will be given to the probability of occurrence of these various design trajectories or to any of the important parameters which define these trajectories.

For life design conditions typical mission profiles will be used to establish thermal effects since it is recognized that every flight will not involve a critical

design trajectory. No conditions are imposed on the relationship between design trajectories and mission profiles, but the need to consider the time involved in each maneuver, because of the transient thermal effects, is recognized and quantitative data is given.

b) Space Vehicles

For most space vehicles, whether manned or unmanned, launch and ascent provide the most important structural design conditions. While the heating effects during re-entry of recoverable vehicles may be critical also for the re-entry structure, nevertheless the majority of the airframe weight will be designed by launch and ascent. During launch and ascent the important loads arise from (a) environmental effects such as winds, atmospheric turbulence, gusts, wind shears, etc.; (b) engine thrust loads including transients during engine start and cutoff and including the load components resulting from engine gimballing for control purposes; (c) aerodynamic loads due to vehicle angle of attack and changes in angle of attack; (d) loads produced by the relatively large volumes of liquid propellants, including both steady state inertial loads and dynamic loads due to fluid motion; (e) dynamic effects resulting from coupling and interactions between all of the above and the structure. The probabilistic treatment of loads is used more extensively for space vehicles than for conventional aircraft, but at the present time the treatment is still incomplete and contains some inconsistencies.

A large amount of statistical data has been gathered on the atmospheric environments at space vehicle launch sites. Consequently, winds, gusts, turbulence, etc. are treated probabilistically to the extent that the environmental conditions are selected based on a specified non-exceedence probability level. However, having selected the individual wind, gust and turbulence conditions, these are generally combined into a synthetic wind profile which is then used to determine specific critical load conditions in a deterministic manner.

To determine aerodynamic loads, dispersed trajectories are used in which the dispersions reflect probable variations in guidance and control equipment and any other factors which control the trajectory. Similarly, performance variability is considered to a specified probability level in determining engine thrust and the associated transients, load components, etc. When all of these loads are combined, however, to determine the critical bending moments, axial loads, etc. at each station, they are generally treated deterministically and combined by simple superposition.

6.2.2 Design Methods

In order to follow a structural design procedure entirely based on the recognition of the statistical basis for all aspects of the design, it is necessary to recognize also the statistical nature of all of the design data which is used to predict the structural response to the prescribed design loads. Such data includes physical and mechanical material characteristics, aerodynamic characteristics for describing total aerodynamic forces and pressure distributions, and all of the experimentally determined physical data involved in predicting external and internal heat transfer and the resulting temperatures. All of this data will normally show variability, either inherently in the properties being determined as in the case of the mechanical properties of materials, or as a result of variabilities in test technique, instrumentation, test facilities performance, etc.

A similar variability is implied in theoretical methods used to predict structural response, to the extent that all such methods have been verified by correlation with experimental data. Since this experimental data is statistical, the accuracy of the analytical methods is only known to a certain probability level. Thus, all data used in predicting structural response to a specified system of loads is again statistical in nature, and the contribution of this variability to the overall probability of failure should be included.

Here again current practice is inconsistent. The establishment of material properties is generally treated statistically and the allowable properties to be used for design are determined from experimental data by prescribing the acceptable level of probability that the properties will be exceeded in the actual structure. There is also some recognition of this situation with respect to the methods and data used to determine the effects of aerodynamic heating on high speed vehicles. Various methods are in use for predicting external heat transfer characteristics, and these methods show differences in results and variability in the correlation with experimental data. As a consequence, in the discussion of criteria for high temperature structures it is frequently proposed that factors of conservatism be used on predicted heat transfer coefficients. In this case, there is no real attempt to statistically examine the scatter in experimental data but only to apply a rather arbitrarily selected factor to the predicted values.

In all other areas of technology involved in the prediction of structural response to the design conditions, there is no attempt in any structural design criteria to treat experimental data statistically, or to consider the accuracy of prediction methods on a statistical basis. For instance, statistical procedures are not generally followed in reducing aerodynamic data measured in wind tunnel testing. The usual practice is to leave the interpretation of experimental data, and the conservatisms to be added to predicted values, to the judgment of the specialists in the various areas of technology. The chance of this practice leading to an inadequate structure is not very great since the technical

specialists are generally conservative in assumptions, methods and experimental data as used for strength predictions.

Almost equally significant, however, in high performance weight critical vehicles is the fact that these conservatisms may be excessive, leading to overdesign. The extent to which this situation exists in typical vehicle structures is not generally known, since many engineers are usually involved in various parts of the design activity, such as the determination of aerodynamic pressures, the determination of heat transfer coefficients, stress analysis, dynamic analysis, etc. Each applies his conservatisms which, in the nature of a typical engineering organization, are usually cumulative.

This problem is not relieved by the practice of structural testing. The testing of full scale structures or structural components will reveal the weakest point and will show whether this point is strong enough to meet the design conditions, but such tests will not generally show the level of excessive strength throughout the remainder of the structure. Quite frequently ground testing will show that even at the weakest point the structure is much stronger than is necessary, and frequently this is the result of the accumulation of conservatisms during the design. Usually however, at this stage the design work is complete, tooling is finished and a number of production articles are being fabricated. Consequently, any large effort to reduce weight by a redesign to eliminate the excess strength is usually discouraged. Some organizations anticipate this situation by prescribing artificially reduced loads for design purposes and though unscientific this procedure is usually satisfactory where there is considerable experience with similar structures in similar applications.

In defense of the situation described above it is usual to argue that the structural criteria should not constrain the designer by prescribing particular methods of analysis or particular experimental data with associated factors of conservatism. This concern is particularly valid in the case of general criteria, such as Reference 6.1, expected to remain in use with the minimum of revision for many years. The problem is not so much one of prescribing methods of analysis, however, as it is one of properly reducing all experimental data to produce data for design purposes.

6.2.3 Safety Factors

Factors of safety are customarily used to establish an ultimate load from limit load where the limit load is defined as the maximum load expected in service. It is usual to require that the structure suffer no degradation with respect to its ability to perform its functions when subjected to limit loads and that it not collapse or rupture when subjected to ultimate load.

The current values of factors of safety, for example the 1.5 value specified in Reference 6.1, are quite arbitrary and a considerable amount of effort has been wasted in trying to rationalize these arbitrary factors. They apparently evolved, at least in the U. S., by choosing values of safety factor and limit maneuver load so that previously satisfactory strength levels were retained. (Reference 6.5). Subsequently, some reduction in safety factor level has become acceptable practice in unmanned systems. Higher risks are generally acceptable in unmanned systems though whether the reduced factor of safety results in a higher failure probability depends on the extent to which the selection of limit load levels has been influenced by the single mission, and the resulting lower probability of experiencing extreme operating conditions.

Some organizations consider that the safety factor covers variations in the strength of the fabricated structures due to processing variabilities, residual stresses, etc. Others consider the factor of safety to cover the possibility of exceeding limit load. In the past the factor of safety was often supposed to have covered the effect of repeated loads, but the importance of fatigue in most modern aerospace structures has required that this subject be properly analyzed rather than buried in a safety factor.

When repeated loads are considered it is usual to apply the factor of safety not to the load levels but to the number of load repetitions. USAF practice, Reference 6.6, applies the factor of 4 to the number of load repetitions, whereas the U. S. Navy applies a factor of only 2 but uses a load spectrum which emphasizes the high load levels. This latter procedure is a concession to the practical problem of fatigue testing of complete airframes. It recognizes the large cost and time involved in simulating a realistic load history and repeating this a number of times, and it attempts to produce an accelerated ground fatigue test.

Again, there is no clear rational basis for applying a factor of safety to the number of load cycles. With the exception of certain critical components such as helicopter blades, it is not usual to retire an airplane from service because its design life has been reached. For vehicles from which a long service life is expected the use of tough construction materials in conjunction with periodic inspection, frequently involving quite sophisticated methods, and with repair or replacement where cracks are detected, is the basis for continued utilization of the vehicle. Retirement will then result when the rate of degradation is such that this process becomes uneconomical.

With respect to aerodynamic heating effects the question of factors of safety arises because of the precedence established by the use of safety factors on loads and life. Many parameters are involved in determining all of the effects which are present under aerodynamic heating conditions. Since these effects act simultaneously consistent values of these parameters must be used to avoid meaningless margin of safety assessments.

The introduction of arbitrary factors of safety into a complex and otherwise rational analysis produces difficulties, and no generally acceptable approach has yet evolved. It is generally recognized, however, that factors applied to heat transfer coefficients or temperatures or thermal stresses, etc. will each have significantly different consequences and that the significance of such a factor will be quite different from applying the same factor to load. There is some agreement that factors of safety should not be applied to temperatures and that factored stresses due to load and unfactored stresses due to temperature gradients, should be combined. However, the Federal Aviation Agency in Reference 6.10 requires a factor of 1.25 on thermal strains for supersonic commercial aircraft, while some recent USAF considerations of criteria for high temperature airframe structures have seriously considered using a factor of 1.15 on velocity. The latter is intended to produce a more rational design condition than the simple application of a factor to heat transfer coefficient or temperature, and it has some precedence in the treatment of factors on airframe stiffness requirements.

Despite the confused situation with respect to loads and safety factors, it must be recognized that the practice has generally led to structures having satisfactory strength levels. Where limit loads are prescribed probabilistically, the usual values are 99% or 99.9% which implies a relatively high probability of load occurrence. Similarly with manned aircraft, since the pilot is permitted to fly to the load factors which are used to establish limit conditions, it can be assumed that these conditions arise relatively frequently. On the other hand ultimate loads are permitted to develop stresses which have a 1% chance of exceeding material allowables so that the chances of failure under ultimate loads are relatively high.

Evidently, therefore, the typical 1.5 factor of safety has, in the past, been sufficient to ensure either that ultimate loads have an extremely low probability of occurrence or that the stress levels associated with limit loads result in an extremely low probability of material failure. Effectively, the factor of safety has made it unnecessary to seek the vast amount of statistical data, on both loads and strength, that would be necessary if the criteria were expressed as a total failure probability of extremely low value. On the other hand the failure probability that is achieved with the factor of safety is not known until large numbers of vehicles have been in operational use for extended periods of time. This may be unimportant when a new design involves a small extrapolation of past experience but with a major change in vehicle configuration, operating conditions, structural concept, or materials, the situation may be quite different.

6.2.4 Qualification Testing

It is current practice to qualify any airframe by ground testing, in which all of the critical conditions are imposed on a full size airframe, in conjunction with flight testing to measure loads and load distributions and to demonstrate structural integrity at critical design conditions. Where this is not done, usually for economic reasons when only very few vehicles are to be built, either higher factors of safety are used by restricting the flight envelope, or an increased risk is assumed. The latter is often the case with research aircraft.

The qualification test program is thus an integral part of the assurance of structural integrity. It is an element in current practice which has contributed to the generally successful performance of airframes and it must, as a consequence, be considered in any attempt to formulate new criteria.

Qualification testing serves two purposes. It verifies that the airframe has the predicted strength under the applied loads and environmental conditions, and for the expected failure modes. However, it also verifies that other critical loading conditions have not been neglected and that failure modes have not been missed. Strength analysis of structures requires a prior knowledge of potential failure modes, it does not predict them. Imagination, coupled with experience, can usually suggest what the failure modes will be, but there is no absolute proof except in test. For this reason also, full scale ground testing on the complete vehicle is invariably used for qualification purposes, since there can be no absolute proof that scaled models or partial structures are revealing every failure mode.

The typical ground test program will include two airframes; one is exposed to each of the potentially critical discrete flight conditions, the so-called "static" test, while the other is subjected to "life" testing. Both test programs are conducted with discrete loading conditions; the "static" test is exposed in turn to each of the critical limit load conditions; life testing is conducted by exposing the airframe to loads of various levels and distributions, representative of various flight conditions. Specific numbers of load cycles at various levels of load intensity are applied to approximately represent the distribution of load levels expected in service, and loads of various levels are intermixed in sequence, since there may be sequence dependent fatigue failure modes. Obviously the life testing is an approximation since load level distributions and load sequences are known only very approximately from statistical data on other, and different, vehicles. Also it is not practical to provide for a large number of different load distributions simultaneously, in the loading rig, and only the most critical static conditions are usually represented.

In the examination of current criteria practices, it is interesting to ask why two test articles, or why separate the discrete limit load testing from the repeated load tests since the latter should include the former. One obvious reason is timing. Full scale testing requires a great deal of time during which operational vehicles must be restricted

until it is complete. Static testing takes much less time than life testing and when it is complete restrictions on the operation of the vehicle within the flight envelope can be removed. Restrictions on vehicle life, while life testing is being conducted, are not significant since although the testing takes considerable time, nevertheless the number of cycles can usually be built up faster on the test specimens than on the vehicle in operational use. Thus, the practice is simply to ensure that the life test is always ahead of actual usage in terms of the number of flight hours simulated.

It is also evident, however, that the separation of static and life testing, whether it is conducted with two test specimens or one, reflects again the concept of assessing airframe strength in terms of discrete, static load conditions. There is some justification for this, with typical metal structures, since the bulk of the structure in terms of weight of material and the type of construction used is dictated by load level, while repetitions of load control very local (but nevertheless important) elements of the structure, such as joints, where stress concentrations predominate.

It is conceivable that, with equipment available today, ground testing could be conducted on a probabilistic basis. With a loading rig arranged so that maneuver, gust, landing and other load conditions could be applied by proper selection of loading cylinders, a computer could be arranged to randomly select load levels and sequences within a programmed sequence of operating conditions. For a deterministically designed vehicle this method of testing would have the disadvantage that a complete vehicle life may need to be simulated before all limit load conditions are applied. In fact some of the conditions may never be applied if their probability of occurrence is sufficiently small. Even for a probabilistically designed airframe the time required to demonstrate the integrity of the airframe would be impractically large.

An equally important part of structural qualification testing is the flight test program, which has two parts, a flight load survey and the structural testing in flight, to limit conditions. The flight load survey is a verification, principally by the measurement of stresses, in flight, of the loads and load distributions used in the design and stress analysis. In this respect it parallels the "static" test which is a verification of the failure modes assumed and of the predicted strength levels in each failure mode. The structural flight testing is a final verification in which the limitations in the ground test program, such as the localized application of loads, the use of an incomplete airframe, etc., are finally eliminated.

Again, it is clear that both aspects of structural flight testing must be conducted on a deterministic basis. Specific conditions must be established for the measurement of loads and for the demonstration of structural integrity and the aircraft must be deliberately flown to these conditions. To conduct a flight test probabilistically implies flying until critical loads arise statistically, obviously impractical since it may require all of the flying of all vehicles of the type.

The flight testing of space vehicles for structural purposes is far less common than the flight testing of conventional airplanes. Each flight requires a vehicle, and for economic reasons each must be used to accomplish a mission or at least to accomplish a large number of system tests. Possible compromise of the mission or systems test programs in order to apply critical structural design conditions, either for load measurement or to verify structural integrity, will rarely be tolerated. Similarly, the use of a vehicle specifically for these purposes can rarely be justified economically.

This situation is perhaps tolerable with a space vehicle since a greater risk of failure is usually acceptable than is the case with a vehicle intended for many thousands of hours of operation. This is true even with manned space vehicles. It is a trade-off between the cost resulting from a vehicle failure and the cost of increasing structural reliability by structural flight testing. Such trade-offs are probably not made in fact, but only as a judgment by the program management.

6.2.5 Changes to Current Practice

From the above discussions it appears premature to propose, as a basic criteria, that an overall airframe failure probability be specified, since this would require major changes in the approach to load determination and to ground and flight testing concepts, particularly for manned aircraft. Such a development is beyond the scope of this Handbook, which is concerned only with differences in design practice due to the use of brittle materials. Broad use of the probabilistic method is not peculiar to brittle material structures but is desirable generally, from an idealistic point of view, to permit more rational design conditions.

Furthermore, a change of this type, to be effective, requires the concurrence of structural engineers, airframe designers and the managers of major vehicle programs, if it is to be effective. Such an activity requires committee type action, so that all interested parties can be represented and so that economic and legal implications, which are quite as significant as the technical, can be given proper consideration.

It is quite certain that the structural community is not at all ready to consider departing from the well established deterministic methods of establishing design conditions and the associated loads for conventional manned aircraft. It is probable that they never will be, because the probabilistic method is not convenient or even feasible for initial design, only for subsequently checking a completed design. Even for space vehicles,

where probabilistic concepts are used in the structural criteria, particularly for the important launch and ascent conditions, it is usual to pick some specific deterministic conditions for design purposes, and as already discussed, the statistical wind and gust data is turned into a specific synthetic wind profile. Thus the actual approach to design is the same as is used for aircraft; the differences are only in the manner in which the criteria is written.

It must also be recognized that brittle ceramic structural parts are likely to be used only in very local areas of the airframe. It is unlikely that these materials will ever be competitive, on a strength to weight basis, with metals wherever the latter can be used. Ceramics will be used where their special characteristics, primarily their refractoriness, are essential. Consequently, the majority of the airframe design will undoubtedly follow conventional practices, and this will include the loads analysis. Loads for those structural elements fabricated from brittle materials must be related to and developed from the load system for the entire vehicle so that to insist on a completely probabilistic approach for the brittle elements would require a complete duplication of a major section of the design activity.

In view of these considerations it will be assumed here that conventional practices will be followed in the determination of design conditions and limit loads, when brittle materials are used in the airframe. Differences in design procedure required by these materials will be accommodated by changes in the treatment of factors of safety and in the method of selecting allowable stresses. Structural integrity assurance will also require reconsideration of qualification testing and changes will be necessary in the ground testing phases.

The flight test phase of qualification testing will be unchanged from present practices. Flight loads measurements are obviously related to the system of load determination so that if no changes are made in the latter, the approach used for flight loads measurements will also be unchanged.

6.3 SUGGESTED CRITERIA PRACTICE

6.3.1 Discrete Load Conditions

In the discussion of suggested criteria practice for structures fabricated from brittle materials, it is appropriate to begin with the consideration of discrete load conditions. Strictly speaking, it should only be necessary to consider design for repeated loads using the spectrum of loading which will include all of the loads expected to occur during the life of the vehicle. However, the use of discrete load design conditions is well established practice, it is particularly convenient for preliminary design and it provides the designer with a good appreciation of the capabilities of his structure. Furthermore, the lack of experimental data on the fatigue characteristics of typical brittle materials and the lack of any significant experience with practical hardware design leaves us without knowledge of the relative significance of discrete maximum loads and the whole spectrum of loads, to the design. Consequently, it may only be necessary in many cases to consider designing for discrete loads with attendant simplification of the design process.

It is suggested that current practice be followed as closely as possible with brittle materials in order to exploit as much as possible the generally successful experience with airframe structures. Accordingly, two discrete load conditions should be used corresponding roughly to the limit and ultimate conditions of conventional metal airframe design. Limit conditions should include the maximum loads which are likely to occur in the life of the vehicle.

For those aspects of the vehicle operation which are controlled by the pilot, or in the case of missiles and space vehicles by an automatic flight control system, the limits of the operating envelopes or the extremes of the programmed flight conditions should be used to define limit loads. This applies to maneuvers, and it includes the forces introduced to maintain a vehicle on a programmed flight path. It includes also the dispersions caused by variability in the functioning of guidance and control systems, variations in the thrust performance of rocket engines, tolerances in the gimballing of rocket engines, etc. Where variability in the performance of vehicle subsystems is involved or where loads may fluctuate randomly or uniformly, the objective should be to select levels such that the probability of occurrence of the combined system of loads is 1% or less. Stresses due to loads from various sources, both probabilistic and deterministic, can be combined as follows:

$$\sigma = \sigma_{1m} + \sigma_{2m} + \sigma_{3m} + \ldots n\sqrt{\sigma_{1f}^2 + \sigma_{2f}^2 + \sigma_{3f}^2 + \ldots}$$

σ_{1m} etc. are the mean stresses from each individual loading.

σ_{1f} is the r.m.s. value of the fluctuating part of any random or uniformly varying load. (Note that σ_{1f} is zero for a deterministic load)

n is selected so that the probability of σ being exceeded is 1%, or less.

For those load sources which are not under the control of the pilot or an automatic flight control system and which include principally natural phenomena such as winds, turbulence, gusts, etc., the limit conditions should be based on a 1% probability of occurrence; that is, the condition selected should be sufficiently extreme that there is only a 1% probability of the combined loads being exceeded in the life of the vehicle. The relationship given above can again be used to combine loads or stresses.

Associated thermal effects should also be based on extremes of likely occurrence. Aerodynamic heating should be determined for trajectories which are the extremes permitted for vehicle operations in conjunction with dispersions due to variations and tolerances in guidance, control and sensing equipment where the variation selected should be of such a degree that there is only 1% probability of exceedence of the combined condition in the life of the vehicle. Where sufficient statistical data is available similar levels of tolerance and variation should be applied to the data used to determine aerodynamic loads and heat transfer coefficients and to determine such factors as surface emissivity, material conductivity and other factors which control the temperatures and temperature gradients in the structural materials.

If the above practice is followed in defining loads, the resulting critical discrete load levels will have a probability of occurrence of 1% or less. This is a relatively high probability, in other words the resulting design loads are almost certain to occur. Accordingly, the probability of structural failure under the load condition defined above must be very small. In order to ensure this and also to avoid the problems discussed elsewhere of costly statistical programs to define material properties, it is recommended that the basic philosophy used for structural design be to conduct a proof test on each structure to the stress levels produced by the critical limit loads. This, in simple terms, will reduce the structural failure probability to zero, and the result should be essentially the same level of structural integrity as is present in conventional metal structures.

The use of a proof test to impose on each element of material the maximum stresses expected in service is an ideal situation which is not likely to be realized in practice. Most structures are designed by more than one loading condition and it is not likely to be economically practical to conduct a proof test on each component for each critical design condition. Even if only one loading is important it is not likely that the proof test will be a precise simulation, again for economical reasons. As a result the proof test will subject some elements of the structure to stresses greater than those expected in service, and other elements to smaller stresses. Nevertheless the benefits of the proof test in reducing the overall failure probability are retained as will be evident from the analytical relationships given in Section 3. In Section 3 equations are given from which the effect of the proof test in reducing the probability of failure for any given load condition can be obtained by a summation over all elements of the complete structure. Since negative values, such as might be produced by compressive stresses or by elements in which the proof stress is greater than the applied stress, are not included in this summation. The only way to make the result zero is to ensure that the proof stress exceeds the maximum operational stress from any condition at every point throughout the structure. This may well result in the imposition of relatively high and damaging proof stresses over some elements of the structure for the sake of exceeding operating stresses in a few other locations. Consequently, it is much more practical to assign a finite but nevertheless small probability of failure value to the structure under limit loads rather than to expect the value to be zero. Probability of failure values of the order of 10^{-6} or 10^{-7} are suggested and should generally be readily obtainable with a well chosen proof test.

It should be noted that in order to meet such a requirement it is not necessary that the proof test simulate any of the design conditions, and this point should be exploited to make the proof test as cheap and economical as possible. On the other hand a proof test which deviates too far from the simulation of the critical stress distribution and from reproduction of the maximum stresses will not establish confidence in the structural integrity. This situation should be automatically taken care of however by the methods presented in Section 3. If there is an element of the structure where the operating stresses are large and the proof stresses are small the probability of failure, even if only a few elements are involved, will be large and it will not be possible to meet the proposed criteria. This point is illustrated in Section 7 where very small volumes of highly stressed material in stress concentration areas such as fillets, are shown to contribute significantly to the overall structural failure probability.

The other complication that enters into the question of a proof test is the question of how much damage is done to the material by the proof test itself. In most practical cases the fact that the structure must sustain many load repetitions suggests that the damage in the form of flaw growth or crack propagation due to the proof test must be small. If it were not, it is unlikely that the structure would be able to sustain repeated loads. However, there may be applications such as single flight vehicles in which the critical design loads are not repeated, and where the critical loads are substantially greater than the loads expected during the remainder of the operation. A tentative criteria for selecting proof stress levels so that material damage is not significant has been evolved and leads to the following requirement.

$$\frac{1}{\sqrt{1 + \frac{1}{a}\left(\frac{da}{dN}\right)_{\sigma_p}}} \geq 0.99$$

where a is maximum flaw size in sample $\left(\frac{da}{dN}\right)_{\sigma_p}$ is flaw growth rate under application of proof stress. The derivation of this relationship is given later, in conjunction with a fracture mechanics approach to the establishment of criteria for the design of brittle material structures to sustain repeated loads.

Any criteria philosophy which requires a proof test of each article may be imposing significant cost considerations. Certainly to proof test a complete airframe would be a very serious consideration which would severely limit the interest in materials of this class. In the nature of nonmetallic refractory materials however, the manner in which they are fabricated and the structural weight penalties that will be imposed by their use is expected to limit this use, for aerospace applications, to those areas where high temperatures make metallic materials impractical. Such areas are expected to be limited in size and because of the requirements for hot pressing, sintering, diamond grinding, etc., in their fabrication, it is expected that these materials will always be used to make relatively small parts. As a result it is not expected that a requirement for proof testing of each part will impose a serious restriction on the use of these materials, or introduce a large cost increment.

It is believed that if the approach described above is followed with respect to discrete load conditions, the result will not be completely acceptable to the airframe designer and user because the question of safety factors has not been introduced. The procedure will lead to a structure which has a factor of safety, because larger loads than those described above as limit loads could be sustained with an increased probability of failure. Nevertheless, the level of this factor of safety is not defined and it is not related to past and current experience in the assessment of airframe integrity. Furthermore, the factor of safety implied in the material strength capability will be a function of the variability in material properties and may therefore vary from one structure to another depending on material characteristics.

For the reasons given above it is considered desirable to also continue the practice of designing the structure for factored loads, but rather than continuing with a completely arbitrary situation a more rational basis for the factor of safety can be established by accepting an increased probability of failure over that existing under limit conditions.

It is considered that factors of safety applied to loads should rationally reflect only uncertainties in those loads. Currently, there are many who consider that the factor of safety covers inadequacies in stress analysis, uncertainties in material properties or variability in strength of the complete structure as a consequence of variables in the manufacturing process. Evidently, however, if there are uncertainties in these areas, they should be accommodated by introducing appropriate factors or conservatisms or margins into the stress analysis or the material properties or the structural strength assessment as appropriate. It is certainly no rational to accommodate uncertainties in material strength prediction by an increase in the applied loads. In the case of brittle materials many of the uncertainties mentioned are necessarily considered with the more sophisticated design and analysis procedures that are required. As is indicated in other sections of this Handbook the stress analysis must reflect local concentrations, built in fabrication stresses, deformation constraints, etc., and the material properties used for the strength assessment must reflect variability caused by processing variables throughout the manufacturing activity from the starting powders to the finished component.

It is also clear that there is no significant question of yielding with brittle materials, which is another justification with metallic structures for considering two distinct design conditions. This is not to dismiss the possibility of creep deformations in high temperature ceramic structures nor does it imply that deflection limitations may not be a design criteria, but clearly these latter considerations again do not justify arbitrary factors on design loads. Similarly it is very unlikely that buckling considerations will ever arise in a ceramic structure because the length to thickness ratios of the various elements of each component are likely to be very small. As a result the uncertainties with respect to straightness and flatness, which are important in thin sheetmetal structures because of their effect on buckling, will not be of concern with ceramic structures.

Returning to the consideration of safety factors on loads, there appear to be two considerations which may justify factors of safety. The first situation arises when the maneuver envelopes and the range of flight trajectories including the effect of dispersion and variability in guidance equipment, sensors, etc. are exceeded, either by emergencies in the case of manned vehicles or by system failures in unmanned vehicles. Again, these situations should be examined rationally and appropriate design limit loads developed, but it is easy to argue that every possible emergency or malfunction can never be anticipated; thus, an arbitrary factor of safety on those load conditions produced by pilot initiated maneuvers or by the functions programmed into guidance and control equipment may be justified. At the same time since these are unpredictable emergency conditions with a very low probability of occurrence it is reasonable to accept a higher probability of structural failure. It seems reasonable to take advantage of past experience and retain factors of safety between 1.4 and 1.5 for manned vehicles and 1.2 to 1.3 for unmanned systems. In conjunction with these factors of safety the probability of structural failure can be raised to 1% or 10^{-2}. This is consistent with the use of "A" values from Mil-HDBK-5 for the design of metal structures.

Where brittle materials are used in high temperature airframe structures, which will generally be the case, the question of factors on temperatures and temperature gradients arises. If the principal reason for using factored loads is to cover emergency or malfunction conditions, it is reasonable to expect that these emergencies will also cause flight path deviations, taking the vehicle beyond the band of dispersed trajectories and flight paths within which it is supposed to operate. Hence more severe heating than that produced during limit load conditions can be expected. Unfortunately, there is no significant background of experience from which arbitrary factors on temperatures and temperature gradients can be based and there is certainly no justification for using the same factor on temperatures as is used on loads. The application of a factor to heat transfer coefficients for a certain period of time might be more reasonable. It would properly reflect the dependence of radiant heat dissipation on the fourth power of the temperature and it would properly relate the increase in temperature with a change in temperature gradient and the associated thermal stresses. No reasonable suggestions can be made, however, about the magnitude of this factor since it is likely to vary considerably from one vehicle to another. It, therefore, becomes necessary in each application to make some studies of the effect of emergencies and malfunctions on vehicle trajectories in order to make a judgment of the factor to apply to heating effects.

The other justification for a factor of safety applies to those design loads which result from the effect of statistically defined natural environments such as winds and gusts. If the limit loads are derived from environmental conditions which have a probability of occurrence of the order of 1%, it is reasonable to be concerned about the very slight possibility that much more severe conditions might be encountered. Again, it would be more rational to impose a specific probability level, for instance, to specify gusts which have a .01% probability of occurrence but generally to do this with any accuracy would require far more statistical data about the natural environments than is presently available or than is likely to become available for many years. Consequently, it is justified to apply an arbitrary factor to those loads produced by winds, gusts and other natural environments, but again to use a higher failure probability in selecting material and structural strength properties. The factors mentioned above are appropriate since they reflect past experience and again it is considered appropriate to use 1% failure probability levels in selecting material properties. Generally, the question of factors on temperatures and temperature gradients, and hence thermal stresses will not arise in this situation since significant changes in velocity and altitude are not likely to occur nor is the vehicle likely to be disturbed in its attitude for any significant length of time.

6.3.2 Repeated Loads

In considering the question of criteria for repeated load conditions the specification of loads and thermal effects follows directly from the previous discussions with respect to discrete loads. Load spectra should be generated to include all of the loads, trajectories and associated thermal effects to the levels which have been defined in 6.3.1 as limit values. How this is to be done in the case of gusts, random vibration, propellant sloshing forces and many other sources of loads which are statistical and random in nature is not a subject for this handbook since the methods will be the same as those currently used for metallic structures. These will generally be routine procedures in any aerospace company or airframe research organization.

The question of safety factors on loads, temperatures and temperature gradients to be used in the repeated load analysis does not arise if the reasoning given previously is used. With a concept that factored loads arise from emergencies, malfunctions or extremely rare environmental conditions, only a single occurrence of such loads should be anticipated and this will be covered in the discrete load analysis. Typically, however, in the design of metal structures, factors are applied to the number of load cycles. In other words, having developed a spectrum of loads representative of the complete life of the vehicle, the designer assumes this spectrum to be repeated a number of times, typical factors ranging from 2 to 4. Again, these are arbitrary factors variously considered to cover inadequacies in the prediction of fatigue strength of the structure and its materials, the lack of precise knowledge of the number of repetitions of some of the loads and the desire to have some life left in the structure at the time when it would be retired from service.

It does not seem rational to use a factor on life to cover any inadequacies in knowledge of the properties of the structure and its materials under repeated stresses. Such inadequacies should be covered by appropriate factors and conservatisms in selecting allowable stress levels. On the other hand, in the prediction of the load spectra uncertainties may exist in both the number of cycles of any particular load level or in the sequence of loads. This is particularly true for all of the various load sources which are probabilistic in nature since levels, numbers of cycles and sequences are all statistical. In general, however, statistical knowledge of load frequencies and sequence is much less developed than statistical knowledge of load intensities. In fact, the significance of load sequence in the fatigue of metal structures has only recently been recognized and there are no available methods for discussing the probability of any particular sequence.

The requirement for a reserve in structural life so that it is not at the point of failure during the last moments of operational use is realistic, but whether in the case of brittle materials this requires a factor on the design life may depend on the methods used to predict the life under repeated loads.

If these methods involve probabilistic values of material properties then the reserve may be inherent in the material allowable values used. At the present time, however, the available knowledge about the response of brittle materials and structures fabricated from brittle materials to repeated loads is so small that at least a factor of ignorance is justified.

On the basis of the above discussion it is considered that for the design of brittle material structures under repeated loads, a factor on life should be used and values between 2 and 4 which have been commonly used for metallic structures seem appropriate.

In discussing the problem of predicting material and structure life under repeated loads, as a basis for establishing appropriate design criteria, we begin with essentially no data in the literature except a few isolated examples (Reference 6.7) which are sufficient to show that the flaws and microcracks inherent in most of the materials considered in this Handbook do, in fact, grow with stress repetition. As a consequence what follows is speculative and would require substantial experimental verification before application to the design of flight hardware.

There are at present two methods used by structural designers for describing the behavior of materials under repeated stresses, and it is reasonable to attempt to extend the same methods to brittle materials. The first of these involves a generation by experimental means of data describing the number of cycles to failure for a material under repeated stresses of a particular level. Information is presented as a plot of stresses vs number of cycles, the typical S-n curve. Most materials are sensitive not only to the peak stress but to the range of stress imposed during stress cycling so that S-n curves are usually presented for various values of the factor R which defines the ratio of maximum stress during each cycle to mean stress. Obviously also the response of materials to repeated stresses is a function of temperature so that the test data must be repeated for each temperature of interest.

In almost all structural applications successive stresses applied to an element of the structure will vary in intensity, frequently in a random manner. Since it is not practical to conduct fatigue tests in which all possible sequences of stress levels are examined, methods have been developed for assessing the material damage accumulated by the number of cycles of stress at each level. Miners rule is generally used and is given below.

$$\frac{n_1}{N_1} + \frac{n_2}{N_2} + \frac{n_3}{N_3} + \ldots\ldots = 1$$

In this relationship N_1 represents the number of cycles to failure at a particular stress level σ_1. The data is obtained from the S-n curve. n_1 is the number of cycles actually applied at this stress level and it is assumed that the ratio $\frac{n_1}{N_1}$ is the fraction of material life used by the stresses of a level σ_1. It will be evident that this rule attributes no significance to the sequence of stresses, which is inconsistent with observation.

If this method is applied to brittle material, it will be necessary for each material and each temperature, to generate Weibull curves for material subjected to various cycles at the particular stress level. What is required is shown in Figure 6-1 where a point such as A shows the probability that the material will fail when subjected to 10,000 cycles of the stress σ_4.

When the fatigue tests are conducted it will not generally be possible to select a stress level such that failure occurs precisely at the required number of cycles, particularly since the stress level will vary from sample to sample in a statistical manner. Accordingly, the tests must be conducted at preselected stresses, the number of cycles to failure observed and extrapolation used to obtain the stress level at which the particular piece of material would have failed for a particular number of cycles. This extrapolation should be done using an S-n curve but it is not possible to obtain such a curve for a single piece of material. Accordingly, it is necessary to plot all of the test data on an S-n plot, to draw a representative line through these points, and then extrapolate each test point to the required number of cycles by extrapolating parallel to the average S-n curve. This procedure is illustrated in Figure 6-2. Having obtained a series of failure stresses for a given number of cycles the procedure described in Section 3 for determining the appropriate Weibull curve is then followed. As with metals the process requires variation not only of peak stress in each cycle but mean stress.

Curves such as Figure 6-1 can then be used to determine the permissible stress level for a given number of cycles if a specified probability of failure is to be achieved. The procedure can be extended to a cumulative damage summation where the applied stresses vary, by again using Miners rule and where the number of cycles to failure (N_i) are determined for the various stress levels at a particular failure probability level. If the summation gives the value 1 then the failure probability of the structure is the value used to determine the values N. If the summation gives a value less than 1 the probability of failure is less than that assumed or alternatively all stress levels can be increased.

If it is intended to proof test the structure it may be possible to analytically truncate the curves of Figure 6-1 similar to the Weibull expression used to truncate

the curve for a single cycle of stress. It is not evident at this time how this might be done, however, and the alternate is to subject each specimen to the proof test before conducting the fatigue tests.

The consequence of the above procedure is as follows:

a) Miners rule needs verification for brittle materials.

b) The suggested method of extrapolating test points to obtain the probable failure stress for a particular test specimen and for a specific number of cycles may be considerably in error. The accuracy will depend primarily on how closely the number of cycles to failure for a particular specimen agrees with the selected cycle value. This in turn depends on the stress level selected for each particular test specimen, but in view of the variability of the material and the impossibility of determining before testing what the strength level is and there is no evident method of exercising control.

c) The need for statistical data, which has already been discussed with respect to the generation of Weibull curves for a single stress cycle, is now extended to cover such additional parameters as number of cycles, ratio of peak stress to mean stress and proof stress. Thus, in most cases to conduct such programs may be economically impractical.

Assuming however that this procedure can be followed, the only criteria statement required is the specification of acceptable failure probability under the complete spectrum of stresses. A value of 10^{-6} or 10^{-7} is suggested since the spectrum will include stresses (limit values) with a relatively high probability of occurrence.

In all of the above discussions it is assumed that appropriate conservatisms will be introduced into the selected stress levels to cover the uncertainties already described in the method and also to reflect the statistical significance of the number of test points used to generate each curve.

The second method in current use for the determination of the life of metallic structures involves the use of fracture mechanics principles and again it is reasonable to pursue the same ideas in the design of brittle material structures as used in the design of metal structures. The stress level which will cause a crack to propagate catastrophically to a complete failure is proportional to a critical stress intensity factor, and to the crack size, and to a function of the geometry of the structure. The critical stress intensity factor is a material property which can be measured in tests utilizing simple specimens, and the results can be used to predict the onset of failure in a complex structure with a complex stress distribution. Since the initial crack size must be known it is usual to apply this technique to determine allowable stresses, and it is assumed that the material contains a flaw which is as large as the minimum size detectable by whatever inspection techniques are used.

In its current state of development fracture mechanics is limited to relatively simple structures and flaw configurations by the difficulties of analytically determining stress intensity factors for structures of complex geometry and such complex flaw shapes as a partially through-crack.

In extending fracture mechanics principles to the prediction of structural performance under repeated loads, it is necessary to determine experimentally, crack growth data as a function of stress level, temperature, etc. With this information predictions can be made of the growth of an initial flaw as a result of the various stresses applied during the life of the structure, and the requirement is to determine whether the critical size flaw is reached for the maximum stress level during this life.

In metallic structures the question of crack initiation also arises and experimental data is generally needed on the combination of stress and numbers of cycles required to initiate a crack in an otherwise homogeneous piece of material. This complication, however, is not likely to arise with ceramics since these materials are assumed to contain large numbers of flaws initially.

At this point virtually nothing has been done to study fracture mechanics as applied to ceramic materials. Reference 6.8 indicates tests to measure critical stress intensity factors on a number of materials such as aluminum oxide and silicon carbide, but there is no evidence in the literature of efforts to measure crack growth rates, and it is not known at this time whether it is practical to make such measurements.

If we assume that critical stress intensity factors and crack growth rates can be measured for brittle materials, it is reasonable to assume that these characteristics are not statistical in nature. The cause of the variability in the mechanical properties of ceramics is assumed to be the flaws and defects which are the result of processing, together with the sensitivity of the material to the stress concentrations produced by the flaws. The variability is not, in other words, inherent in the material itself. If we consider the growth of a flaw where the crack is propagating through homogeneous material, this crack growth should be reproducible, as a function of the applied stresses, from any material sample. Since a typical ceramic material is likely to contain large numbers of flaws, crack growth characteristics might be affected by the propagation of the crack from one flaw to the next, and in this respect the size and shape distribution of flaws may influence crack growth characteristics. However, the volume of material affected by a growing crack is very small, particularly since critical flaw sizes in ceramics at the stress levels of interest will be only a few thousandths of an inch.

In using the fracture mechanics principles with ceramic materials we begin with the assumption that failure is produced by propagation of a flaw of critical size and that the variability in strength properties is the result of variability in the size and distribution of flaws. Thus, the Weibull strength also gives the probability of a flaw of a certain size in any individual sample. Conversely, if a probability level is given the corresponding maximum flaw size can be determined. The two relationships necessary to do this are as follows:

$$S = \left[\left(\frac{\sigma - \sigma_u}{\sigma_o}\right)^m - \left(\frac{\sigma_P - \sigma_u}{\sigma_o}\right)^m\right] V \qquad (1)$$

$$a = \left[\frac{K_{IC}}{\sigma f\left(\frac{a}{w}\right)}\right]^2 \qquad (2)$$

Where

- m σ_u and σ_o are Weibull constants. (See Section 3)
- σ = specimen failure stress
- σ_P = proof test stress level
- V = specimen volume
- S = specimen failure probability associated with stress σ .
- a = maximum flaw size in specimen that fails under stress σ .
- K_{IC} = stress intensity factor
- w = width of tensile specimen
- $f\left(\frac{a}{w}\right)$ = geometric factor

With the Weibull parameters determined by strength tests, the first equation can be used to determine the failure stress for any given probability level and the second equation will determine the maximum flaw size in the test piece that fails at that stress level, assuming that the critical stress intensity factor K_{IC} has been measured in separate tests. Note that the effect of proof testing the material can be easily included. The proof test eliminates material containing flaws greater than the critical size associated with the proof stress, but in so doing it decreases the probability value associated with a flaw of any size.

In order to determine flaw size information from the Weibull cruve, it is necessary to know the value of the geometric factor in Equation 1 for the configuration of the test specimens used to conduct the strength tests. This will usually consist of either a square cross-section bend specimen or a round bar tensile specimen. In either case we require the geometric factor for an essentially infinite volume of material with a small crack in the surface. Geometric factors for this crack configuration are available in the literature Reference 6.9. From the above procedure, if the allowable material failure probability under repeated loads is specified, a maximum "allowable" flaw size can be determined. The flaw size will depend on the proof stress level so that in a complete structural component the allowable flaw size will vary throughout the component as the stress produced in the proof test varies. Having determined allowable flaw sizes throughout the component, the flaw growth at each point is calculated from flaw growth data, which will be given as a function of stress level and temperature, and from the spectrum of stresses and temperatures expected throughout the component life. When the maximum size flaw at the end of the structural life has been determined for each element of the component in this manner, a check can be made to determine whether the critical flaw size has been reached at any point for the maximum stress to be experienced at each point. If the critical size is just reached in some location, the probability value used to determine the initial flaw sizes is the probability of failure of the component. If a critical flaw size is not reached, either a margin of safety is indicated or the stress levels throughout the component life can be raised by reducing material thicknesses, etc.

This method of determining the effect of repeated loads on a brittle material structure offers the promise that no additional statistical data other than that required to develop the Weibull curve is needed. It does, however, involve numerous assumptions which, with the present limited knowledge and experience, still require verification. These include, (1) flaw growth data and critical stress intensity factors are not statistical; (2) that it is practical to determine flaw growth data and critical stress intensity factors for brittle materials; (3) that flaw growth data can be applied without regard for the sequence of stresses. This is not a good assumption for metallic structures because plasticity effects at the crack tips change the crack shape as a function of stress--such effects are likely to be absent or very much reduced with brittle materials; (4) that geometric factors can be determined for all structural and flaw configurations of interest. This has not presently been done because of mathematical complications for metallic structures although there is considerable activity involving various methods of attack, and there is no reason to doubt that the necessary information will eventually become available.

Again, the only criteria statement required is the specification of acceptable failure probability under the complete spectrum of stresses and environments. If the suggestions of this section have been followed and the spectrum includes stresses associated with loads up to limit level, then a failure probability of 10^{-5} or 10^{-7} is suggested. In addition, however, appropriate conservatisms must be introduced to cover the uncertainties indicated above in the method and particularly the uncertainties in crack growth and critical stress intensity factor data. These conservatisms will depend on the state of knowledge and the amount of test data available at the time the analyses are conducted. It is not presently feasible to offer any suggestions in this respect since the available data base is zero.

Returning to the discussions earlier in this section on factors of safety for discrete loads, the question arises whether provision should be made to accommodate these factored loads at the end of the vehicle life, after the structure has been exposed to the full spectrum of repeated loads. If, as suggested, a factor of safety is used to cover unpredictable emergencies or severe environmental conditions which available statistical data is inadequate to predict, then the probability of these occurrences diminishes as the vehicle life proceeds. Since the factors are arbitrary, however, there is little purpose in trying to add the refinement of reducing the factors as the life of the structure is used. Consequently, it seems most practical to require that the structure sustain one cycle of factored loads at the end of the structural life. However, the probability of failure can be raised to 10^{-2} which indicates that the predicted flaw growth will be applied to a smaller initial flaw size if "ultimate" loads are to be sustained.

Previously in this section the subject of material damage due to proof testing has been raised and an expression was given from which the proof stress level which would not cause significant material damage could be determined. This relationship can now be derived by application of the fracture mechanics principles described above, but it also involves the assumptions and limitations which have been described. It is determined by equating two values for the expression of the critical stress intensity factor; the one containing the initial flaw size before proof testing, and the other including the growth in flaw size due to the proof stress. This development is presented below.

Assume that flaw growth rate under proof stress is given by $\left(\frac{da}{dN}\right)_{\sigma_P}$ then the critical stress of material before proof testing is given by $\sigma_1 = \frac{K_{IC}}{f(\text{geometry})\sqrt{a}}$. Critical stress of material after proof testing is given by $\sigma_2 = \frac{K_{IC}}{f(\text{geometry})\sqrt{a+\left(\frac{da}{dN}\right)_{\sigma_P}}}$.

If material is not to be damaged significantly by proof testing, then σ_2 must be close to σ_1. Say $\sigma_2 \geq .99\,\sigma_1$

Then $\frac{1}{\sqrt{a+\left(\frac{da}{dN}\right)_{\sigma_P}}} \geq \frac{.99}{\sqrt{a}}$ or $\frac{1}{\sqrt{1+\frac{1}{a}\left(\frac{da}{dN}\right)_{\sigma_P}}} \geq 0.99$

6.3.3 Qualification Testing

In the discussions of 6.2 it was stated that qualification testing includes flight testing to measure loads and verify structural integrity, and ground testing to verify the assumed failure modes and the predicted strength under these failure modes. It was shown that flight testing must remain unchanged when brittle materials are used since flight testing to achieve critical loads on a probabilistic basis is impractical. Ground testing, however, requires further discussion.

If a typical "static" test is conducted on an airframe containing elements fabricated from brittle materials, a failure in one of these elements will be a test result that is of little value. Whether the failure occurs above or below the design "ultimate" load, it will not generally be possible to decide whether or not the design was satisfactory. A failure above ultimate may be due to the combination of an inadequate design with a material strength on the high end of the scatter band, or conversely a premature failure may be due either to "low" strength material or an inadequate design.

Some design verification is possible, in this situation, by using strain gages but if some failure mode or some high local stress has been neglected the strain gages are not likely to be located in the proper location. With brittle materials the failure initiation point will probably not be detectable after the failure has occurred since the part will generally break into many pieces. Consequently, it will not be possible to locate strain gages at the failure initiation point on a second test specimen.

Obviously, what is necessary is a statistically significant sample size. While this is completely impractical with complete airframes it is expected, as has been discussed elsewhere in this Handbook, that brittle materials will be used in relatively small structural elements each mounted in an unconstrained, nonredundant manner from the primary structure. It thus becomes quite feasible to conduct the qualification tests on individual elements with little concern that failure modes will be missed because of the absence of the complete airframe. Alternatively, it may be possible to test numerous

brittle elements using a single metallic primary structure with the loads adjusted slightly to ensure failure first in the brittle element.

Thus, it is feasible to consider a statistically significant sample if only a relatively small brittle element is destroyed each time. However, a statistically significant sample size may still involve a large number of samples, particularly if proof testing is not used. Fortunately, however, the problem is now the reverse of trying to determine a Weibull curve sufficiently accurately to predict the failure stress for a very low probability of failure. Now we are given an accurate Weibull curve and the problem is to determine the probable response of a small number of specimens. Since the results of say three or four tests should lie near the mean strength level, where the Weibull curve is most accurate, there should be little difficulty.

In order to put this concept into practice we first determine, for the structural element to be tested, a curve of failure probability against some reference stress σ_x. Stresses at all other points throughout the element are expressed as a ratio of the stress σ_x at the reference point. The equation for determining failure probability is:

$$S = 1 - e^{-B} \qquad (3)$$

where $$B = \frac{\sigma_x{}^m}{\sigma_o{}^m} \sum V^m \left[\left(\frac{\sigma}{\sigma_x} - \frac{\sigma_u}{\sigma_x}\right)^m - \left(\frac{\sigma_P}{\sigma_x} - \frac{\sigma_u}{\sigma_x}\right)^m\right]$$

where m, σ_u and σ_o are Weibull constants, explained in Section 3.

$\frac{\sigma}{\sigma_x}$ is the ratio of the stress in any element V, for the design condition of interest, to the corresponding stress at the reference point.

σ_p is the proof stress in any elemental volume V.

Note that the exponential form of the equation is used since we will be concerned with a small number of test results and relatively high failure probabilities.

The summation is extended over all elemental volumes V, in the structural element.

Values of S are calculated for various assumed values of σ_x and the result is plotted as shown in Figure 6-3.

Now suppose that three test specimens are used and they fail at three different values of σ_x. The results are ranked, the lowest representing a failure probability of 0.25, the next an S value of 0.5 and the highest a value of 0.75. These results are plotted, as shown in Figure 6-3 at the appropriate failure probability levels. By comparing the resulting curve with the predicted curve, observations can be made about both the element design, and also about the similarity of the material in the structural element to the material of the specimens used to determine the Weibull constants.

If the experimental curve parallels the Weibull curve but is displaced, the material is material is satisfactory but the design is either better or worse than predicted depending whether the experimental values of σ_x, for a given failure probability, are greater or less than the predicted values.

If the experimental curve passes through the predicted curve at S = 0.5, but does not parallel the predicted curve, the design is as predicted but the material characteristics differ from the material defined by the Weibull parameters. Combinations in which both the design and the material differ from prediction are also, of course, possible and likely, but those results can be broken down into the two cases given above. Figure 6-3 explains this procedure diagrammatically.

It should also be possible to follow a similar procedure for repeated load testing by combining the above with the previously described method for predicting failure probability under repeated loads. This suggestion is offered very tentatively, however, in view of the fact that so many of the previously described steps are presently unverified by experiment. The procedure would be as follows:

a) For each elemental volume in the structural element under consideration, establish the stress due to the proof test.

b) Select a probability of failure value and from Equations 1 and 2 of this Section determine the corresponding probable failure stress and hence the probable maximum flaw size, in each elemental volume of the component.

c) For the sequence of repeated loads, and associated stresses, determine the growth of these flaws. Determine the number of load sequences required until, at some point in the structure, a flaw reaches critical size.

d) Repeat (b) and (c) for other probability values and plot the results as a curve of failure probability versus number of load sequences to produce failure.

e) Conduct repeated load tests on a small number of specimens, applying a sufficient number of load sequences to each specimen to produce failure.

f) Present and use the results as shown in Figure 6-3 except that the parameter σ_x is replaced by number of loading sequences.

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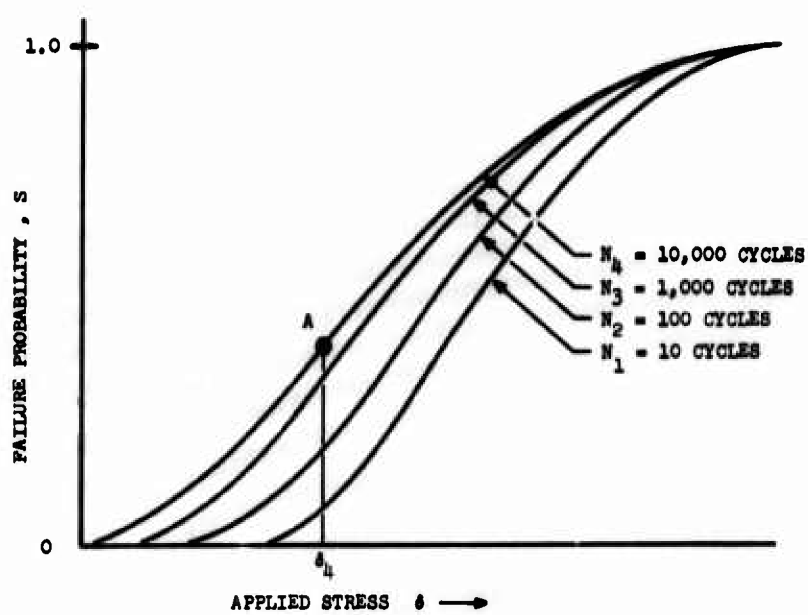

Fig.6.1 Weibull curves from repeated stress testing

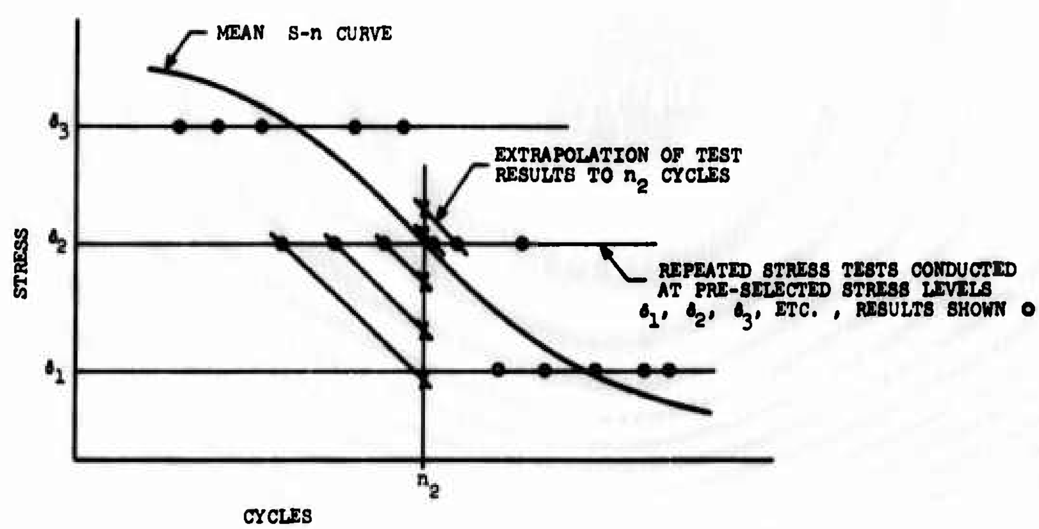

Fig.6.2 Method for generating Weibull curves for repeated stress conditions

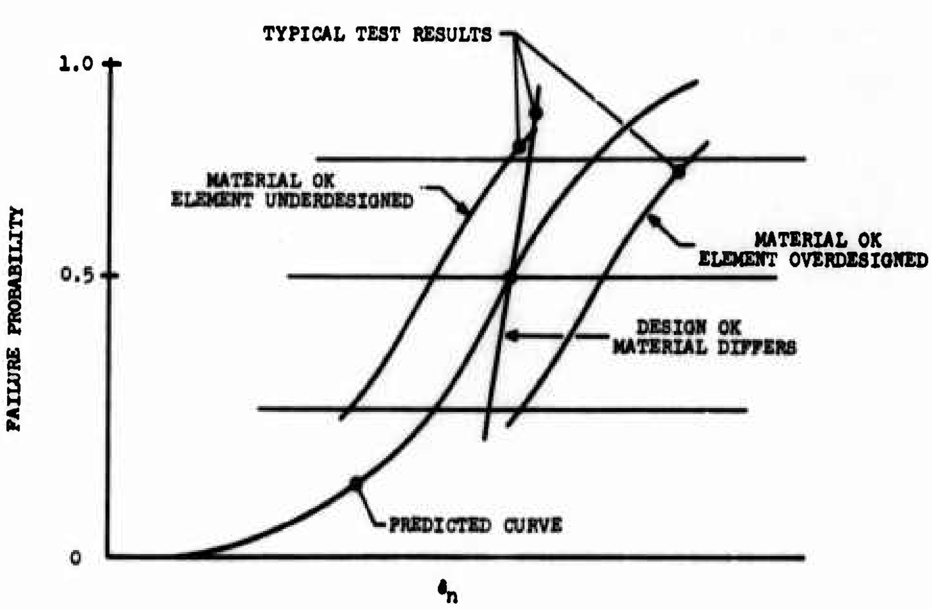

Fig.6.3 Qualification testing of brittle elements

7. DESIGN TECHNIQUES

7.1 BRITTLE MATERIAL APPLICATIONS

Brittle materials find many engineering applications where particular characteristics such as electrical insulating qualities, refractoriness or low cost are necessary. Such applications include furnace linings, electrical insulators, etc., where the limitations of weight and the requirements for high structural reliability, which are typical of aerospace applications, do not apply. The discussions of this chapter, however, will be limited to applications where the material must be used in an efficient structural manner, and consideration will be limited to very high speed airframe components and components of rocket engines and gas turbines.

In aerospace applications it can be expected that ductility is a characteristic that will always be desired by designers and one for which some compromise of the structure in other respects, such as weight, will be made. None of the minor advantages of refractory, nonmetallic materials such as low cost, high stiffness to weight, etc., are likely to pursuade a designer to sacrifice ductility. Consequently, applications for brittle refractories will almost certainly be limited to those where temperatures, and the environments in which they occur, exceed the capabilities of metallic materials.

Within the above limitations the possible aerospace applications for refractory nonmetallic materials include small lifting surfaces, lifting surface leading edges, engine inlet leading edges, heat shield elements, insulative surface elements and nose caps, all for lifting re-entry and hypersonic cruise applications. Nonmetallic refractory materials may also be used as primary structure for radomes and antennas and small very high speed missile bodies.

In addition to the above airframe applications these materials have found and will continue to find use as rocket nozzle inserts and possibly for the construction of complete nozzles and thrust chambers. Many elements of gas turbines and other types of air breathing engines may also use nonmetallic refractory materials where freedom from impact loadings can be assured. The use of these materials in compressor and turbine blades of conventional gas turbines has been quite unsuccessful due to impact from ingested material, but advanced applications can be visualized where the high temperature requirements are so severe that special provisions to eliminate ingested material on or near the ground are justified.

All of the above applications are expected to involve relatively small parts. Furthermore, parts made from nonmetallic refractory materials are generally produced from powder by hot pressing, pressing and sintering or slip casting and sintering techniques so that geometrically complex shapes are relatively easy to produce. Generally, therefore, nonmetallic refractory components will be designed with integral stiffening and internal structure in which case the only detail design consideration is the method of attaching nonmetallic elements to internal metallic structure. All of the applications mentioned offer the opportunity for the use of one-piece nonmetallic parts from the highest temperature regions of the application to some point where the internal environment is cool enough for a metallic primary structure. Thus, a discussion of detail design practice reduces primarily to a discussion of joining methods for attaching nonmetallic elements to supporting metallic structure.

The present early development stage of brittle material design technology is mentioned elsewhere in this handbook. Methods of joining are even less advanced than the technology generally. No more than half a dozen unclassified U.S. publications exist at present which describe original work on joining, and all of the work reported to date is dependent on the use of metallic elements within the joint. Methods for joining ceramic to ceramic where high structural strength at high temperature is required have seen little development, with the exception of one British program, described in Section 7.6. This situation emphasizes the need for one-piece elements to the point where the environment permits metallic materials to be used in the joint. Furthermore, because of the very limited experience with the joining of brittle materials, it is not possible to present in this chapter well substantiated design rules or even empirical methods. Accordingly, the chapter will present concepts and principles and a suggested design practice which it is believed will lead to satisfactory joints. Whether the degree of design refinement suggested is sufficient or necessary can only be determined by experience.

The chapter is supplemented by design charts which are chiefly concerned with the application of statistical methods to failure prediction for the particular design situations encountered in joints. Again the lack of experience makes it difficult to assess the significance of the absolute values given by these charts, but the relative values are extremely useful in choosing the best design configurations for particular joints.

7.2 JOINT DESIGN PRINCIPLES

The important principles which should be applied in the design of joints in brittle materials were mentioned briefly in Section 2.3. This Section reviews these principles in more detail as a basis for the discussion of their application in joining concepts.

Typically, in metallic construction, balanced internal load systems of relatively low stress level can be accepted. Such load systems arise from deformations under load of

the supporting structure, from assembly when the parts do not fit accurately and from modest temperature gradients. When these built-in stresses are combined with stresses due to the external loads applied to the element, a local yielding or buckling may occur where the material yield strength is exceeded. This will generally relax the internal load system and not materially affect the ability of the element to carry the externally applied loads. The same situation is not true with brittle materials where yielding and general buckling is not possible. Thus, any built-in stress system must be known and must be included in the assessment of the total stress picture. An important principle in joint design, therefore, is to attempt to arrange the joints so that such internal load systems are eliminated. This requires that an element be supported so that it is constrained against motion only in three mutually perpendicular directions and against rotation about three mutually perpendicular axes. Note, however, that this can be accomplished with a single attachment. Any other constraints than these will introduce internal stress systems which are not statically determinate. When this principle is applied rigorously, it will frequently indicate the need for some type of spherical bearing within a joint to eliminate rotational constraint about all axes. A typical pin joint for instance, which is commonly used to attach metallic substructures to the primary structure, provides rotational freedom about one axis only. A three-point attachment for a nonmetallic leading edge will require three spherical bearings to avoid all local rotational constraints and two of the three bearings must also be free to translate along one axis.

The concept of using a statically determinate support arrangement for any "brittle" structural element would appear to eliminate the possibility of redundancy in the structure, whereas the characteristics of these materials makes redundancy even more desirable thatn it is with a metallic structure. Again, this is a subject for which practical experience is completely lacking. Conservatism then requires that if a redundancy in attachments is included, the part should be checked for the various statically determinate attachment load systems which are possible when various attachments are assumed ineffective.

Nonmetallic refractory materials can generally be used to much higher stresses in compression than in tension, with the same degree of reliability. Thus, if only compressive stresses can be produced by internal load systems these may be beneficial. To effect this benefit the balancing tension loads must be reacted in the metallic primary structure. This can frequently be done quite conveniently where the principal source of stress is due to temperature gradients. Since the nonmetallic element will be located to accept the highest temperatures, it will normally expand relative to any supporting metallic structure. Restraints of this expansion may introduce beneficial thermal stresses, and this restraint must be built into the joints between the nonmetallic and metallic components. This technique must be used very cautiously, however, since the use of refractory materials implies very high temperatures such as 3000°F to 4000°F which in turn suggests large temperature gradients. Under such circumstances thermal stresses in the high modulus refractories can quickly become very large notwithstanding the very high compression strength of most of these materials.

The considerations mentioned above with respect to the joints between metallic and nonmetallic components apply similarly within the joint itself. Any type of multiple connection implies an unknown load distribution since in a material lacking yielding the load distribution will depend on the tolerances and degree of fit at each attachment point, on the smoothness of the surfaces in contact, and on the deformations in the surrounding material, in which a very complex stress system will generally exist. These conditions are generally too complex for a practical analysis of elastic deformation and consequent load distribution to be made. Thus, if multiple connections are used conservatism requires that various possible load paths be examined with only the minimum statically determinate number of connections effective in each.

Another principle which must be followed in the design of joints in brittle materials is attention to stress concentrations, a consideration which has been emphasized repeatedly in this handbook. In a typical structural component the consideration of stress concentrations will usually imply concentrations due to changes in the geometry of the structure and the absence of constant cross sections. In the case of joints and attachments, however, there is another important source of stress concentration and this results from dimensional tolerances in the detail parts. For instance, if a bolted connection is used in a metallic structure, bearing pressures between the bolt and the surrounding material are computed on the basis of a uniform distribution across the bolt diameter. Experience shows that this assumption predicts the ultimate strength of connections satisfactorily. Presumably whatever the initial form of the distribution of bearing pressure it becomes uniform as a result of local yielding before the ultimate strength of the connection is reached. When this yielding capability is absent, the fact that bolt and hole diameters will never match precisely results in the concentration of load along a line.

The consideration described above can be carried further because in general, and again as a result of manufacturing tolerances, the axis of a hole in a structural part will not coincide directionally with the axis of a bolt or pin which fits into the hole (see Figure 7.1). Again, as a result and in the absence of yielding, this could lead to the concentration of load at a point at one edge of the hole. In the case of a metal pin, attaching a ceramic part this concentration may be modified by local yielding of the pin, and in general there has been insufficient experience to know to what extent it is necessary to consider all such possible tolerance effects. The examples mentioned here are intended only to illustrate possible problem areas. If these considerations are pursued in a particular application, they will probably lead either to the selection of a joint configuration in which manufacturing tolerances are not important or it will be

necessary to assume that some degree of stress concentration can be accepted by the material and to confirm this assumption experimentally. Experimental confirmation of an assumption is not easy, however, because the concern is not with a failure level but rather with probability of failure, which implies a sample size sufficiently large to have statistical significance, which in turn implies a costly experimental program.

7.3 DESIGN CONCEPTS

It would be desirable, in order to assist designers concerned with the application of brittle materials, to present examples of joining and attachment methods which incorporate the principles that have been presented and which have been extensively developed and verified with actual hardware. At the present time this is not possible because new joining concepts particularly appropriate to brittle materials have not been evolved, even on paper, and the limited amount of joining that has been done has been based on adaptations of metallic joints. It is believed, therefore, that joining of brittle materials will begin with the application of joint configurations and joining concepts which have been used successfully with metallic materials and that new configurations will evolve as experience is gained.

There are two considerations which can be expected to provide the basis for the evolution of new joint configurations for brittle materials, both of which result from attention to stress concentrations. In any component fabricated with a material having no yielding capacity, stress concentrations will be significant since they will force only a small part of the total material available to work at stress levels for which the material is capable, while most of the material contributing to the weight of the structure will be working at very low stresses. This problem is made much more severe when variability of material properties is considered. The effect is illustrated by Figure 7.5 which is a simple tension member of thickness t subjected to a stress σ with a change in cross section involving a fillet of radius r. The figure shows the ratio of allowable stress with the stress concentration effects considered, to the allowable stress if the stress concentration is zero, for any probability of failure and for a range of values of fillet radius. The effect of the material constant m is included and two combinations of proof testing and zero probability of failure stress, (σ_u) are considered. An increase in the fillet radius from 0.2 t to 1.0 t can increase the permissible applied stress for a given failure probability by a factor which can be as high as 2 . Joint configurations are expected to evolve from studies of this type, and as this example illustrates the method will involve either reducing stress concentrations by careful selection of local geometry, such as fillet radii, or removing material which would otherwise be operating at low stresses, which means reducing the thickness t.

The type of design study work just mentioned has not been attempted at this time so that it is not possible to project even approximately the configurations that well developed brittle material joints will take. Consequently, examples to be presented in this section will be confined to typical metallic connections.

Another approach to stress concentration reduction is applicable to concentrations which arise in mechanical connections involving a pin or a bolt through a hole. In this situation the combination of manufacturing tolerances and the inability to yield result in a line contact between the pin and the plate, which again leads to a severe stress concentration. A possible approach to stress reduction in this situation is the introduction of a thin ductile metallic liner between the pin and the hole in order to provide yielding capability and load distribution. Although in general nonmetallic refractory materials will be used only where temperatures prevent the use of metals, the choice is primarily one of economics, since there are precious metals having good oxidation resistance at very high temperatures. While it is not economically feasible to use these materials for structural elements, or even their attachments, it is possible to use them in very small quantities for applications such as that mentioned. This concept has been tried experimentally, however, with little success. More details are given in 7.6 which summarizes current experience in brittle material joining.

Some typical joints which might be used for brittle structures are illustrated in Figures 7.1 through 7.4. As explained these are similar in principle to conventional metallic connections and they all anticipate, for reasons already mentioned, that all joints will involve connection of nonmetallic components to metallic components. The figures illustrate some of the features of the various joints, and they show the possible stress concentration areas which must be considered in properly refining the configuration.

Figure 7.1 is a standard shear lug connection which is useful for joining major structural components where subsequent disassembly is anticipated, or where freedom for component deformation is required. The joint provides rotational freedom about the pin axis and translational freedom along the pin axis. This latter is accomplished by bending the metallic support bracket. A metallic spherical bearing can also be incorporated to provide rotational freedom about each of the two axes which are perpendicular to the bolt axis. The joint will then provide four degrees of freedom and two degrees of constraint.

The connection shown in Figure 7.1 contains many sources of stress concentration which must be considered in the stress analysis. There are stress concentrations around the hole and stress concentrations at the fillets where the lug becomes an integral part of the nonmetallic component. Concentrations arise at the contact between the lug and the pin as a result of dimensional tolerances on diameter, pin bending which will tend to concentrate the load at the edges of the hole, and eccentricities in the direction of

loading due to tolerances in the location of the two parts of the attachment. There is not presently sufficient experimental evidence to indicate whether it is indeed necessary to consider all of these effects, but it seems reasonable to minimize them by appropriate joint configuration design and then establish the absolute size optimistically and to verify the joint integrity experimentally.

Where significant temperatures arise and a close tolerance metal pin is used in a nonmetallic lug it may be necessary to make some provision for the thermal expansion differences between the pin and the hole. The proper use of a tapered pin and a tapered hole accomplishes this and is based on the fact that although dimensional changes resulting from temperature are different from materials with different thermal expansion coefficients, angles remain unchanged. Thus, if the mating surfaces are tapered so that the projections of these surfaces meet at a single point, then the tolerances between the pin and the hole will not be changed with temperature.

Figure 7.2 shows a number of tension attachments between nonmetallic and metallic components, nd they include versions which are useful for connecting cylindrical sections such as rad 's or small missile body sections. This type of joint provides no accommodation for relative motion between the nonmetallic and metallic components such as may be caused by deformations under load or temperature gradients. Accordingly, arrangements for avoiding restraints on the deformation of the nonmetallic component must be made by building flexibility into the metallic supporting structure.

Figure 7.2 shows the principal stress concentrations which can arise in tension attachments and which must be minimized by the proper selection of fillet size and the proper selection of thickness and thickness taper in the various elements of the joint. Other stress concentrations arise at the point of contact of the bolt head and the surface of the nonmetallic material. These concentrations might be minimized by soft metal inserts or by using spherical contact surfaces to allow for bolt misalignment, etc. Accurate pretensioning of the bolts may be necessary in a multiple bolt connection to ensure that each bolt takes its share of the load.

Figure 7.3 shows some typical shear joints which are useful for transmitting tension, compression, shear or a moment in the plane of the splice plates. Symmetrical construction is shown to minimize stress concentrations from eccentric loadings and tapered joint elements can be anticipated to minimize the stress concentrations at the ends of the splice elements. So far as possible the proper choice of stiffness characteristics and thickness of the bond material should also be made with stress concentration reduction as the objective.

It should be noted that most of the literature which is concerned with stress distributions in joints such as those shown in Figure 7.3 emphasize the stresses in the bonding material. For present considerations where the bond will generally be a metallic braze, the important stress concentration is that which occurs in the nonmetallic plate elements.

Figure 7.4 is a typical mechanical splice again using double splice plates to avoid eccentricities. This type of joint is unlikely to be successful unless some type of yielding insert material can be used between the bolts and the holes in the splice plates, or alternatively if the bolts are of soft material or have plated surfaces or are hollow or include some other device to provide a slight amount of local deformation. If this is not done the load distribution between the bolts is extremely dependent on manufacturing tolerances, surface finish, etc., which, even if these characteristics are controlled, cannot be included in the load distribution analysis.

7.4 SUGGESTED DESIGN PRACTICE

7.4.1 Selection of Material

For a structural component to be fabricated from brittle nonmetallic refractory materials the selection of materials will involve the usual considerations of temperature capability, oxidation resistance, availability, fabrication considerations, cost, etc., and since these topics are not particularly unique to brittle materials they will not be discussed here. With respect to mechanical properties, however, the basis for the selection of material is quite different from that used with metallic materials where yield strength and ultimate strength and possibly fatigue characteristics are the primary considerations.

The important mechanical property considerations in material selection when brittle materials are involved are the average strength of the material and the shape of the strength distribution curve. These considerations are described by the three Weibull parameters m, σ_0 and σ_u. Furthermore, the significance of these parameters is dependent on the acceptable failure probability level and also on the structural configuration. Thus, these parameters will have a different significance with respect to material selection at a joint with its usual preponderance of stress concentrations than in regions of the structure where the structural cross sections do not change.

In Section 7.5 which follows, analytical relationships are developed to determine the probability of failure for an element of structure containing a stress concentrations, and these relationships can be rearranged to describe the permissible stress in regions away from the concentration for a specified probability of failure. Such relationships show that, for very low failure probabilities, the allowable stress is directly proportional to the Weibull material parameter σ_0. Thus, the significance of this parameter

in selecting materials is straightforward, the greater the value, all other material parameters remaining constant, the greater is the allowable stress.

The material parameter m is b[illegible] most important consideration in material selection, however, and using [illegible]lionships given in 7.5 numerical studies have been made for a fillet at a change [illegible] section for a range of values of both geometric and Weibull parameters. The st[illegible] shows that approximately

$$\frac{\sigma_{m_1}}{\sigma_{m_2}} = S^{\frac{1}{m_1} - \frac{1}{m_2}},$$

where S is the allowable failure probability and m_1 and m_2 are the values of m for two different materials which are being compared and σ_{m_1} and σ_{m_2} are the corresponding allowable stresses. The studies show that the geometric characteristics of the stress concentration have little affect on this relationship. If a typical failure probability of 10^{-6} is assumed and $m_1 = 9$ and $m_2 = 3$ which covers the range of values of m which are likely to be specified, then the above relationship shows that the allowable stress can be increased by a factor of 20 if a material with m value of 9 can be used rather than an m value of 3. These effects are increased as the stress concentration is increased and as the proof stress and the zero probability of failure stresses are reduced. For a proof stress and zero probability of failure stress of zero, and a fillet with an r/t of 0.2 the above effect can be doubled.

While m is possibly a material characteristic it is certainly also affected by the material processing since well controlled processing with the minimum of variations from batch to batch will minimize variability in the resulting material. Since the value of m is so significant in determining the allowable stress and hence weight, it is important to impose rigid controls on the material processing after having first selected a material with an inherently high value of m. More discussion of this topic is given in Section 5.

The third material parameter is σ_u, the zero probability of failure stress. Figure 7.6 shows the significance of this parameter to the allowable stress for a typical stress concentration effect of a fillet at a change of cross section. The curves show the effect on allowable stress of both σ_u and the proof test and curves are given for two extreme values of fillet radius.

From an examination of Figure 7.6, it is evident that the effectiveness of σ_u increases as the stress concentration decreases and as the proof test level increases.

From the above discussion it will be evident that of the material parameters which describe mechanical characteristics m is by far the most significant and considerable efforts should be made to obtain and use materials with high m values. σ_o is next in importance with allowable stresses and hence weight being directly related, and σ_u is of least significance since it is useful only if the stress concentration effects are not severe and in the absence of a proof test.

7.4.2 Selection of Proof Test

Figure 7.7 shows the results of studies conducted to determine the significance of proof test level in joint areas. Again the example consists of a change of cross section including a fillet and two fillet radii are considered. The allowable stress in terms of the allowable stress with zero proof test is presented as a function of proof stress level. Clearly, from these results the proof test can be significant with materials of low m value but is of little use for m values around 9. Even with low m values the proof stress needs to be at least 90% of the applied stress to produce significant benefits.

It should be noted that these conclusions are derived by studying a structural element containing a stress concentration which is typical of a joint. The results for both a proof test and σ_u are related to the fact that the proof test bears the same relationship to the local stress at every point in the element. Thus, its benefits are approximately independent of geometry. σ_u on the other hand may be a large proportion of the applied stress σ, but it does not change with local stress. Thus, its benefits in the areas of high stress which control the design are small to a degree which is dependent on the severity of the stress concentration.

It should also be noted that the considerations above, with respect to the significance of the proof test, assume that the Weibull curve for the material is known equally well, whether or not a proof test is to be conducted. The improvements noted from proof testing result from truncation of the strength distribution curve, not from improvements in its accuracy. The proof test also has benefits, however, either in the reliability of the Weibull curve for a given number of material test samples, or, conversely, in the number of samples required to achieve a distribution curve of specified accuracy.

Unpublished studies of this aspect of proof testing have been made by establishing a computer program to generate 10,000 random numbers ranging from 0 to 1 and using these numbers to represent to the results of 10,000 hypothetical material tests. This represents essentially an infinite population so that the corresponding Weibull curve can be accurately defined. The program also permitted any number of these hypothetically test points to be selected randomly and the corresponding Weibull curve generated. Thus, allowable stresses for a given failure probability could be determined from the "exact" Weibull curve and from the Weibull curve determined from a limited number of test points. By this method the effect of the number of test points on the accuracy of the allowable stress was examined.

An additional refinement was the repeated selection of a specific number of test points so that for each number the mean values, standard deviations and confidence limits of the Weibull constants were determined. Finally, the method permitted an exact determination of the effect of proof stress on the number of test and specimens needed to achieve an allowable stress of specified accuracy.

The number of samples required to determine allowable stress within 25% of the true value, with 90% confidence, when a proof test is not used, is summarized below as a function of m.

Failure Probability	10^{-2}	10^{-3}	10^{-7}
No. of samples for m = 2	350-400	500	>500
No. of samples for m = 8	5-10	60-100	>500
No. of samples for m = 20	<5	<5	65

When a proof stress is introduced into the above study the number of samples required to predict allowable stresses within 25% of the true values is reduced to between 5 and 20 specimens depending on the value of m, the level of proof stress, etc. Such values are much more practical, particularly for very low values of failure probability. However, the study also confirmed that for very low values of failure probability, say 10^{-7}, the difference between the allowable stress and the proof stress is negligible. The significance of this fact on structural design criteria is covered in Section 6.

7.5 DESIGN CHARTS

In other parts of this handbook the importance of conducting a stress analysis sufficiently refined to reveal local stress concentrations has been emphasized. The requirement for predicting failure probability based on a statistical distribution of material strength characteristics has also been indicated. When this design procedure is applied to a typical joint the resulting analytical work is laborious for a number of reasons. The geometry of a joint is typically complex, introducing numerous stress concentrations and requiring an elaborate stress analysis to obtain reasonably correct stress predictions. The typical engineering theory will not produce even crude approximations to the stress distribution in many typical joints, and finite element methods (see Section 4) applied with a very refined element breakdown are necessary to obtain good stress predictions. Furthermore, the prediction of failure probability is laborious as a direct consequence of the rapid variations of stress that occur throughout a joint, again requiring a very fine element breakdown if the methods of Section 3 are to be applied.

The complexity described above applies when it is necessary to analyze a joint that is already designed. The problem is significantly more difficult when it is necessary to carry out the design process, since this usually involves a number of analyses of different joint arrangements until a design which meets all of the requirements is established. Furthermore, experience in designing when failure probability, rather than strength limitations, becomes the basis for an acceptable design is virtually nonexistent so that the establishment of a satisfactory design will probably require many more trials than is usual in designing joints in metal structures.

This section is intended to relieve this situation by providing design charts which should facilitate the design of attachments in brittle materials. Some approximations have been made in the interests of facilitating the rapid analysis of joint designs, since, where necessary, a refined analysis can always be made using the methods presented in Sections 3 and 4 after the preliminary design work has been completed. The approach used to joint analysis in this section is as follows:

a) A number of typical joints and attachments have been examined and a small number of typical stress concentration problems have been selected as representative of most of the stress concentration effects to be found in actual joints.

b) The failure probability theory has been linearized for small values of failure probability and this permits the total failure probability of a structural element to be determined by summating contributions from various effects.

c) The various stress concentration problems mentioned under a) above have been studied parametrically to determine the resulting stress distributions, and also to assess the increment of failure probability due to the stress concentration for a range of values of the Weibull material parameters.

d) Charts are presented in this section to give the increment of failure probability for various stress concentration effects in terms of the geometric parameters associated with the stress concentration and the Weibull material parameters.

The procedure in using this section to analyze a typical joint is therefore as follows: First, a simple stress analysis of the joint is made neglecting stress concentration effects, and the failure probability of the joint is assessed on the basis of this simple stress analysis. Next, each of the stress concentration effects present in the joint is examined and an increment of failure probability is determined for each, using the charts presented in this section. Finally, these increments of failure probability are added to

the failure probability determined from the simple stress analysis, to obtain the probability of failure for the joint.

The generalized stress concentration problems which are believed typical of those experienced in joint design are summarized in Figure 7.8. The fillet is considered typical of most changes in cross section which occur within a joint.

The typical hole shown in Figure 7.8 is self-explanatory except for the fact that the design charts to be presented cover only the case of a uniaxial stress since, as will be shown presently, the problem of failure probability prediction has been reduced to a linear solution and the principle of superposition can again be used for any complex stress state.

The third concentration problem is the pin in a hole, which is considered particularly significant in brittle material joints since the lack of yielding can lead to line contact with very high local stresses. In this latter case consideration is given to both a metallic pin in a nonmetallic plate and also the case where both the pin and the plate are nonmetallic.

In all of these examples of stress concentrations, geometric boundaries have been eliminated where possible in order to minimize the number of geometric parameters to be considered. Generally speaking, this can be done quite effectively since stress concentrations are local effects. The assumption is even more valid when a failure probability is being determined since the exponent in the Weibull distribution function greatly emphasizes the significance of the peak stresses, which of course are extremely localized.

Consider an element of structure in the region of a stress concentration, and denote by σ_x the maximum principal stress at any given location in the element. Then the probability of failure of the entire element, using the linearized form of the failure probability expression given by equation 21 of Section 3, and including only the maximum principal stress at any point is:

$$S = \Sigma V \left[\left(\frac{\sigma_x - \sigma_u}{\sigma_o} \right)^m - \left(\frac{\sigma_p - \sigma_u}{\sigma_o} \right) \right]^m$$

Assume that at a remote distance from the stress concentration the stress is σ. Then define a stress concentration factor:

$$K_1 = \sigma_x / \sigma$$

And define a proof test factor:

$$K_o = \sigma_p / \sigma_x = \frac{1}{K_1} \frac{\sigma_p}{\sigma}$$

Then $$S_1 = \left(\frac{\sigma}{\sigma_o} \right)^m \Sigma V \left[\left(K_1 - \frac{\sigma_u}{\sigma} \right)^m - \left(K_1 K_o - \frac{\sigma_u}{\sigma} \right)^m \right]$$

This expression assumes that the proof load produces the same stress distribution as the operating stress, which will usually be valid for a local concentration.

Now if the stress concentration is neglected, by using some simplified analytical approach, we have:

$$S_2 = \left(\frac{\sigma}{\sigma_o} \right)^m \Sigma V \left[\left(1 - \frac{\sigma_u}{\sigma} \right)^m - \left(K_o - \frac{\sigma_u}{\sigma} \right)^m \right]$$

The increment of failure probability Sc, due to the stress concentration, is therefore:

$$Sc = S_1 - S_2 = \left(\frac{\sigma}{\sigma_o} \right)^m \Sigma V \left\{ \left(K_1 - \frac{\sigma_u}{\sigma} \right)^m - K_o^m \left(K_1 - \frac{1}{K_o} \frac{\sigma_u}{\sigma} \right)^m + \left(1 - \frac{\sigma_u}{\sigma} \right)^m \right.$$
$$\left. - K_o^m \left(1 - \frac{1}{K_o} \frac{\sigma_u}{\sigma} \right)^m \right\}$$

In general if V is the volume of any element of material and the results are to be expressed in terms of the fillet or hole radius, and the structural thickness, then V must be numerically evaluated for the case of unit radius (r) and unit thickness (w) and with this understanding

$$Sc = w r^2 \left(\frac{\sigma}{\sigma_o} \right)^m \Sigma V \left\{ \left(K_1 - \frac{\sigma_u}{\sigma} \right)^m - K_o^m \left(K_1 - \frac{1}{K_o} \frac{\sigma_u}{\sigma} \right)^m + \left(1 - \frac{\sigma_u}{\sigma} \right)^m - K_o^m \left(1 - \frac{1}{K_o} \frac{\sigma_u}{\sigma} \right) \right\}$$

In order to evaluate this expression, finite element analyses of the various stress concentrations have been made, to determine K_1 for each element. Then the term $\left(K_1 - \frac{1}{K_o} \frac{\sigma_u}{\sigma} \right)^m$ is summated over all elements for various values of m and $\left(\frac{1}{K_o} \frac{\sigma_u}{\sigma} \right)^m$.

The remaining terms in the expression for Sc are constants for particular values of σ_u, K_o and m. In the evaluation of Sc, $\left(K_1 - \frac{1}{K_o}\frac{\sigma_u}{\sigma}\right)^m$ and $\left(K_1 - \frac{\sigma_u}{\sigma}\right)^m$ is not included if the value, for a particular element, is zero or negative.

Figures 7.9 through 7.12 give the increment of failure probability Sc due to the stress concentration effects at a fillet in a brittle material element. The data is expressed in terms of element width, w, and fillet radius r, and the Weibull material parameters m and σ_o. The constant is included to facilitate plotting of Sc on a logarithmic scale. Note that the geometric parameters in these figures must be in inch units.

The increment of failure probability Sc is to be added to the failure probability determined by neglecting the stress concentration. Referring to Figure 7.8 this simple stress distribution involves the stress σ, in the portion of the element of thickness 2t, and it is assumed that this stress changes abruptly, at the change of cross section, to a uniform value determined by the increased section depth.

In Figures 7.9 through 7.12 K_o defines the proof test level and is equal to $\frac{\sigma_p}{\sigma}$ where σ_p is the stress produced by the proof test, in the element of thickness t. The ratio $\frac{\sigma_u}{\sigma}$ expresses the ratio of the material "zero probability of failure stress" to the applied stress.

Figure 7.13 gives increment of failure probability due to the stress concentration produced by a hole of unit radius in an infinite plate of unit thickness under a uniaxial stress σ.

The stress concentration condition illustrated by item 3 of Figure 7.8 requires a different treatment since no tension stresses are produced locally as a result of the contact between the pin and the hole. Since the Weibull approach to the determination of failure probability assumes that compression and shear stresses do not contribute, the stress concentration effect shown does not increase failure probability. However, it is not certain, at this time, that compression and shear do not contribute to failure probability so that some information on the effect of this type of stress concentration is desirable for the designer.

In Section 5 of this handbook the subject of material failure under compression stresses is discussed and will be evident that the compression strength of nonmetallic refractories does have a limiting value. Since this is many times the tensile strength it is not normally critical but in the situation illustrated by Figure 7.8/3 the compression stresses can be so large that some check must be made. Accordingly, Figure 7.14 gives data on the maximum stresses.

7.6 EXPERIMENTAL RESULTS

A review of the literature shows that very little work has been done to join ceramic materials to each other in a manner which would permit the high temperature capabilities of the material to be fully exploited. One outstanding exception to this statement is work on silicone nitride, jet engine combustor cans by the British Navy. Typically, these cans are a complex sheetmetal part, and the work referred to has reproduced these in silicone nitride by cementing many complex pieces of material together.

In Reference 7.1 a number of bolted joints in graphite and alumina were evaluated. These were tensile specimens containing a single bolt. Tensile control specimens were used to determine the material strengths and joint efficiencies were quoted based on the gross cross sectional area with no provision for the stress concentration effect of the hole. The joint efficiencies obtained during these studies are low, but unfortunately since a stress analysis of the joints is not given the effectiveness of the design techniques used cannot be assessed.

Some of the alumina test joints included copper and Apocast inserts around the pin. The copper insert joint was no better than the plain pin joint, but the use of Apocast inserts resulted in the highest joint efficiencies.

Reference 7.2 investigated joining methods for attaching alumina to a metallic structure. Two types of bolted joints were evaluated: (1) bolted joints with nominal holes, and (2) sleeved or potted bolts using copper and brass sleeves, Sairset cement, Presstite, and silver braze potting. The specimens were a simple rectangular shape, concentrically loaded. Before conducting tensile tests with the above specimens, the authors conducted torque-down tests with countersunk and hexhead fasteners in 1.4-inch thick alumina plate. All of the specimens containing countersunk fasteners failed during torquing, presumably due to the wedging action of the countersunk head, although the authors indicate the mismatch of the bolt head and hole taper to be the cause. For the hexhead torque-down tests, either the nut was stripped or the bolt broke. The tensile tests show large scatter in the results from simple bolted joints, significant benefit from metallic sleeves (by a factor of at least 2.0), but surprisingly poor results from the use of silver braze. Potting with cements produced various results; Presstite, for instance, produced significant improvements, but Sairset cement did not. The tensile strength of all the joint specimens, however, was substantially less than would be predicted by taking the net section strength (0.10 to 0.20 times the net area strength).

Two types of bolted attachments in ATJ graphite were evaluated in Reference 7.3. Most tests were made with a standard bolt in a standard hole, but some are made with specimens such as Figure 7.1 which included a spherical metal bearing on a tapered bolt. The bolt taper mates with a similarly tapered graphite surface such that the thermal expansion differences are accommodated.

The single point attachment involves a connection between metal plates and a graphite block using two metallic bolts. Differential axial growth between the bolt holes in both members of the joint is compensated by slotting one of the bolt holes in the metallic member of the joint. The joints were tested in tension, and the results were compared with the predicted failure loads for both attachments based on (a) the ATJ graphite strength for one failure in one hundred, and (b) the mean strength of the ATJ graphite. In addition to making predictions based on two material strength levels, two analysis methods were also used to predict the failure loads for the standard lug. The first method considered the lug as a plate loaded through a hole with a stress-concentration factor for the effect of the hole, while the second method employed the theory of elasticity. Stress-concentration factors only were used for the single point attachment stress analyses.

Two standard lugs were loaded to destruction in tension and, in both cases, the failure load was 1800 lb. One of the lug specimens had a prior loading history. This lug was subjected to 300 repeated load cycles which varied from zero to 1200 to 0 lb. Evidently, this prior loading history did not cause material damage. An additional standard lug, which had been coated with an oxidation-resistant coating, failed at 850 lb. It was noted from the material test bars, however, that the coating lowered the strength of the ATJ graphite, because the coating contained microcracks due to the differential expansion of the coating and the graphite substrate.

A comparison of the two methods of analysis indicated the failure load predicted by the theory of elasticity method (640 lb. for one failure in one hundred strength and 1050 lb. based on the mean strength) and the loads predicted by the stress-concentration method for one failure in one hundred (700 lb.) were conservative. The mean strength and the stress-concentration method gave good predictions (1320 lb.) for the uncoated lug; for a small sample size, the mean strength should, therefore, be realized. However, the mean strength and the stress-concentration method were unconservative for the prediction of the coated lug failure load (1240 lb.), even though the strength data were obtained from coated bars. It is believed that these latter results are quite questionable due to the cracks in the coating.

Five "single point" attachments were subjected to tension loads, and five specimens were loaded by differential shear loads by a loading bar. Four of the tension specimens were loaded by one bolt, and one specimen was loaded through two bolts. The predicted failure load for one failure in one hundred was 914 lb., while the mean strength prediction was 1550 lb. The test failure loads ranged from 1640 to 2380 lb. Evidently, predictions including stress-concentration effects and the use of the mean strength, since only a few samples were involved, gave very satisfactory results. The highest failure load (2380 lb.) was obtained by the specimen loaded through two bolts; this would be expected, since the stress-concentration factor for a multiple connection is lower. This effect was not included in the predictions. The predicted failure loads for the single point attachments loaded by differential shear loads were 340 and 580 lb., based on the one failure in one hundred strength and the mean strength, respectively. These are the predicted loads which, when applied to the loading bar 3.5 inches from the outboard hole, would cause failure. The actual failure loads ranged from 390 to 415 lb. In this case, the prediction using stress-concentration factors and the material mean strength is a little unconservative but still in remarkably good agreement.

Another series of tests were conducted by Anthony, Reference 7.3, to obtain experimental indications of the effect of lug proportions on load carrying capability. These specimens were essentially with lugs machined at the end of the specimen. There were four types of lugs:

a) 0.250-inch hole dia., 1.250-inch lug dia., 0.500-inch thick.
b) 0.500-inch hole dia., 1.500-inch lug dia., 0.500-inch thick.
c) 1.000-inch hole dia., 2.000-inch lug dia., 0.500-inch thick.
d) 0.500-inch hole dia., 1.500-inch lug dia., 1.000-inch thick.

All the lugs were designed with equal cross-sectional areas except type d, which had a cross-sectional area twice that of the others. Equal areas eliminated the effect of size between specimens a, b, and c. That is, the failure load should be identical for types a, b, and c if concentration factors and material variability could be neglected. With specimen d having the same geometry as specimen b, but twice the cross-sectional area, a direct indication of the effect of size was obtained. If there were no size effect, type d should have had twice the load carrying capability of type b.

Predicted failure loads were calculated, both on one failure in one hundred strength and on the mean strength, by use of the stress-concentration factor method for a flat plate loaded through a pin, and five lugs of each type were tested in tension. Both sets of predictions and the test results are shown in the following table. The predicted failure loads based on the one failure in one hundred strength tended to be conservative. The predicted failure loads with the use of the mean strength showed excellent agreement, except for lug type d. This discrepancy may have been a size effect of the ATJ graphite, which was not used in the predicted failure load.

TEST RESULTS OF LUG FAILURE LOAD

		Predicted failure load, (lb)		
Lug type	No.	1 failure in 100 strength	Mean Strength	Average failure load (lb)
A	5	Conservative	350	434
B	5	Conservative	520	581
C	5	Conservative	730	732
D	5	Conservative	1040	813

Since the cross sectional area of specimen type a, b, and c were equal, the only variable (other than the variability of the material) was the stress-concentration factor due to the geometry of the lug. As the hole-diameter-to-lug-diameter ratio increases, the stress-concentration factor is reduced. The test results also incidate this. Type a lugs, which had the smallest ratio of hole-to-lug diameters, had the lowest average failure load, while type c lugs, with the highest ratio of hole-to-lug diameters, had the highest average failure load. The results from this series of tests provide considerable confidence in the approach of including the effects of stress concentrations in designing reliable brittle material joints.

Reference 7.2 also reports investigations of bonded joints with a bond in shear and the results are promising but the testing limited. Each specimen consisted essentially of an alumina plate bonded to a metal plate and loaded in tension. The alumina plate ends were either parallel or modified by machining a shoulder or a taper. Bonding was supplemented in the joints by use of a bolt or clamping. The test failures for the clamped-and-bonded shoulder end and the clamped-and-bonded tapered end specimens did not occur in the bonded area but at points of stress concentrations that were introduced into the alumina element of the specimens by the machining of the filleted shoulder or the taper. The bonded-and-bolted parallel end specimens with no bolt load attained the highest joint strength. In fact, a load capability approaching that which would be predicted from the net area strength through the bolt hole was achieved. The results from this series of tests again suggest that adhesively bonded joints designed with stress-concentration factors in mind may be satisfactory. Also, as would be expected, there is no benefit to be gained by combining bolts with a bonded joint.

Clamped joints were also studied by Hofer (Reference 7.4). Specimens were fabricated in Marblette, a brittle organic substance, and also in Hydrostone plaster. The specimens were bars which had a "necked down" region at each end to receive the clamps. Approximately 100 joints in each material were tested in tension. The joint efficiencies ranged from 13.3 to 26.5% for the Hydrostone specimens, and from 18.3 to 38.1% for the Marblette specimens, based on the gross cross-sectional area. The specimens failed in the necked down area, the point of maximum stress concentration, and minimum cross-sectional area.

Frye and Oken (Reference 7.2) conducted evaluations of three clamped joints in alumina. The test joints were somewhat different from Hofer's joints in that the alumina had a filleted shoulder machined at each end, and the metallic clamps butted against these shoulders. The test joint efficiencies were 14.6, 14.9 and 14.5%.

Unfortunately, these two investigations presented test results as joint efficiencies, and it is not possible to assess the effectiveness of the clamp joints, since a stress analysis of the joints is not given.

Attachments for joining cylindrical shells by internal flanges were studied in Reference 7.4. The test specimens, which simulated one-half of the joint, were cylinders 4 inches high, with an outside diameter of 4 inches and a wall thickness of 1/4 inch, cast in Hydrostone plaster. The specimens were cast in three internal flange thicknesses: 1/4, 3/4, and 1½ inches. The test specimens were mounted by cementing the unflanged end of the cylinder to a plate and applying an axial tension load to the cylinder by loading the internal flange through a loading disk. The results of these studies were presented as joint efficiencies, that is, the failure load was compared to the load carrying capacity of a homogeneous nonolithic cylinder. Based on the mean strength, the joint efficiencies ranged from 5.7 to 14.9% and from 13.3 to 25.2%, for the 1/4- and 3/4-inch internal flanged cylinders, respectively. When the specimens with the 1½-inch flange were tested in the same manner, failure of the adhesive bond at the unflanged end of the cylinder occurred. Therefore, the remaining cylinders were tested as cantilever cylindrical shells, mounted at the flanged end. Joint efficiencies were calculated on the basis of the theoretical bending stress present in a cantilever beam of the same length. Twenty-five specimens were tested with the resulting joint efficiencies ranging from 42.9 to 75.3%, based on the mean strength of the Hydrostone plaster.

The results from the internal flanged cylinders with an axial tension verified analytical studies of this joint by Hofer, in that increasing the flange thickness resulted in a substantial increase in the load carrying capacity of the joint. The

higher joint efficiencies obtained for the joint in bending may have been a result of lower stress-concentration factors for bending relative to axial load, as well as a smaller volume of material subjected to the peak bending stresses.

In summary, the experimental evaluations of joints are very limited and have numerous shortcomings. Adequate support with stress analyses is generally not available, but, in the few cases where consideration of stress concentrations has been made and where adequate predictions are available for comparison with test results, there are remarkably good correlations.

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7.3 Anthony, F. M. et al, "Investigation of Feasibility of Utilizing Available Heat Resistant Materials for Hypersonic Leading Edge Applications, Volume II Analytical Methods and Design Studies", WADC TR 59-744, November 1960.

7.4 Hofer, K. E. Jr., "Utilization of Refractory Nonmetallic Materials in Future Aerospace Vehicles, Part II Study of Attachments for Brittle Components", AFFDL-TDR-64-123, August 1965.

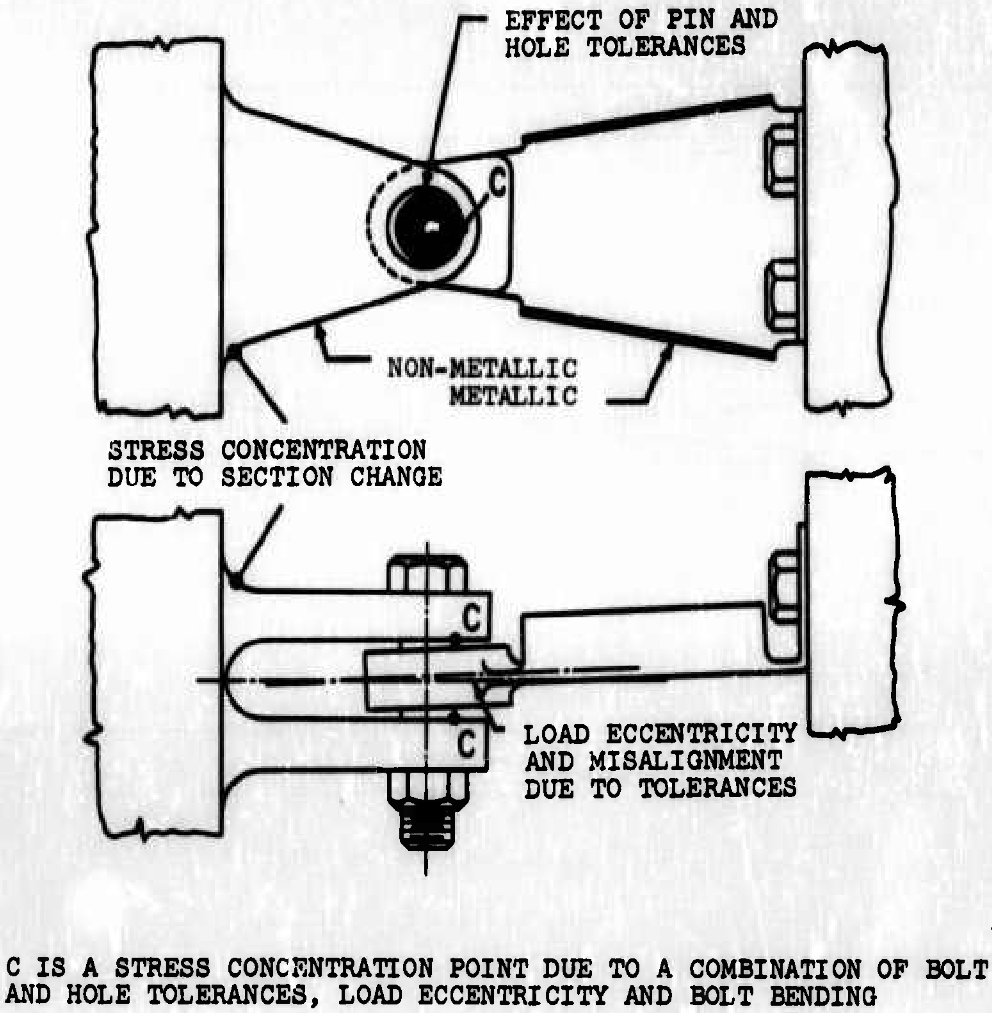


Fig.7.1 Shear lug for attachment of non-metallic to metallic components

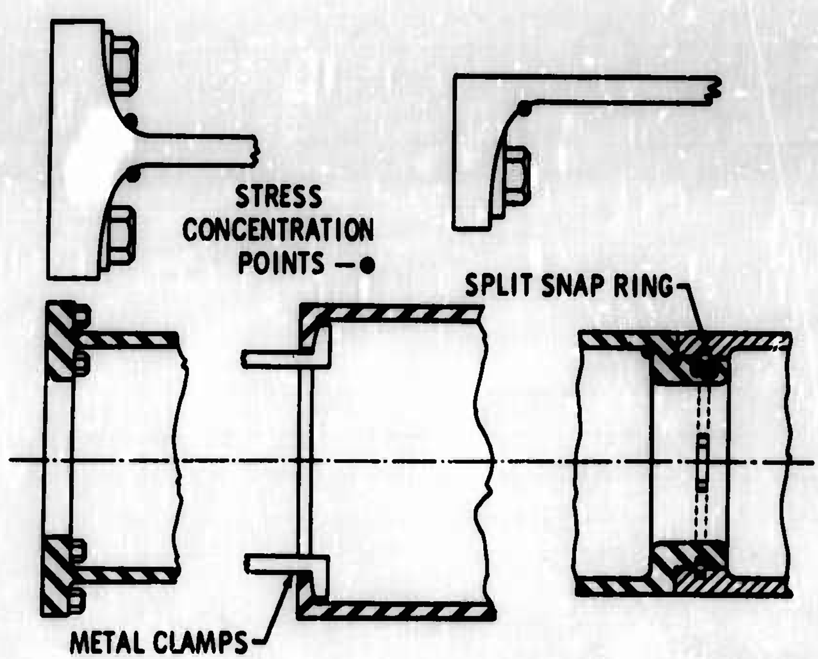


Fig.7.2 Tension attachments between non-metallic and metallic componentr

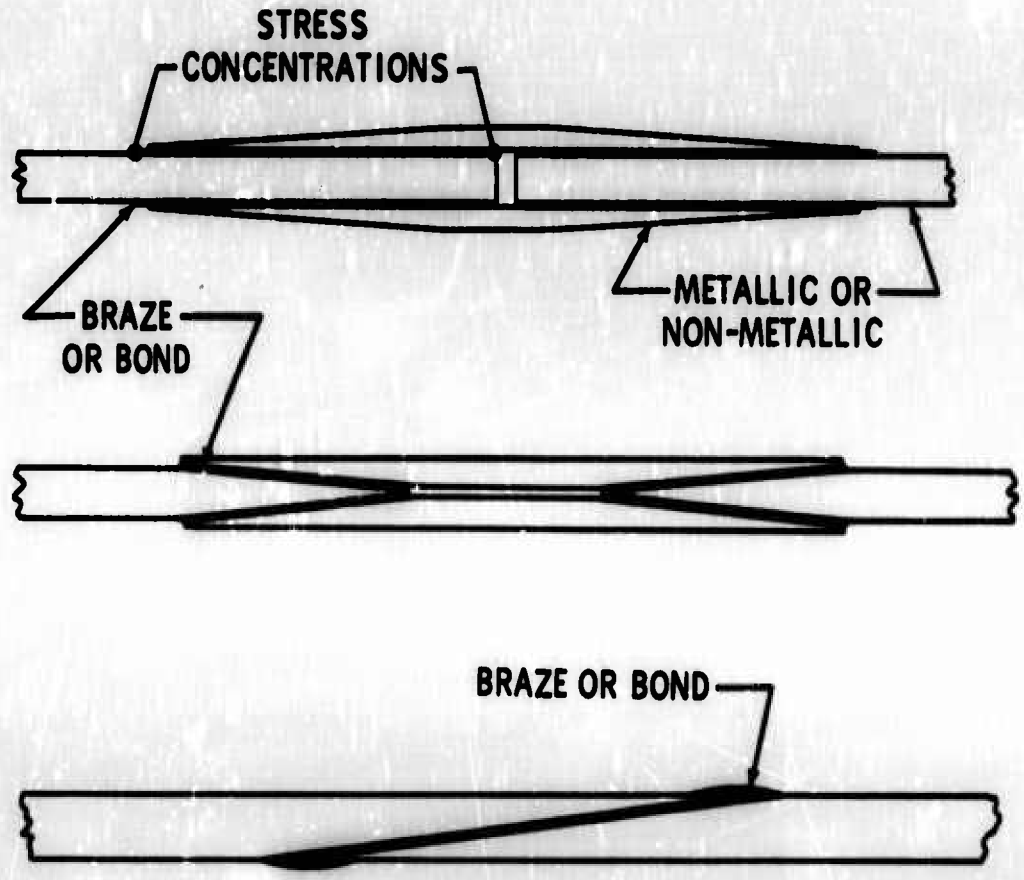


Fig.7.3 Bonded and brazed lap joints

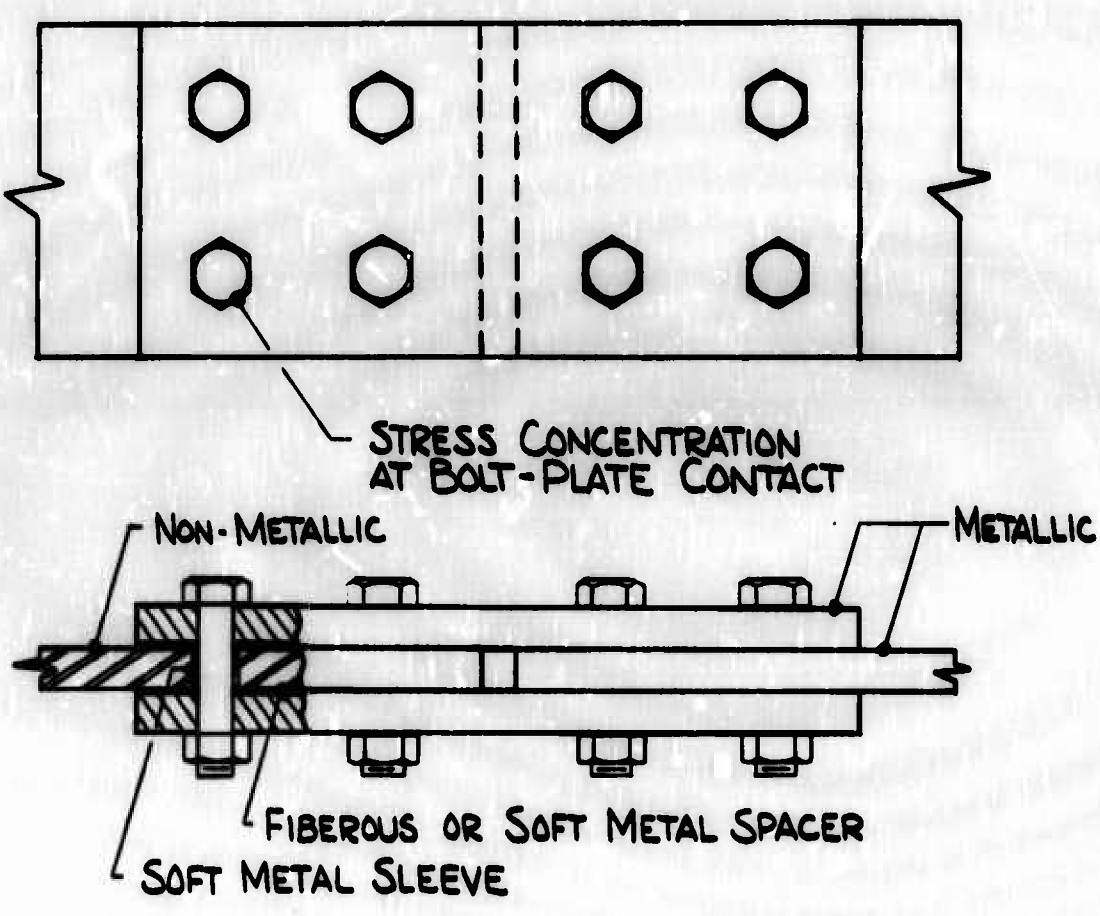


Fig.7.4 Mechanical splice for attachment of non-metallic to metallic components

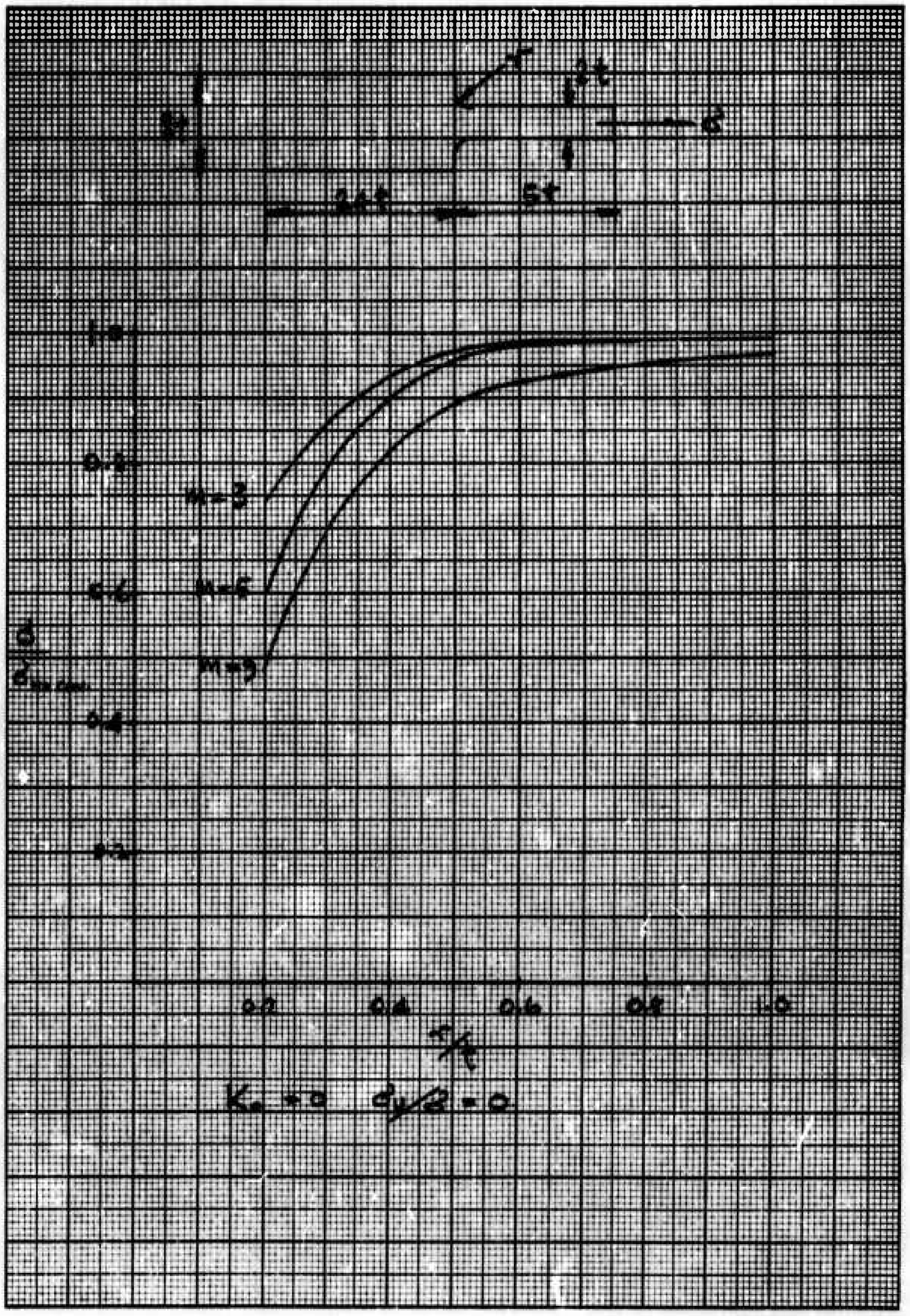


Fig.7.5 Effect of stress concentration on allowable stress

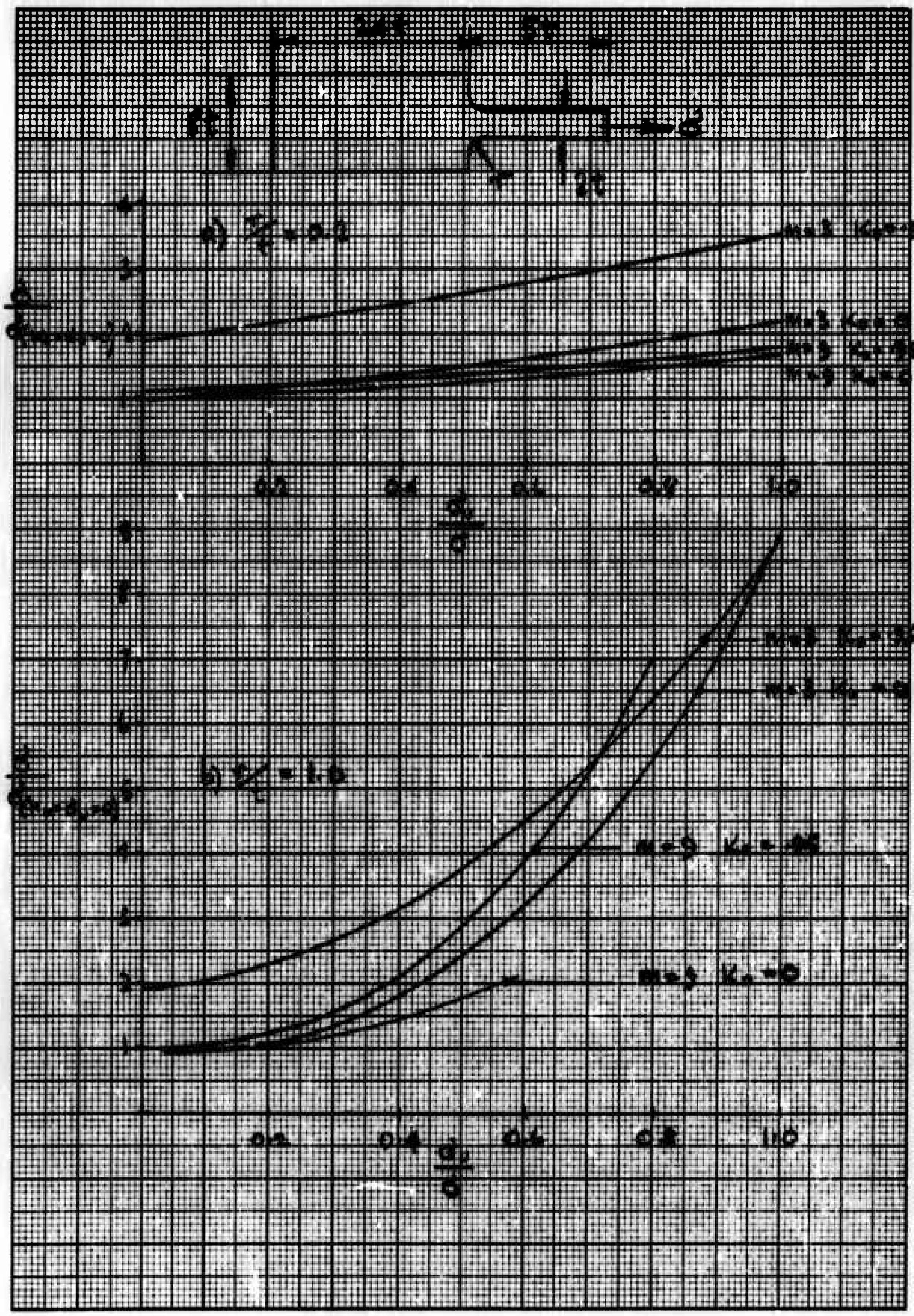


Fig.7.6 Effect of "zero probability of failure stress (σ_u) on allowable stress"

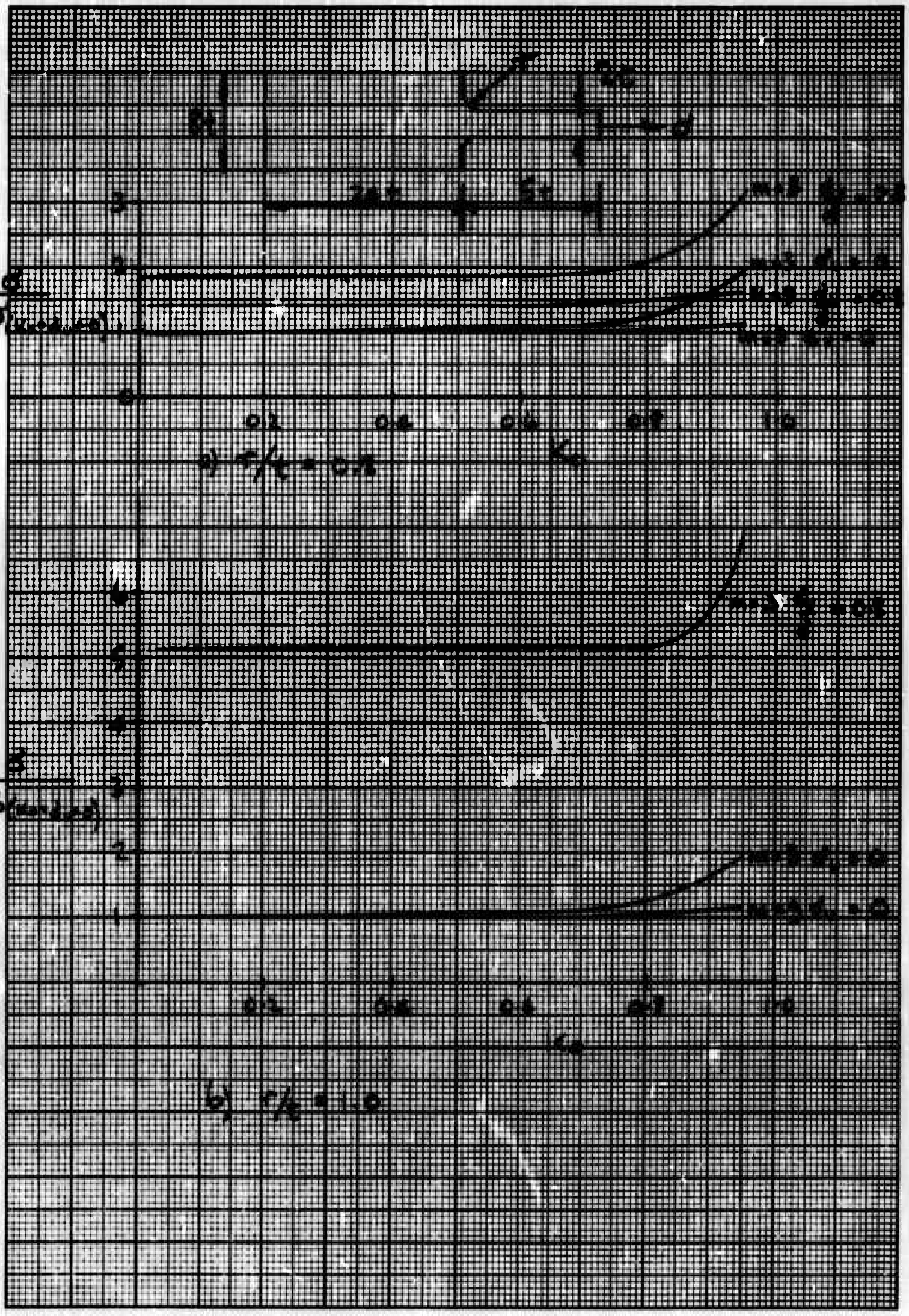
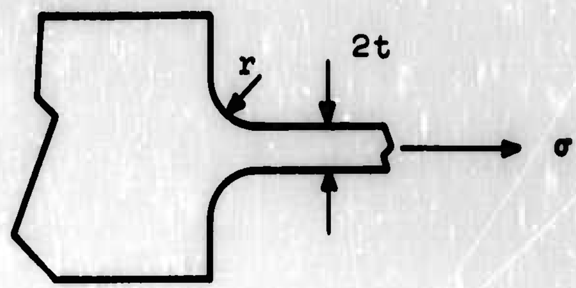
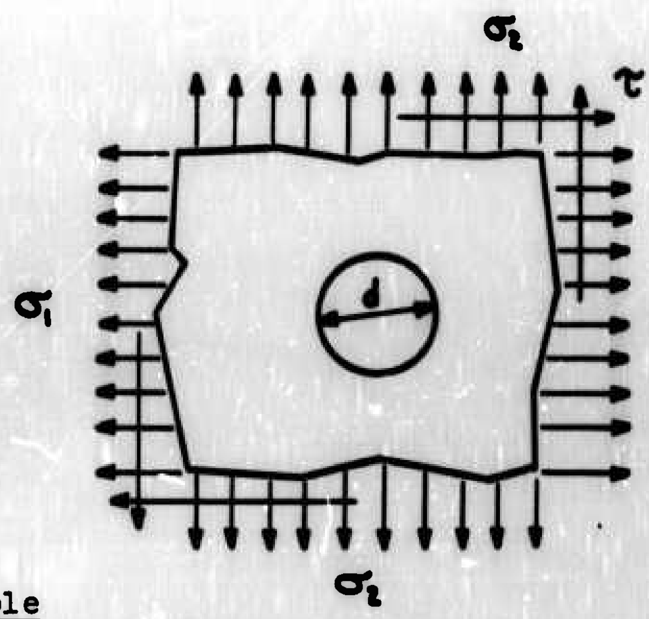


Fig.7.7 Effect of proof stress ratio (K_0) on allowable stress

1. Typical Fillet



2. Typical Hole



3. Pin in Hole

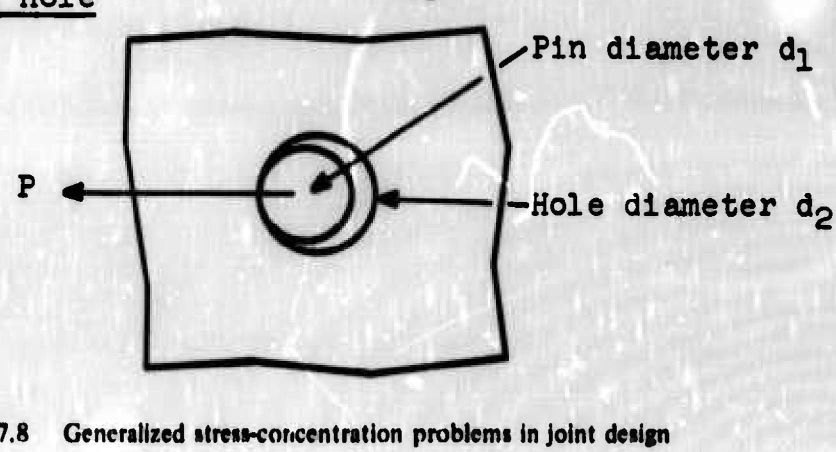


Fig.7.8 Generalized stress-concentration problems in joint design

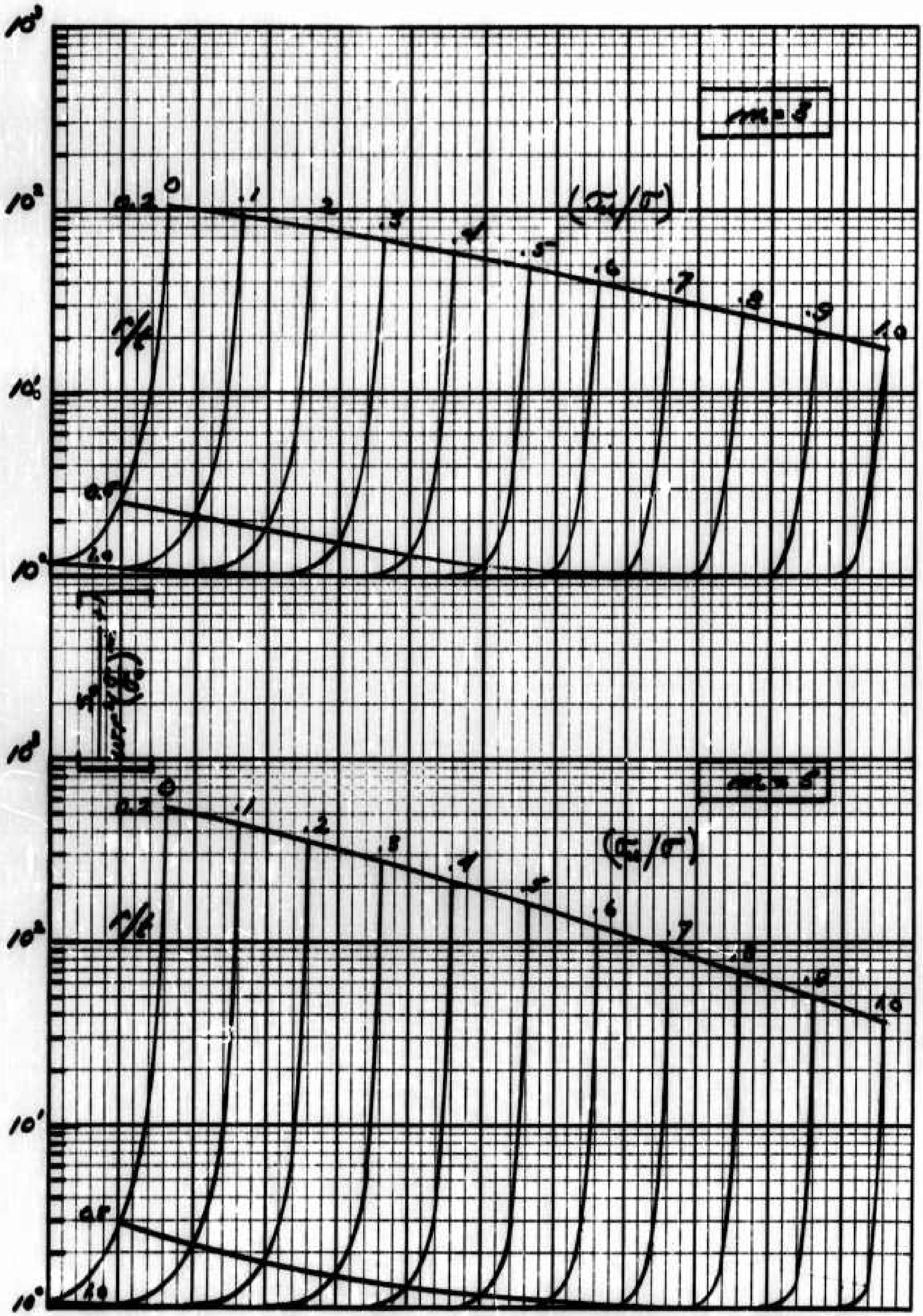


Fig.7.9(a) Increment of failure probability for fillets in brittle materials K_o = 0

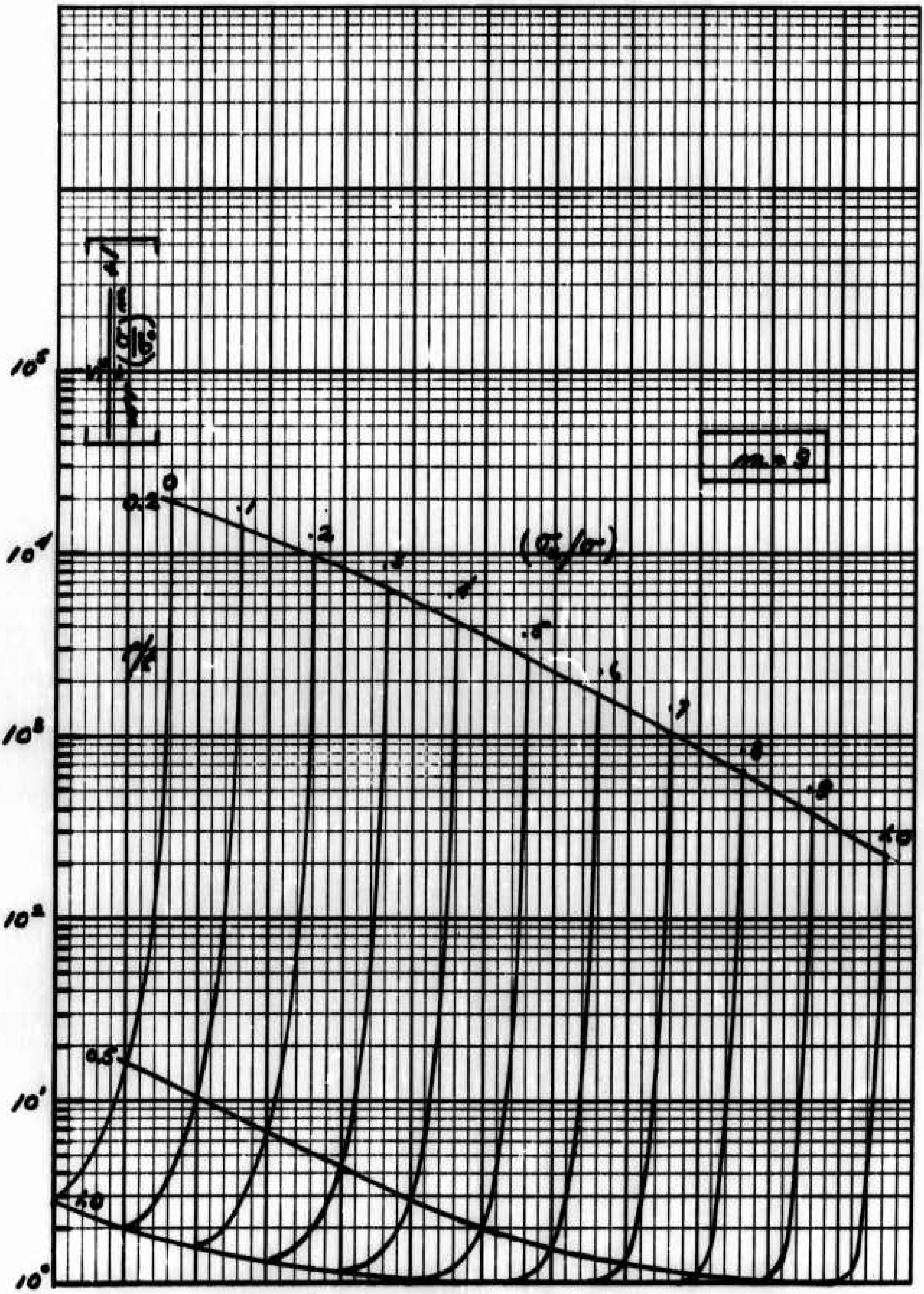


Fig.7.9(b) Increment of failure probability for fillets in brittle materials $K_0 = 0$

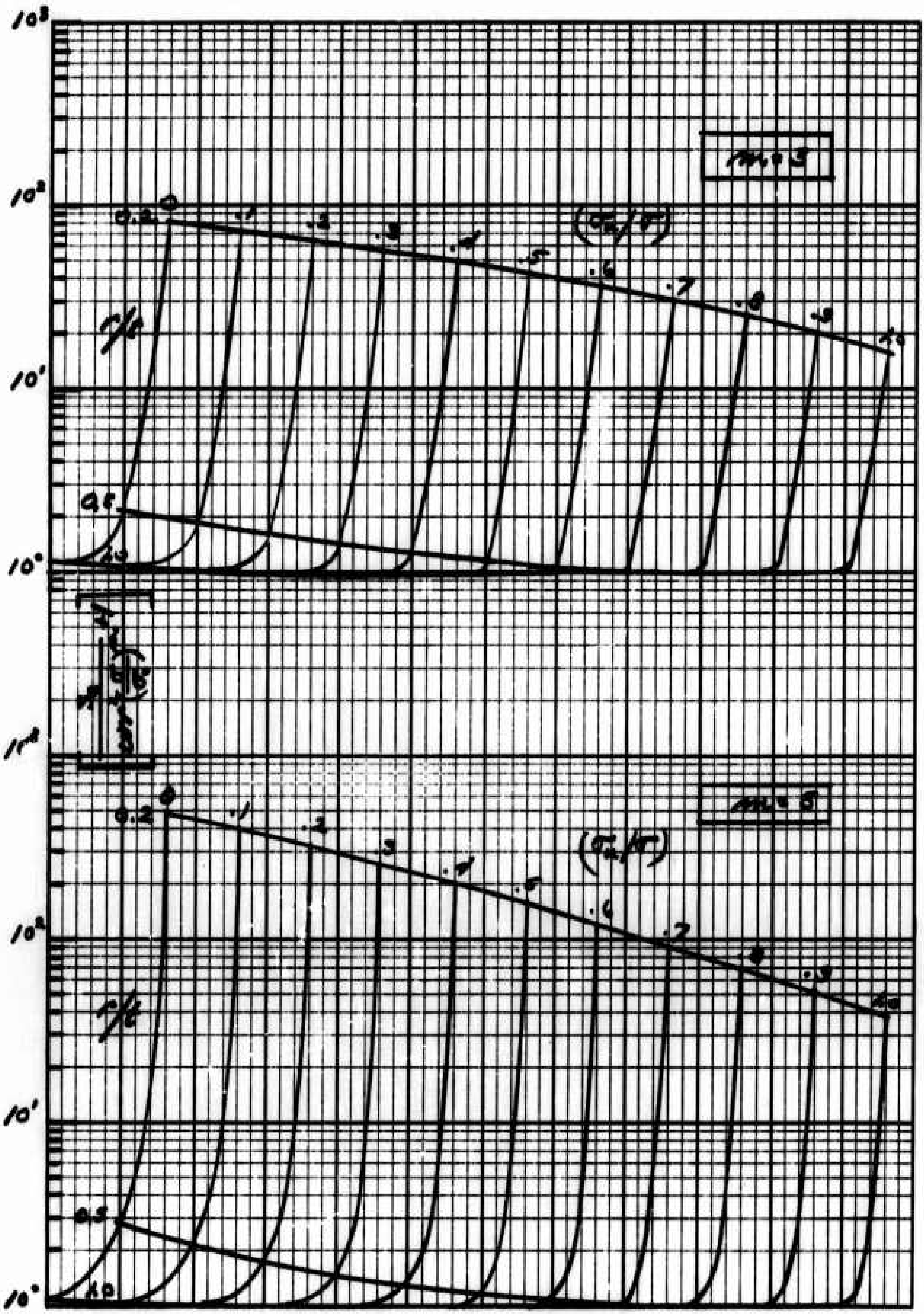


Fig.7.10(a) Increment of failure probability for fillets in brittle materials $K_0 = 0.6$

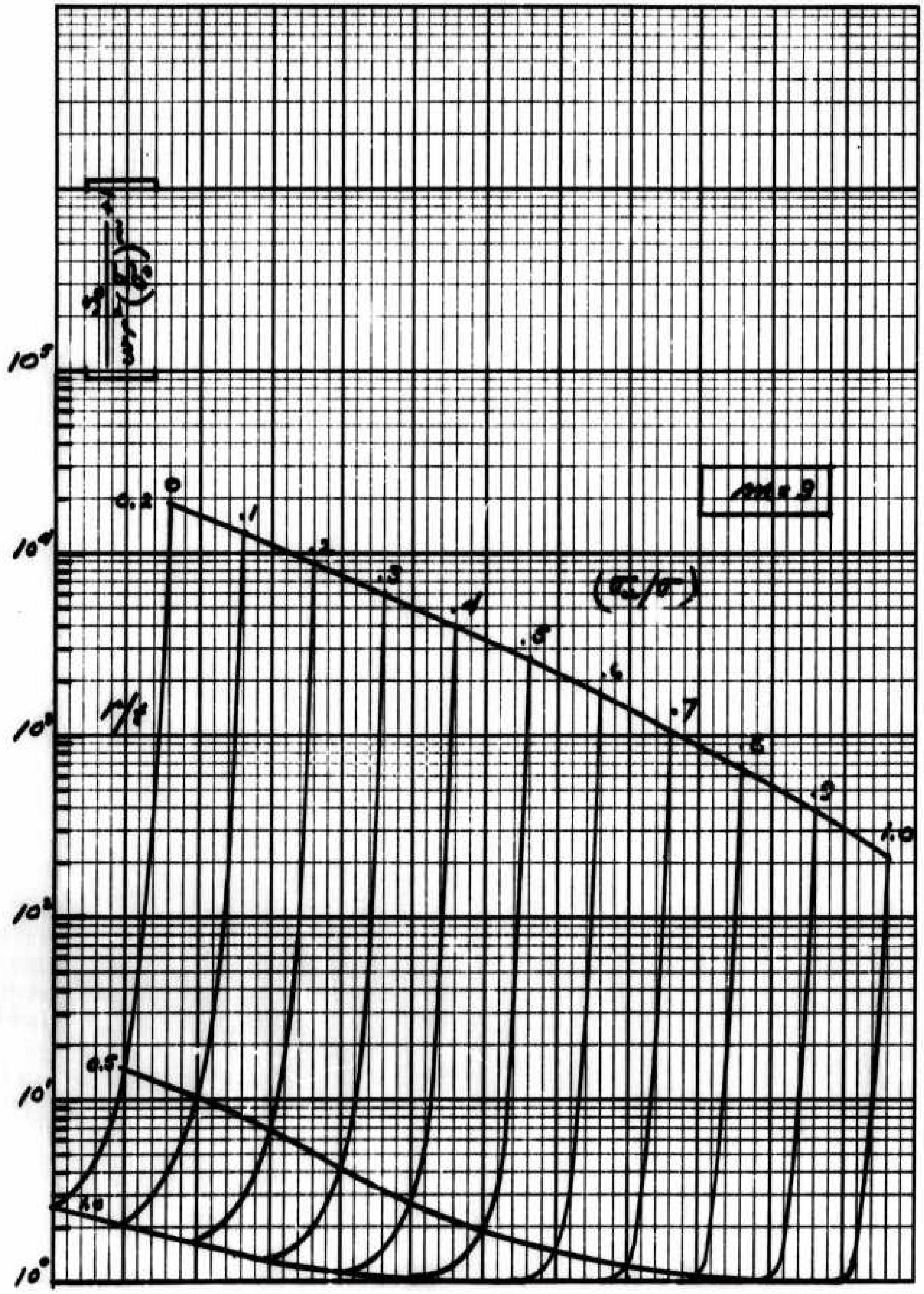


Fig.7.10(b) Increment of failure probability for fillets in brittle materials K_o = 0.6

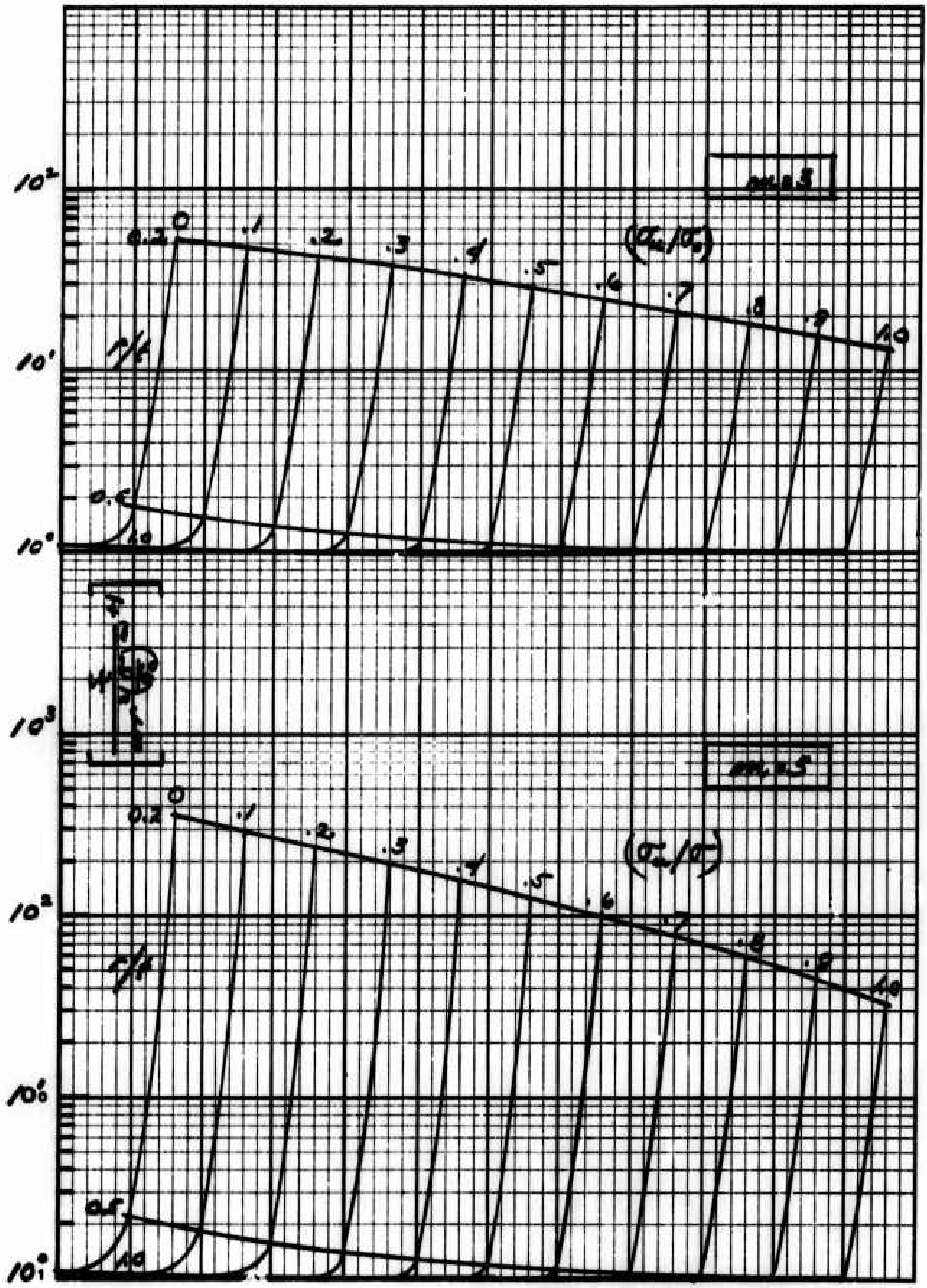


Fig.7.11(a) Increment of failure probability for fillets in brittle materials $K_0 = 0.8$

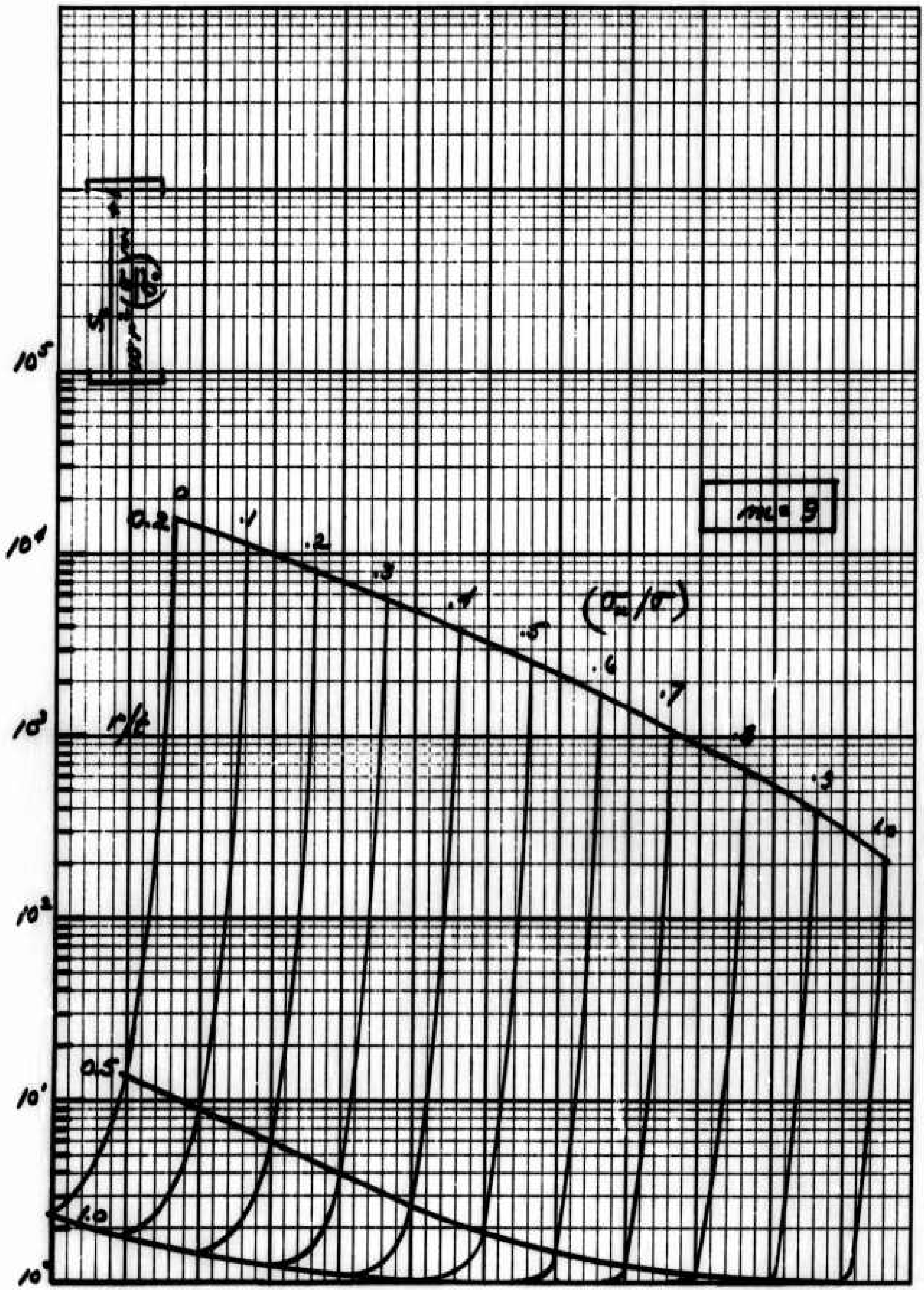


Fig.7.11(b) Increment of failure probability for fillets in brittle materials K_0 = 0.8

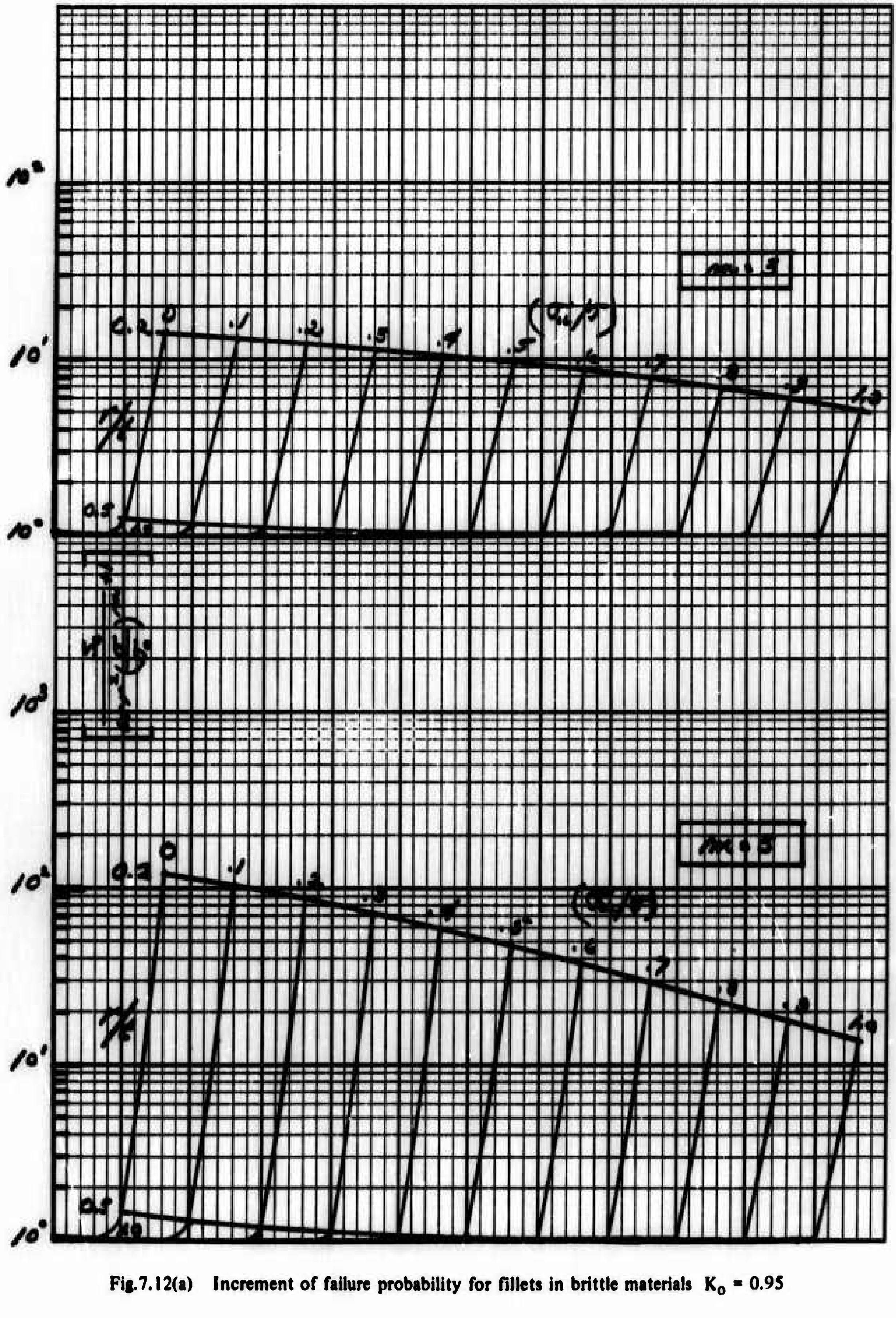


Fig.7.12(a) Increment of failure probability for fillets in brittle materials $K_0 = 0.95$

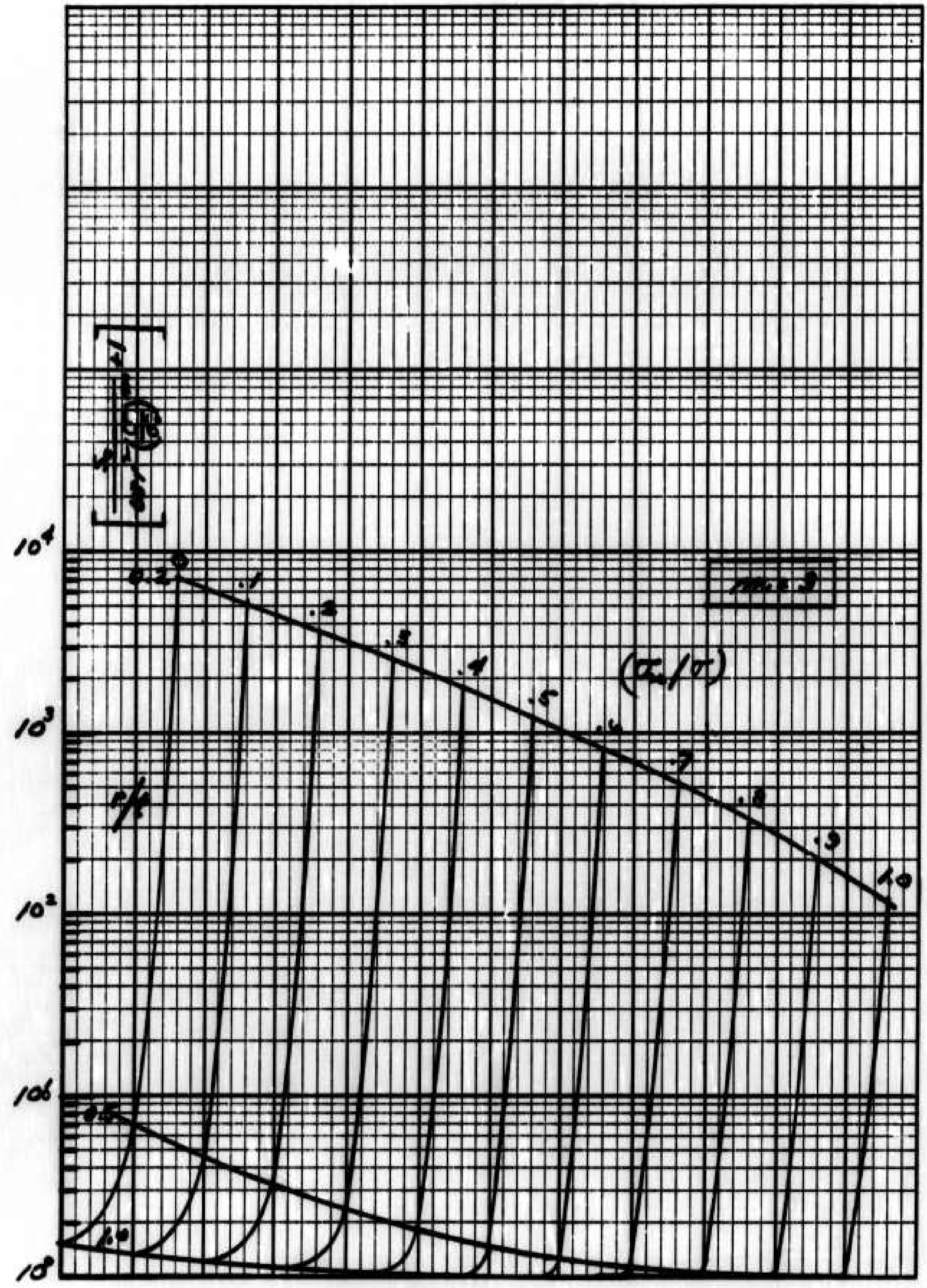


Fig.7.12(b) Increment of failure probability for fillets in brittle materials $K_0 = 0.95$

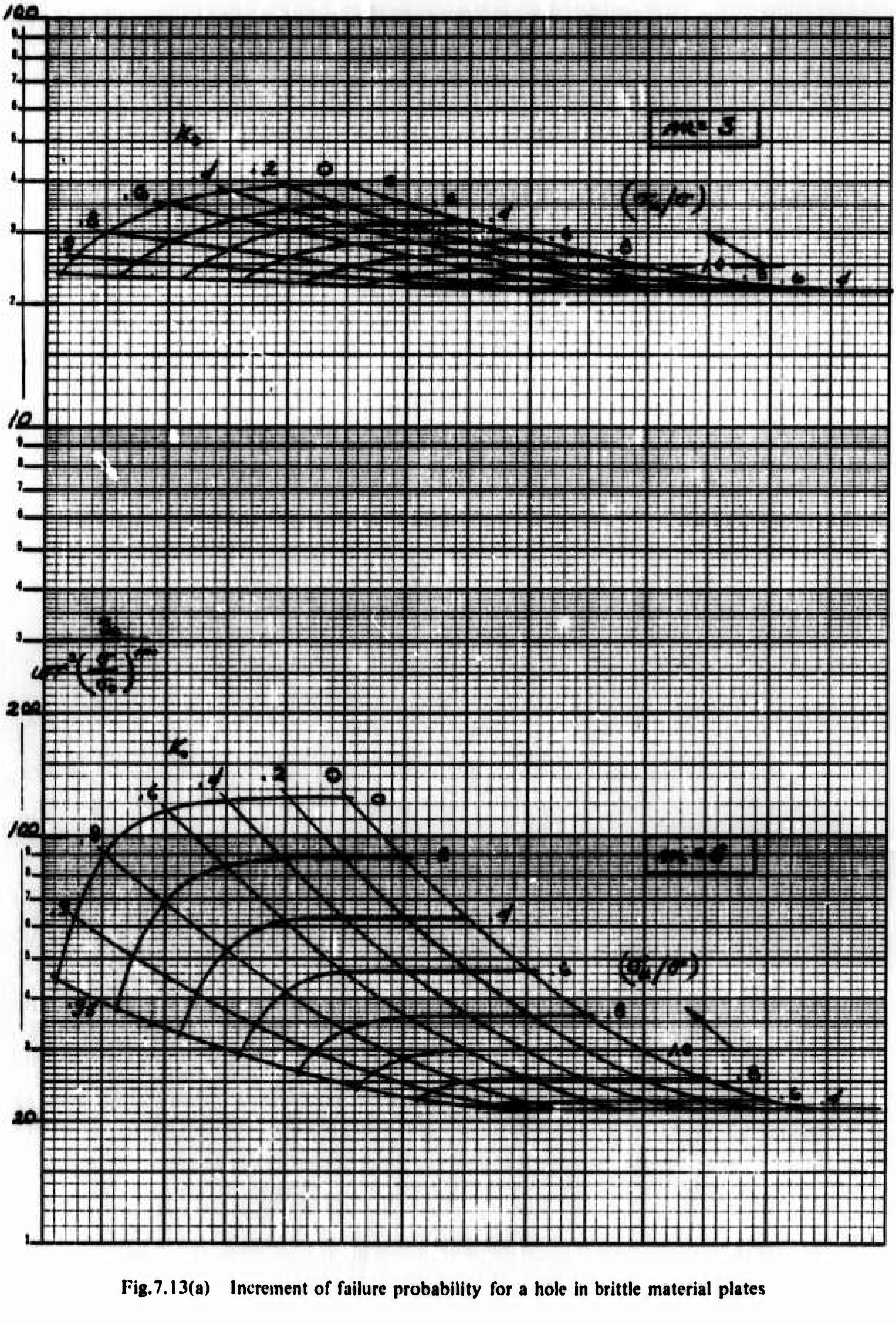


Fig.7.13(a) Increment of failure probability for a hole in brittle material plates

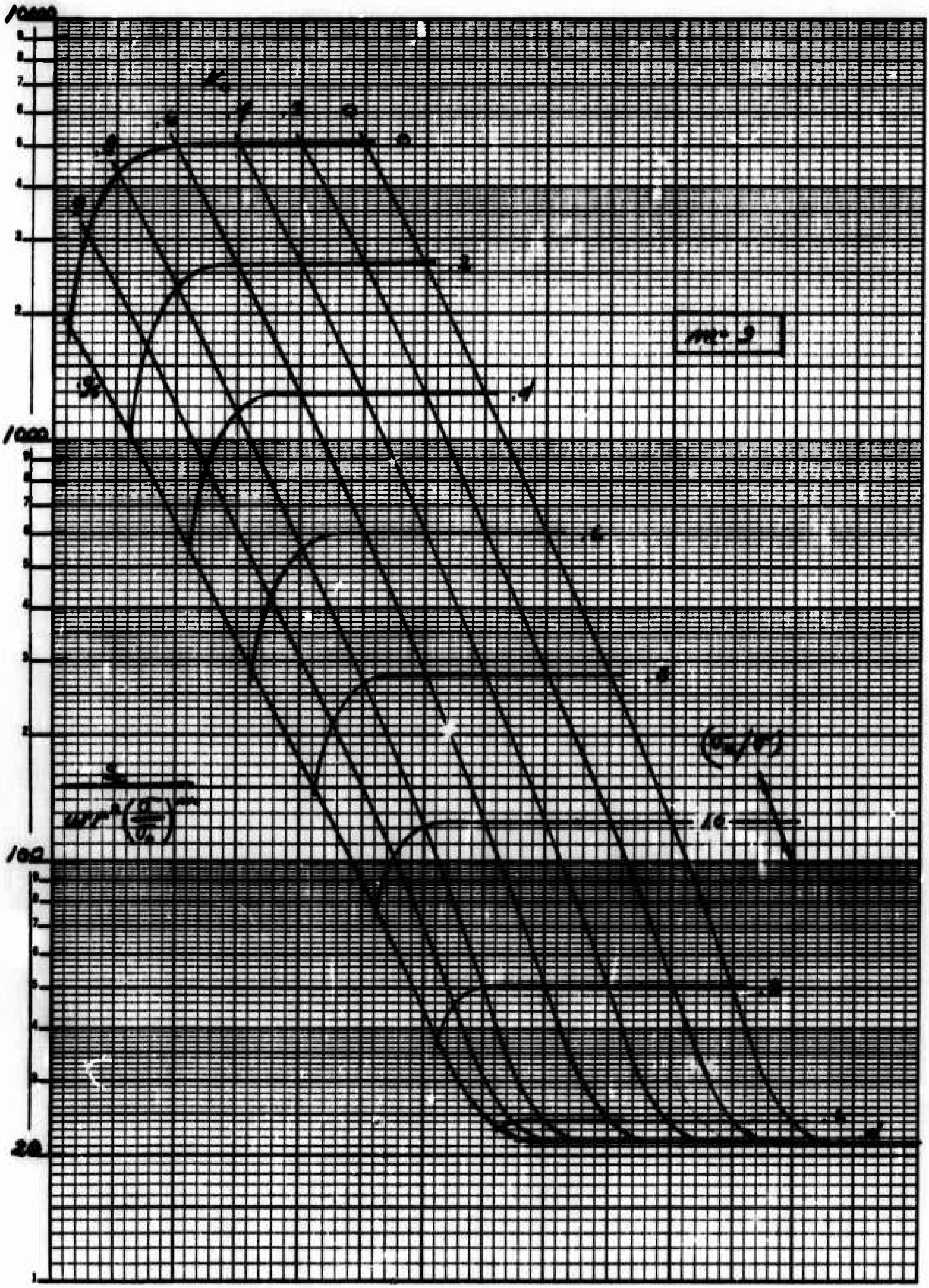
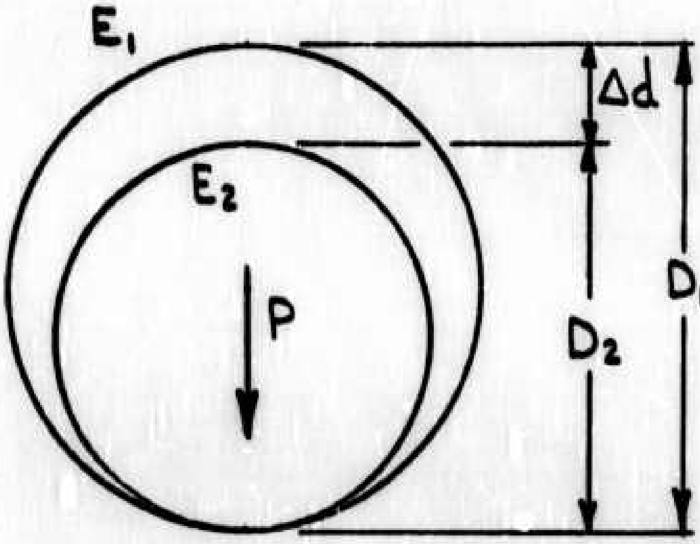


Fig.7.13(b) Increment of failure probability for a hole in brittle material plates



MAXIMUM COMPRESSION STRESS:

$$\sigma_c = \frac{K(p\Delta d)^{1/2}}{D_1}$$

WHERE

$$K = 0.798\left[E_1 \Big/ \left\{(1-\mu_1^2)\left[1+\left(\frac{1-\mu_2^2}{1-\mu_1^2}\right)\left(\frac{E_1}{E_2}\right)\right]\right\}\right]^{1/2}$$

IF PIN AND PLATE ARE OF THE SAME MATERIAL

$$K = 0.798\left(\frac{E_1}{1-\mu_1^2}\right)^{1/2}$$

IF $\mu = 0.3$

$$\sigma_c = 0.836\,\frac{(E_1 p\Delta d)^{1/2}}{D_1}$$

Fig.7.14 Stress concentration effects for a pin in a hole in brittle material plates

8. MECHANICAL PROPERTY TESTING OF BRITTLE MATERIALS

8.1 STATUS OF BRITTLE MATERIAL TESTING

When attempts are made to determine the basic mechanical properties of brittle materials, such as tensile strength, shear strength, etc., using the methods which have been extensively developed for ductile metallic materials, it is a matter of experience that the results are quite unsatisfactory. The principal source of dissatisfaction is a wide scatter in results when apparently identical tests are conducted on identical specimens of the same material. Some of this variability is believed due to an actual variability in the properties of the material, which is a result of the sensitivity of the material to local stress concentrations, caused in turn by flaws, voids, microcracks, inclusions and other defects in the material. This subject is covered extensively in Section 3 since it leads to a statistical description of material strength. Part of the variability, however, is due to test technique which assumes much more importance than it does with ductile materials for the same reason, that is, the sensitivity of brittle materials to local stress concentrations.

Typically, in the mechanical property testing of materials, there are very many effects which lead to disturbances either in what would otherwise be a uniform or at least a simple distribution of stress across the test section of the material test specimen, or in the magnitude of the loads actually applied to the test section. These sources of stress variations are summarized in Figure 8.1 under a number of classifications. They all result from the inability to carry out every step of the test perfectly and from the practical requirement that tolerances must be applied to every test specimen, every piece of test equipment and every activity connected with the test. Some of these variations are very evident, such as the difficulty of aligning the applied load perfectly through the centroid of the cross section of the test specimen, or the difficulty of completely eliminating friction at hinged joints in the load train, or the difficulty of configuring a specimen such that external loads can be applied without local stress concentrations. Some of these effects are less obvious such as density variations in the test specimen, which produce an effective change in the position of the centroid of the critical section. All of these effects, however, lead to localized stresses which are different from the nominal stresses determined from the load at failure, the geometry of the failed cross section, the specimen geometry, and assumptions of a uniform homogeneous material having simple elastic stress-strain properties. Since the material is brittle it will fail when the peak stress reaches the limiting strength of the material at some particular location, regardless of how localized this region of high stress might be. A ductile metallic material, on the other hand will yield locally and will fail only when the average stress over a significant portion of the cross section reaches the limiting strength of the material.

There have been two broad approaches taken in the attempts to resolve this difficulty. The first method is to use the type of test specimen that will produce simple stress distributions which can be readily and accurately determined from the measured loads and geometry, and to concentrate on refinement of test technique, test apparatus and test specimen configuration in order to eliminate or minimize the effects summarized in Figure 8.1. This method leads to relatively expensive testing, with specimens that are expensive to fabricate because of the close tolerances required. Such methods are of interest when accurate material property data is required or when the number of specimens is small or the cost of conducting the test is not of major concern. Such conditions might arise, for instance, if the information is to be used for basic material fracture and failure research, material composition and alloy development, studies of the effect of stress state on fracture, and studies of test methods where an accurate reference point will usually be required.

The second method is to select the test technique for its relative simplicity and to accept a complex stress distribution within the specimen. Reliance is then placed on refined stress analysis to determine maximum stresses and hence material strength from the measured loads at failure. This approach may or may not be low in cost depending on the specimen configuration which is used. Frequently a complex specimen, such as the Theta specimen, which will generate areas of uniform stress, is used to minimize the stress analysis problem by substituting specimens which are expensive to fabricate, although the test technique is simple. Alternatively, simple low cost specimens may be used with complete dependence on stress analysis to determine maximum stresses. The latter is frequently useful for comparative data such as receiving inspection, where large numbers of samples may be tested over a period of time by relatively unskilled technicians. In addition to its dependence on stress analysis this second approach usually involves a stress gradient or a complex stress state at the location of a fracture, so that a simple uniaxial strength is rarely determined.

Numerous methods have been evolved, on the basis of these two approaches, for conducting tensile tests on brittle materials, and most of these are briefly summarized in Figure 8.2. However, at the present time the development of test methods for brittle materials has not reached the stage where completely satisfactory methods can be defined for the development engineer, the structural designer or laboratory technician to follow. Much difference of opinion exists on the respective value of the two approaches mentioned, on the suitability of various test specimen configurations, on the dependability of stress analysis for predicting maximum stresses, etc. A meeting was held in London in September 1967 under the sponsorship of AGARD to discuss a number of questions relating to brittle material design, and much of the available experience within the NATO countries

was represented by a number of carefully selected individuals. The question of brittle material testing was among the topics discussed and in particular the most basic piece of material data, the tensile strength at room temperature, was selected as the object of the discussion. As a result of this meeting the current position with respect to testing can be summarized as follows:

a) There is general agreement that a limited number of simple stress-state test methods are available to provide accurate tensile data. The two methods which currently have the widest acceptance are the gas bearing test and the pressurized ring test.

b) Stress analysis methods for determining maximum stresses are generally acceptable.

c) Recognizing the economic necessity for a simple low cost test for some applications, the three and four point bend tests are generally acceptable as test methods involving simple technique. The acceptance is based primarily on the simplicity of the test method rather than reliability of results, since stress gradients and complex stress states of unknown significance are present. There is disagreement about the usefulness of any other indirect test method.

Based on this opinion the remainder of this section will be devoted to the presentation of information which will assist in the conduct of material property testing by the four methods mentioned. Information will be presented, where appropriate, on test specimen details, test apparatus and test technique. The objective is to present as much quantitative information as possible which will help in the design, preparation and conduct of material property testing. Information in the form of rigid specifications has been avoided because the present state of testing development does not justify such restriction. However, the usefulness of material property data is greatly increased if it can be compared with similar data from other sources and making such a comparison possible requires standardization of testing. For this reason the information presented has been reviewed by a number of individuals to ensure that the recommendations made are generally acceptable.

The information to be presented is limited to tests to determine tensile properties partly because this is the most important and most basic data, and partly because agreement on acceptable test methods is even more difficult to obtain for other properties, than it is for tensile strength. Methods for generating and measuring elevated test temperatures and methods of strain measurement are not discussed since these are not changed as a result of the brittleness of the test material.

8.2 TEST METHODS USING SIMPLE STRESS STATES

a) Gas Bearing Test

This section presents information on specimen and test fixture details to permit the determination of tensile strength properties for brittle materials using the gas bearing test apparatus. The specimen and fixture designs have been taken from Reference 8.1 and small modifications have been made to facilitate accurate alignment of the load train. The consistency and accuracy of test results produced with this apparatus has been confirmed by actual practice, and the dimensions and tolerances shown will permit stresses determined by dividing the failure load by the nominal cross section area of the test specimen to be within 5% and 11% above true values. Specifically, this error considers the specified tolerances on load train alignment and concentricity, alignment of the load train with the geometric center of the gas bearing, alignment of the specimen with the load train, concentricity between the specimen test section and grip section, and finally the small stress concentration due to changes in specimen cross section. The error does not consider the material effects given in Figure 8.1 or errors in the load measuring device.

The information to be presented on fixture and specimen details is not complete, but critical dimensions and tolerances are given from which any competent test agency can design and construct the apparatus.

Figure 8.3 shows the general layout of the spherical gas bearing and the tensile load train. The arrangement shown is duplicated on the other side of the test specimen except that only one load cell is required. Each bearing has a diameter of 9 inches and is supplied with nitrogen gas at approximately 900 psi. Pressure gradients develop in the contact area so that the average effective pressure between the bearing surfaces is 300 psi, which is sufficient to provide a load capacity of 15,000 lbs. This capacity is substantially greater than is necessary for the tensile specimens which will be shown, but the size is dictated by the necessity for using an available and well qualified design.

A simple manual control of the pressurizing gas is used. Some arrangements of the gas bearing apparatus have incorporated electronic devices to control the gas flow as a function of load by sensing film thickness. Satisfactory experience has apparently been obtained without this refinement however so that its use is not recommended unless the quantity of gas used is a problem.

The critical aspects of the load train are the concentricity of the pull rods, the alignment of the center of the pull rod with the center of the spherical radius, and concentricity between the bore of the collet and the centerline of the pull rod. All of these tolerances should be held to .0005 in. total indicator reading which is within the

capabilities of a good machine shop. It will probably be necessary to assemble one side of the load train with the load cell before the final machining cuts are made so that misalignments within the load cell can be properly accommodated.

During assembly of the apparatus it is necessary to achieve the close tolerance shown between the centerline of the pull rod and the center of rotation of the bearing. Since it is not practical to locate the bearing center accurately after the part is removed from the lathe on which it is made, accuracy in assembly is provided by accurately machining the pull rod bore, at the same time that the spherical surface is machined. Maintenance of the close tolerances shown on the pull rod bore and the pull rod diameter will ensure accurate assembly.

The precision collet used in each end of the load train is shown in Figure 8.4. Accurate alignment of the specimen with the pull rod and hence with the spherical bearing, is maintained by holding close tolerances on the concentricity between the pull rod bore and the pull rod centerline, on the pull rod bore and compression nut diameters, and on concentricity between the pull rod bore and the collet bore. Dimensions of the three-piece split collet are given before splitting, since accurate measurements of the collet bore afterwards are not practical.

Clearance is provided in the bore of the compression nut so that load transfer is affected through the three-piece split ring. Accuracy in the surface at the end of the pull rod bore is required to assist in installation of the specimens.

The tensile specimen is shown in Figure 8.5. For most nonmetallic refractory materials diamond cutting tools will be necessary to obtain the required surface finish and close tolerance dimensions. Notice that the gage section contains a slightly reduced portion to ensure that the failures occur away from the transition to the grip diameter. The reduced section of the gage length contains no constant diameter portion since failures would still occur at the end radii. However, the small stress concentration caused by the one-inch radii shown has been considered in describing the accuracy of the test results. Slight corrections will be necessary, because of this local diameter reduction, if this specimen is used with an extensometer across the gage length to measure strain.

b) Pressurized Ring Test

Figure 8.6 shows the arrangement of the pressurized ring test unit together with critical dimensions and tolerances. From this information a suitable apparatus can be designed and built by a competent laboratory. Information is taken primarily from Reference 8.2. The specimen holder consists of two round steel plates containing cavities which are machined and aligned to the close tolerances shown. Accurately sized alignment pins and holes are necessary to maintain the cavity alignment. Hydraulic pressure is applied radially to the ring specimen from the inside through a flexible rubber bulb. The conical plug in the lower steel plate seals the bulb and provides entrance for the owrking fluid. Spacer blocks are provided to separate the steel plates, and tolerances on the spacer block height and the specimen length are arranged so that there is a minimum total clearance between the ring and the specimen plates of .001 in., to avoid constraining ring displacement, and a maximum total clearance of .004 in., to prevent extrusion of the rubber bulb between the ring and the fixture.

Details of the test specimen are shown in Figure 8.7 which gives also the dimensional tolerances which are necessary so that the failure stress can be calculated within ±1½% accuracy from the nominal specimen dimensions and the hydrostatic pressure at failure. The expression for making this failure stress calculation is given below. Accurate pressure gages are needed since errors in pressure readings represent one of the major sources of error in the measurement of tensile strength. Pressure gage error is not included in the values mentioned since the specific gage which will be used in a particular installation is not known. Also, the material effects mentioned in Figure 8.1 are not included in the specified error.

No particular precision in technique is necessary with this apparatus apart from determining the failure pressure accurately. The method however is limited since it cannot be used at elevated temperatures, and rate of loading is difficult to control.

Strain data can be obtained only with strain gages, which is an accurate method, but relatively expensive for large numbers of tests. Furthermore, the strain gages must be placed on the outside diameter, which is not the region of maximum stress.

The tensile stress in a cylinder wall corresponding to an internal hydrostatic pressure is computed from the formula

$$\sigma_r = \frac{P{r_1}^2}{r_o^2 - r_1^2}\left(1 + \frac{{r_o}^2}{r}\right)$$

Where P = hydrostatic pressure in psi

r_1 = internal radius in in.

r_o = external radius in in.

r = radius at which the stress is determined

From this formula the maximum stress will occur at the inside radius. Inserting the nominal specimen dimensions into this expression gives the failure stress as 10.524 x the failure pressure.

8.3 LOW COST TEST METHODS

a) Three and Four-Point Bend Tests

This section presents specimen and test fixture details for conducting three and four-point bend tests on brittle materials. The specimen and fixture designs have been evolved from a very careful consideration of the many sources of parasitic stresses which are important in testing brittle materials, but at the same time consideration has been given to the economics of specimen fabrication, test fixture fabrication and the cost of conducting the tests in terms of complexity of test technique. The objective of bend testing is to obtain relatively low cost data with the understanding that neglect of such effects as stress gradients is the price to be paid.

In evolving the proposed designs the dimensional tolerances for both the test specimen and the test fixture, and other sources of parasitic stress such as frictional effects, have been controlled so that using nominal specimen and fixture dimensions, the stress calculated from the measured failure load is within ±5% of the true value. This does not include tolerances on load measurement since the means for making this measurement are not specified. It also assumes that the material of the test specimens is homogeneous with respect to elastic properties. In particular, it is assumed that the elastic modulus of the test bars in the longitudinal direction is constant across the specimen cross-section and along its length.

The test apparatus for conducting bend tests is based on existing apparatus which has seen considerable use. Some changes have been made, however, in the overall dimensions and in the required tolerances, but these are not considered significant enough to invalidate the previous satisfactory experience with this type of apparatus.

Suggested specimen details for three and four-point bend tests are shown in Figure 8.8. Two sizes of specimen are shown. The smaller size has been selected to minimize the quantity of material and hence minimize cost. The minimum dimensions have been established from the minimum size test fixture which could provide all of the required degrees of freedom and accomplish this using commercially available parts. The larger size specimen is included in recognition of the fact that it is frequently necessary to examine the effect of specimen size in testing brittle materials. The ratio of specimen volumes between the large and small sizes shown is approximately 4, but the true value of this ratio depends on the significance which is attached to the nonuniformity of stresses throughout the test bars.

The dimensional tolerances shown in Figure 8.8 have been established recognizing their contribution to the overall error in stress as determined from nominal dimensions. Grinding of the specimen surface has been specified to reduce frictional effects at the loading points. For many of the harder brittle materials this will require diamond grinding. The cost of such finishing methods has been examined and is considered acceptable by individuals who have carried out extensive testing of brittle materials.

Figure 8.9 shows a test fixture for four-point tests, and dimensions are given so that fixtures can be constructed for both the small and large size specimens. Rollers are provided at all load points except one, in order to minimize friction due to longitudinal motion of the specimen as it bends. These rollers are supported on needle roller bearings. Freedom of rotation about an axis parallel to the longitudinal axis of the specimen is also provided so that twist in the specimen and the fixture, and rotational misalignment between the upper and lower parts of the fixture can be accommodated. To minimize cost, all of the bearings used are commercially available.

Arrangements are made to accurately locate the specimen relative to the rotational axes of the fixture. For the small size specimen this is achieved by locating the specimen in grooves machined in the reaction rollers. For the large specimen the specimen face is aligned with one face of the reaction rollers. The upper and lower parts of the fixture are not located relatively in the longitudinal direction but alignment pins are provided so that when the fixture and specimen are installed in the test machine, the distance between one loading point and one reaction point can be accurately measured. This measurement is made along a slope, and must be corrected for specimen thickness to obtain the projected distance between load and reaction points.

The failure stresses should be calculated from the measured failure load, the measured specimen and fixture geometry, and simple bending theory. Corrections must be applied to allow for the so-called 'wedging' effect which is the effect on the simple bending stresses of the load concentration at the applied load points. From simple bending theory the maximum tension stresses occur under the applied loads and at the face of the specimen opposite from the face on which the load is applied. The local stresses due to load concentration produce a compression stress immediately beneath the load, which is therefore subtractive, but this becomes a tension stress, which is therefore additive, a short distance away from the load. For the four-point test the maximum stress occurs a short distance away from the load and is greater than that given by simple bending theory. The maximum stress occurs at .25 x test specimen thickness from the load point and the increment of stress to be added to that calculated from simple bend theory

is $\frac{0.868\ Pc}{2}$ where P is total load applied to the test specimen and c is one-half specimen thickness.

Figure 8.10 shows a test fixture for three-point tests, again designed to accommodate two sizes of specimen. The three-point fixture design follows closely the four point, except that the single load application roller is fixed rotationally, while both reaction rollers are free to rotate to accommodate specimen twist and fixture rotational misalignment.

For the three-point test the correction to the maximum stress calculated by simple bending theory, to account for the wedging effect, is given by -.1332 Pc where P is total load applied to the test specimen and c is one-half the specimen thickness. In this case the maximum total stress occurs under the applied load and is less than the value given by simple bending theory.

REFERENCES

8.1 _____________: The True Stress-Strain Properties of Brittle Materials to Very High Temperatures. Final Report to United States Atomic Energy Commission (1962-1963). Southern Research Institute. September 1963. (6387-1190-XXXV)

8.2 Sedlacek, R. and Halden, F. A.: Method for Tensile Testing of Brittle Materials. The Review of Scientific Instruments, Vol. 33, No. 3, pp 298-300, March 1962.

FIGURE 8.1 SOURCES OF ERROR IN DETERMINATION OF FAILURE STRESSES IN MATERIAL PROPERTY TESTS

MATERIAL EFFECTS

1. Elastic Properties Not Uniform Throughout Specimen
2. Elastic Properties Different in Tension and Compression
3. Skin or Oxide Layer of Different Modulus to Body of Specimen
4. Material Stress-Strain Curve not Linear
5. Variations in material density throughout the specimen

SPECIMEN GEOMETRY EFFECTS

1. Tolerances in Specimen Cross-Sectional Dimensions
2. Specimen Twist
3. Eccentricity of Grip and Test Sections of Specimen
4. Lack of Parallelism Between Grip and Test Sections
5. Eccentricity of Inner and Outer Diameters - Ring Specimen

FIXTURE GEOMETRY EFFECTS

1. Eccentricities, Symmetrical and Unsymmetrical, in Load Train
2. Tolerances in Position of Load Application Points
3. Angular Misalignment of Load Application Direction
4. Edge Moments Due to Gap Between Specimen and Fixture - Ring Test

CONSTRAINT EFFECTS

1. Moments Due to Friction at Hinges in Load Train
2. Frictional Effects at Load Application Points
3. Deformation Constraints Due to Friction

STRESS DISTRIBUTION EFFECTS

1. "Wedging" Effect Under Externally Applied Loads
2. Stress Concentration at Changes in Specimen Cross-Section
3. Edge Moments - Ring Test

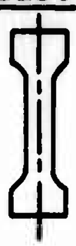
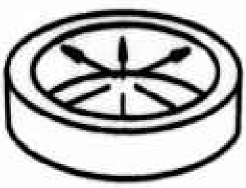
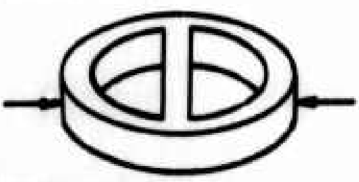
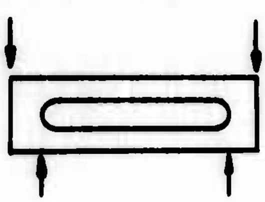
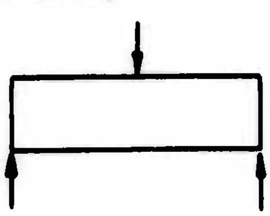
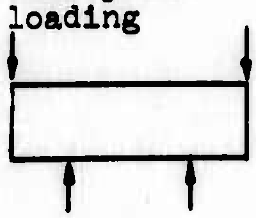
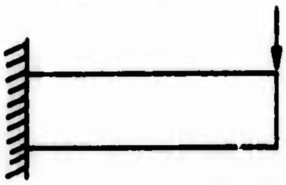
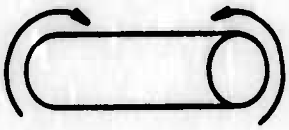
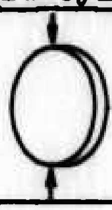
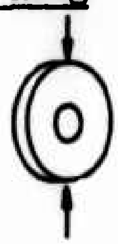
STRESS TYPE AND TEST	MAJOR SOURCES OF ERROR	LIMITATIONS	
Direct Tension  a) Gas Bearing b) Adjustable grips c) Cemented Heads	Eccentric loading	Axiality of loading can only be adequately verified at moderate temps; complex & costly to conduct properly	F 1
Indirect Tension Pressurized ring 	Restraint at ends; biaxial effects & gradients if thick walled	Applicable only at moderate temperatures	C. 1
Theta Test 	Errors in geometry	Difficulty and cost involved in obtain-	T
Trussed Beam 	Errors in geometry, plus those associated with bend testing	Specimen fabrication	D C S
Bend Test: Three-point loading 	Wedging; friction at supports; twisting	Relatively small portion of specimen reaches high stress at fracture	Di Cc oi

Fig.8.2 Summary of tensile test methods. (Taken from material p
on Brittle Materials, London, Sep

ITATIONS	STRESS TYPE AND TEST	MAJOR SOURCES OF ERROR	LIMITATIONS
of loading be ade- ;erified ite temps; ; costly t	Four-point loading 	Friction at supports; incorrect spacing of supports; unbalanced loads; twisting	Remarks similar to those for three-point loading pertain (though to lesser extent)
e only at tempera-	Cantilever loading 	Wedging at support	Difficulties in supporting fixed end; max. stress (at support) may be poorly defined
' and cost .n obtain-	Torsion 	Bending; axial loading	
'abrication	Diametral-Compression of Solid Cylinder 	Nonuniform loading along length	Varying biaxial stress component; poorly defined stress state at ends of loaded dia.
small specimen ;h stress ;	Diametral Compression of Ring 	Nonuniform loading along length; plastic flow at peak-stress position	High Stress gradient in region of max. tensile stress may affect significance of results if specimens of various grain sizes are to be studied; only a small proportion of the total specimen is tested.

ıds. (Taken from material presented at the AGARD Specialist Meeting
'ittle Materials, London, September 4, 1967)

B

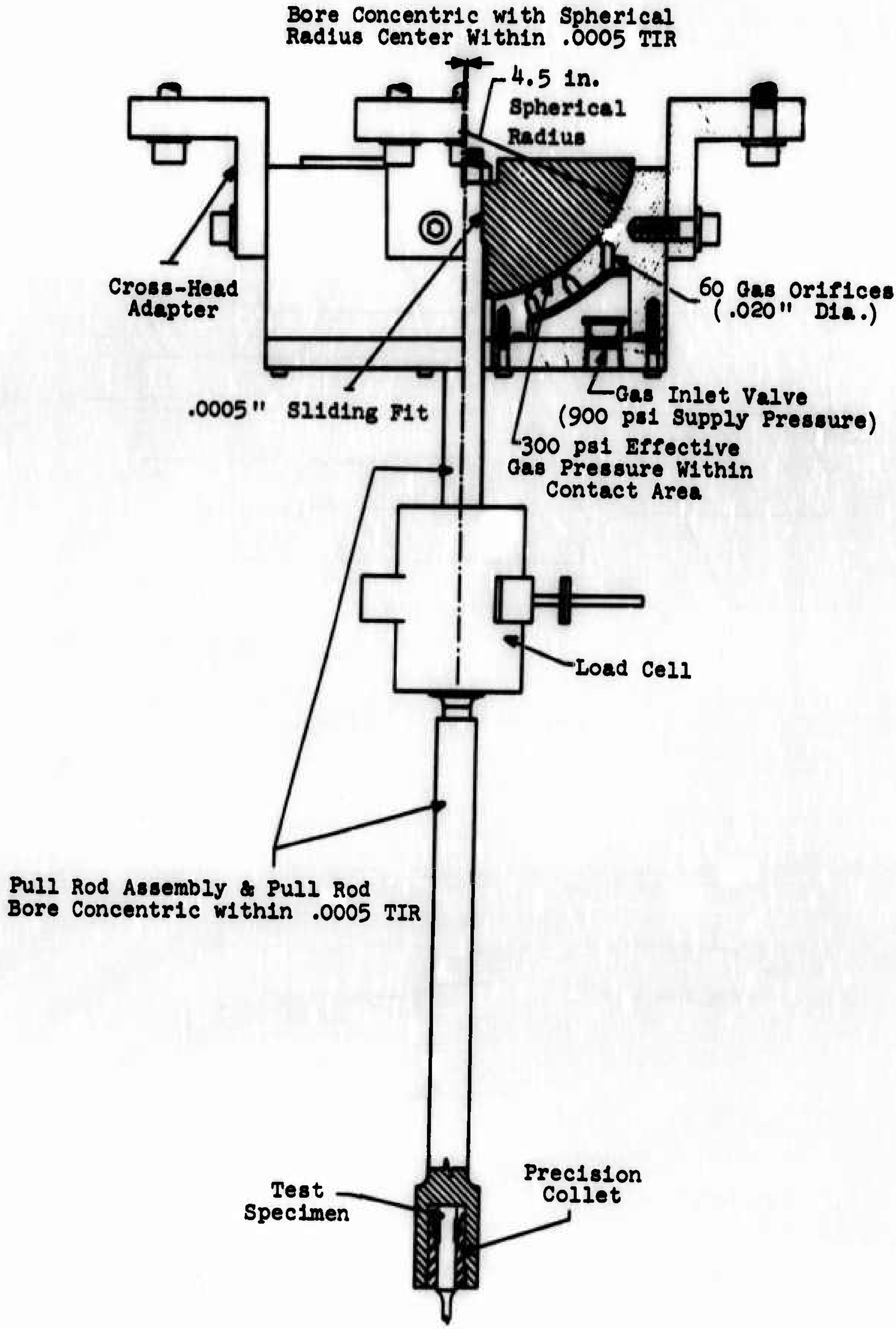


Fig.8.3 General layout of spherical gas bearing tensile test rig

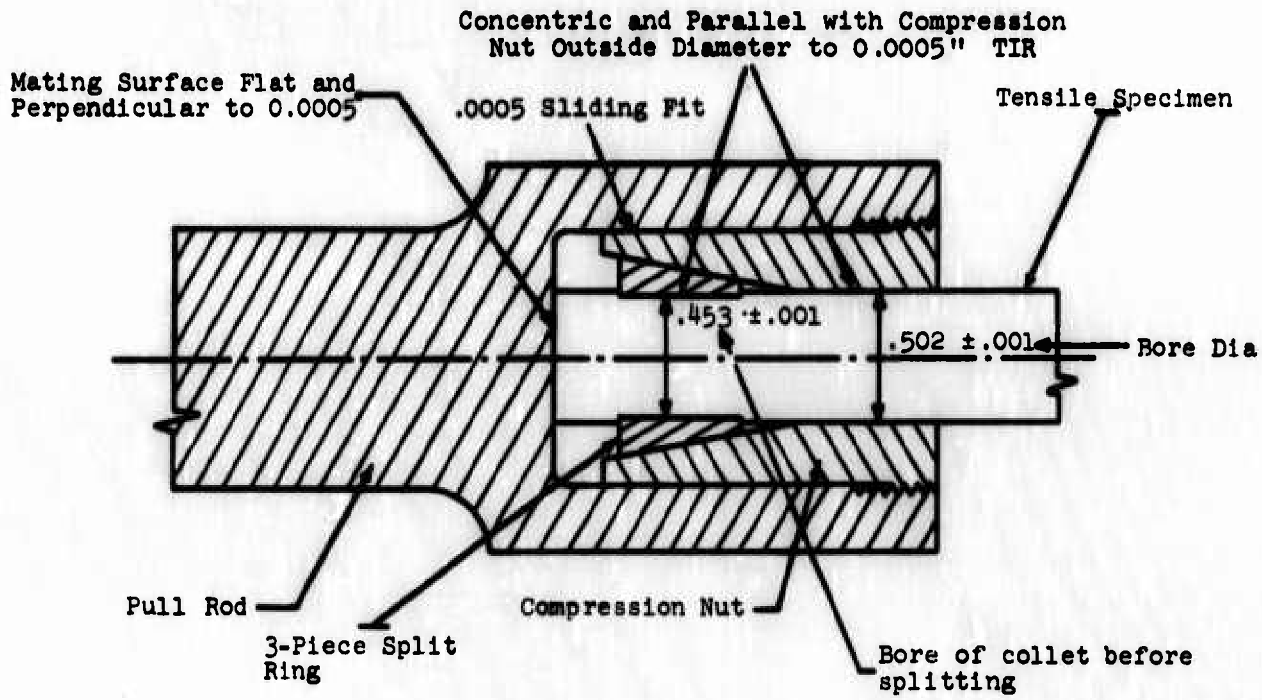
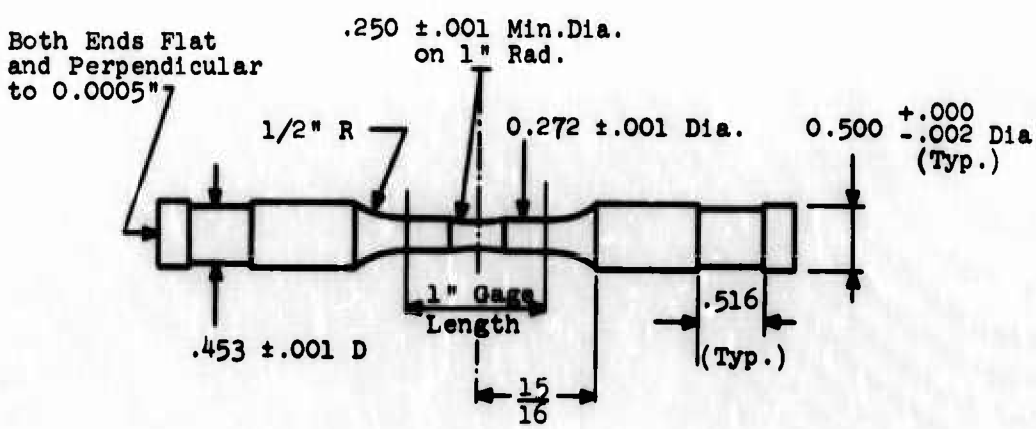
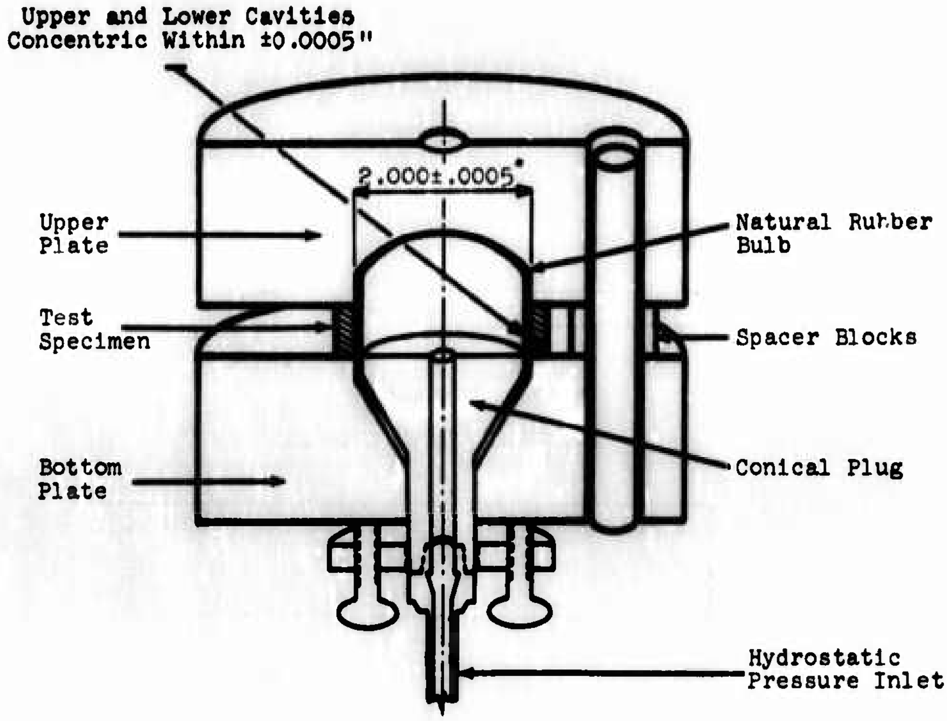


Fig.8.4 Precision collet grip for tensile specimens



NOTES: 1. All diameters true and concentric to 0.0005" T.I.R.
2. Surface finish is 50 RMS

Fig.8.5 Dimensions of tensile test specimen for gas bearing test



Spacer Block Dimensions

Size	Height
A	.252 +.001 -.000
B	.502 +.001 -.000
C	1.002 +.001 -.000

Fig.8.6 Pressurized ring test unit

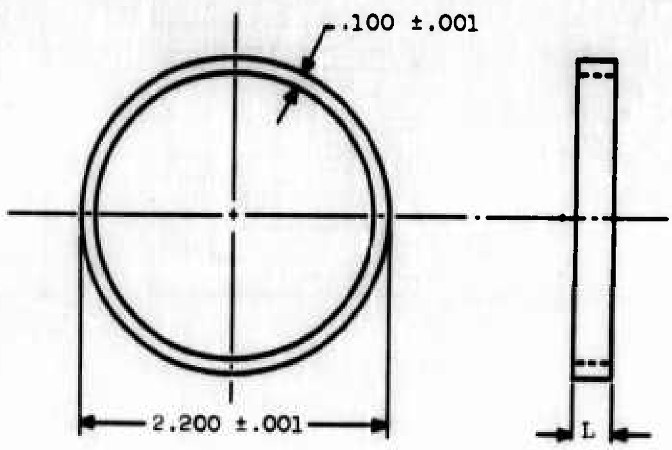


Table of Dimensions

Size	L
A	.250 ±.001
B	.50 ±.001
C	1.00 ±.001

Fig.8.7 Dimensions of test specimens for pressure ring test

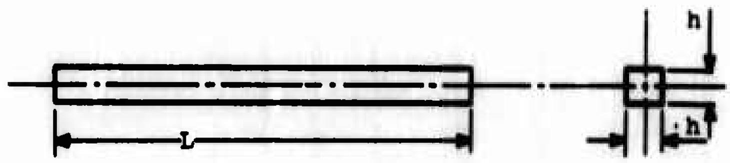


Table of Dimensions

	Size A	Size B
Length, L	3.500 ±.005	4.000 ±.005
Sides, h	0.25 ±.002	0.50 ±.002

NOTE: Finish is 50 RMS

Fig.8.8 Dimensions of test specimens for 3- and 4-point bending test

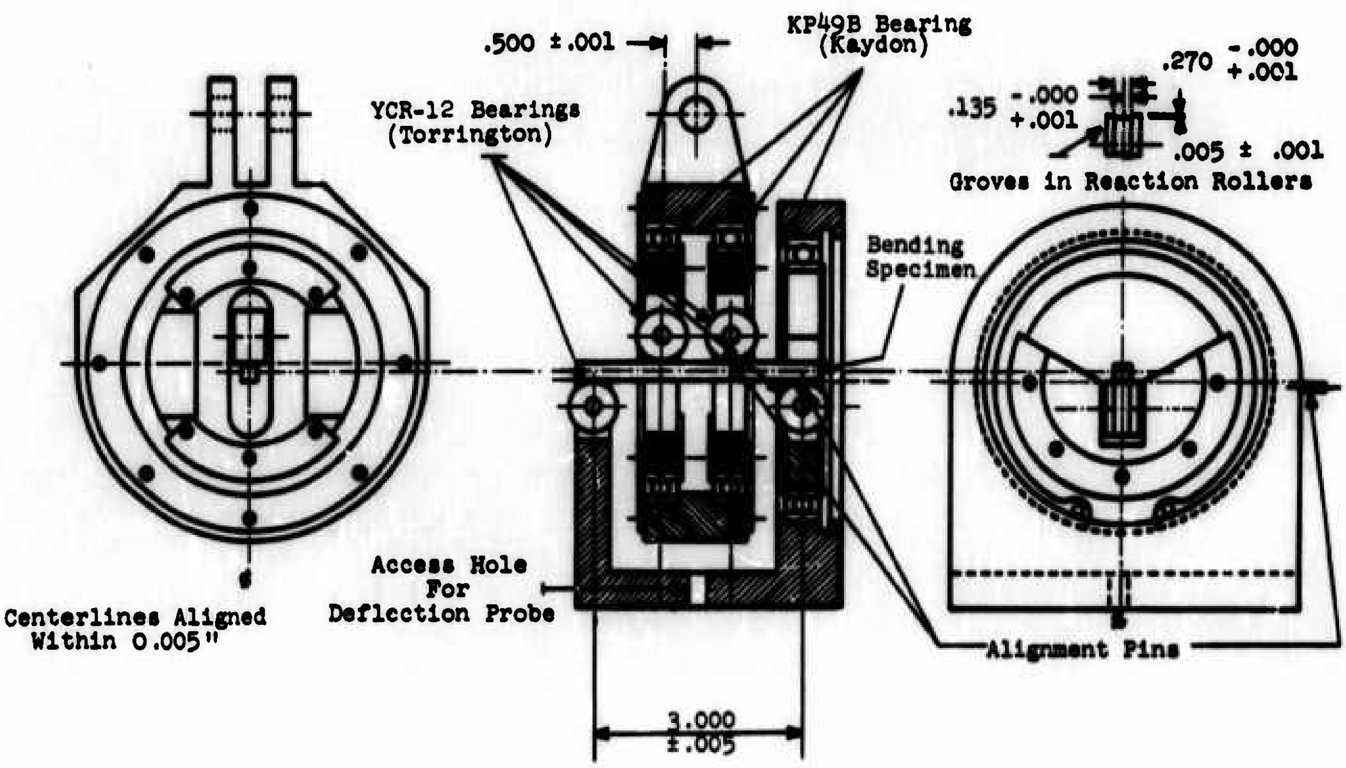


Fig.8.9 4-point bend test apparatus

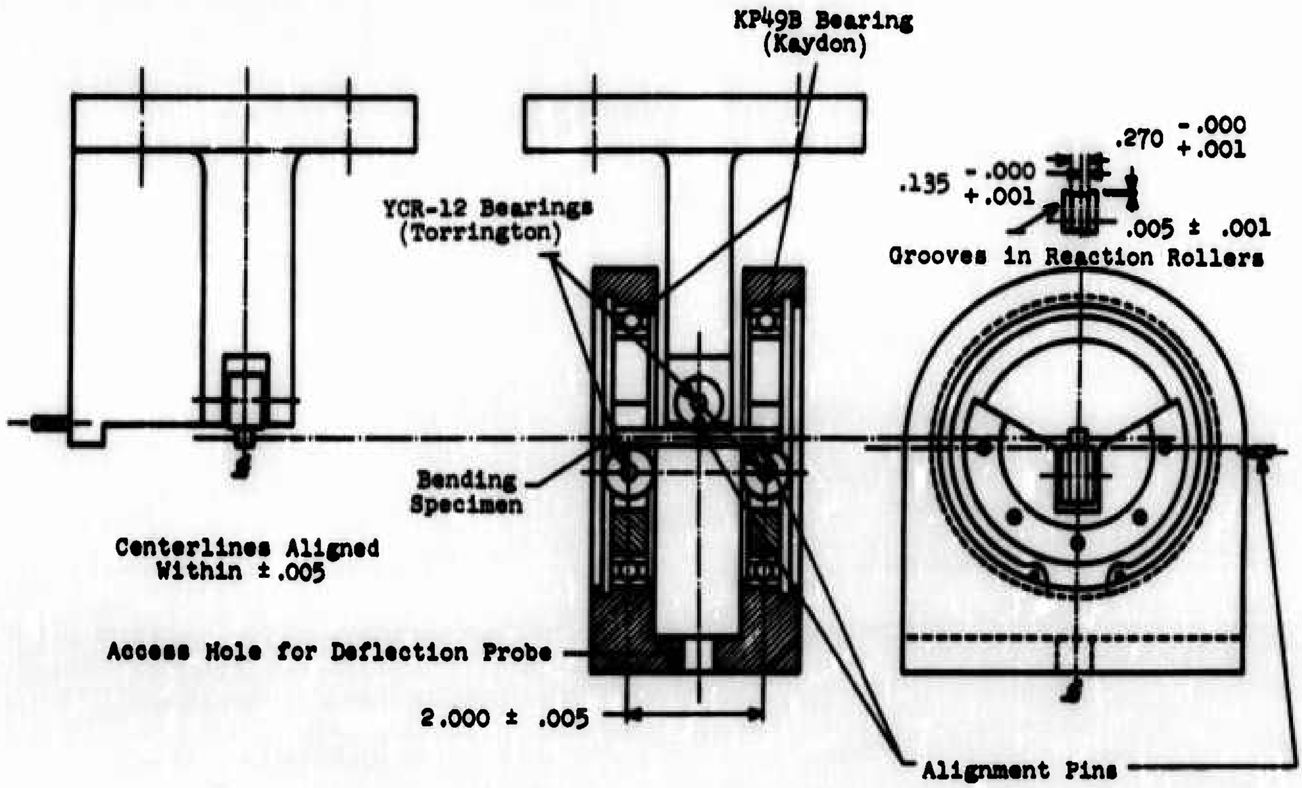


Fig.8.10 3-point bend test apparatus